Definition of First Order Language with Arbitrary Alphabet. Syntax of Terms, Atomic Formulas and their Subterms

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Summary. Second of a series of articles laying down the bases for classical first order model theory. A language is defined basically as a tuple made of an integer-valued function (adicity), a symbol of equality and a symbol for the NOR logical connective. The only requests for this tuple to be a language is that the value of the adicity in \(=\) is -2 and that its preimage (i.e. the variables set) in 0 is infinite. Existential quantification will be rendered (see [11]) by mere prefixing a formula with a letter. Then the hierarchy among symbols according to their adicity is introduced, taking advantage of attributes and clusters.

The strings of symbols of a language are depth-recursively classified as terms using the standard approach (see for example [16], definition 1.1.2); technically, this is done here by deploying the ‘multiCat’ functor and the ‘unambiguous’ attribute previously introduced in [10], and the set of atomic formulas is introduced. The set of all terms is shown to be unambiguous with respect to concatenation; we say that it is a prefix set. This fact is exploited to uniquely define the subterms both of a term and of an atomic formula without resorting to a parse tree.

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The papers [1], [3], [18], [5], [6], [12], [10], [7], [8], [9], [19], [14], [13], [2], [17], [4], [21], [22], [15], and [20] provide the terminology and notation for this paper.

We follow the rules: \(m, n\) are natural numbers, \(m_1, n_1\) are elements of \(\mathbb{N}\), and \(X, x, z\) are sets.

Let \(z\) be a zero integer number. One can check that \(|z|\) is zero.

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1The author wrote this paper as part of his PhD thesis research.
2I would like to thank Marco Pedicini for his encouragement and support.
Let us observe that there exists a real number which is negative and integer and every integer number which is positive is also natural.

Let \( S \) be a non degenerated zero-one structure. Observe that \((\text{the carrier of } S) \setminus \{\text{the one of } S\}\) is non empty.

We introduce languages-like which are extensions of zero-one structure and are systems
\[
\langle \text{a carrier, a zero, a one, an adicity} \rangle,
\]
where the carrier is a set, the zero and the one are elements of the carrier, and
the adicity is a function from the carrier \(\setminus\{\text{the one}\}\) into \(\mathbb{Z}\).

Let \( S \) be a language-like. The functor \( \text{AllSymbolsOf } S \) is defined by:

(Def. 1) \( \text{AllSymbolsOf } S = \text{the carrier of } S \).

The functor \( \text{LettersOf } S \) is defined as follows:

(Def. 2) \( \text{LettersOf } S = (\text{the adicity of } S)^{-1}(\{0\}) \).

The functor \( \text{OpSymbolsOf } S \) is defined by:

(Def. 3) \( \text{OpSymbolsOf } S = (\text{the adicity of } S)^{-1}(\mathbb{N} \setminus \{0\}) \).

The functor \( \text{RelSymbolsOf } S \) is defined by:

(Def. 4) \( \text{RelSymbolsOf } S = (\text{the adicity of } S)^{-1}(\mathbb{Z} \setminus \mathbb{N}) \).

The functor \( \text{TermSymbolsOf } S \) is defined as follows:

(Def. 5) \( \text{TermSymbolsOf } S = (\text{the adicity of } S)^{-1}(\mathbb{N}) \).

The functor \( \text{LowerCompoundersOf } S \) is defined as follows:

(Def. 6) \( \text{LowerCompoundersOf } S = (\text{the adicity of } S)^{-1}(\mathbb{Z} \setminus \{0\}) \).

The functor \( \text{TheEqSymbOf } S \) is defined as follows:

(Def. 7) \( \text{TheEqSymbOf } S = \text{the zero of } S \).

The functor \( \text{TheNorSymbOf } S \) is defined as follows:

(Def. 8) \( \text{TheNorSymbOf } S = \text{the one of } S \).

The functor \( \text{OwnSymbolsOf } S \) is defined by:

(Def. 9) \( \text{OwnSymbolsOf } S = (\text{the carrier of } S) \setminus \{\text{the zero of } S, \text{the one of } S\} \).

Let \( S \) be a language-like. An element of \( S \) is an element of \( \text{AllSymbolsOf } S \).

The functor \( \text{AtomicFormulaSymbolsOf } S \) is defined by:

(Def. 10) \( \text{AtomicFormulaSymbolsOf } S = \text{AllSymbolsOf } S \setminus \{\text{TheNorSymbOf } S\} \).

The functor \( \text{AtomicTermsOf } S \) is defined by:

(Def. 11) \( \text{AtomicTermsOf } S = (\text{LettersOf } S)^1 \).

We say that \( S \) is operational if and only if:

(Def. 12) \( \text{OpSymbolsOf } S \) is non empty.

We say that \( S \) is relational if and only if:

(Def. 13) \( \text{RelSymbolsOf } S \setminus \{\text{TheEqSymbOf } S\} \) is non empty.

Let \( S \) be a language-like and let \( s \) be an element of \( S \). We say that \( s \) is literal if and only if:
(Def. 14) \( s \in \text{LettersOf} \, S \).
We say that \( s \) is low-compounding if and only if:

(Def. 15) \( s \in \text{LowerCompoundersOf} \, S \).
We say that \( s \) is operational if and only if:

(Def. 16) \( s \in \text{OpSymbolsOf} \, S \).
We say that \( s \) is relational if and only if:

(Def. 17) \( s \in \text{RelSymbolsOf} \, S \).
We say that \( s \) is termal if and only if:

(Def. 18) \( s \in \text{TermSymbolsOf} \, S \).
We say that \( s \) is own if and only if:

(Def. 19) \( s \in \text{OwnSymbolsOf} \, S \).
We say that \( s \) is of-atomic-formula if and only if:

(Def. 20) \( s \in \text{AtomicFormulaSymbolsOf} \, S \).

Let \( S \) be a zero-one structure and let \( s \) be an element of (the carrier of \( S \)) \( \{ \text{the one of} \, S \} \). The functor \( \text{TrivialArity} \, s \) yields an integer number and is defined by:

(Def. 21) \( \text{TrivialArity} \, s = \begin{cases} -2, & \text{if } s = \text{the zero of} \, S, \\ 0, & \text{otherwise}. \end{cases} \)

Let \( S \) be a zero-one structure and let \( s \) be an element of (the carrier of \( S \)) \( \{ \text{the one of} \, S \} \). Then \( \text{TrivialArity} \, s \) is an element of \( \mathbb{Z} \).

Let \( S \) be a non degenerated zero-one structure. The functor \( S \, \text{TrivialArity} \) yielding a function from (the carrier of \( S \)) \( \{ \text{the one of} \, S \} \) into \( \mathbb{Z} \) is defined by:

(Def. 22) For every element \( s \) of (the carrier of \( S \)) \( \{ \text{the one of} \, S \} \) holds \((S \, \text{TrivialArity})(s) = \text{TrivialArity} \, s\).

Let us observe that there exists a non degenerated zero-one structure which is infinite.

Let \( S \) be an infinite non degenerated zero-one structure.
Observe that \((S \, \text{TrivialArity})^{-1}(\{0\})\) is infinite.

Let \( S \) be a language-like. We say that \( S \) is eligible if and only if:

(Def. 23) \( \text{LettersOf} \, S \) is infinite and \((\text{the adicity of} \, S)(\text{TheEqSymbOf} \, S) = -2\).

One can check that there exists a language-like which is non degenerated.
One can check that there exists a non degenerated language-like which is eligible.
A language is an eligible non degenerated language-like.
We follow the rules: \( S, S_1, S_2 \) are languages and \( s, s_1, s_2 \) are elements of \( S \).

Let \( S \) be a non empty language-like. Then \( \text{AllSymbolsOf} \, S \) is a non empty set.

Let \( S \) be an eligible language-like. Note that \( \text{LettersOf} \, S \) is infinite.
Let \( S \) be a language.
Then \( \text{LettersOf} \ S \) is a non-empty subset of \( \text{AllSymbolsOf} \ S \). Note that \( \text{TheEqSymbOf} \ S \) is relational.

Let \( S \) be a non-degenerated language-like. Then \( \text{AtomicFormulaSymbolsOf} \ S \) is a non-empty subset of \( \text{AllSymbolsOf} \ S \).

Let \( S \) be a non-degenerated language-like. Then \( \text{TheEqSymbOf} \ S \) is an element of \( \text{AtomicFormulaSymbolsOf} \ S \).

We now state the proposition

\[ (1) \] Let \( S \) be a language. Then \( \text{LettersOf} \ S \cap \text{OpSymbolsOf} \ S = \emptyset \) and \( \text{TermSymbolsOf} \ S \cap \text{LowerCompoundersOf} \ S = \text{OpSymbolsOf} \ S \) and \( \text{RelSymbolsOf} \ S \setminus \text{OwnSymbolsOf} \ S = \{ \text{TheEqSymbOf} \ S \} \) and \( \text{OwnSymbolsOf} \ S \subseteq \text{AtomicFormulaSymbolsOf} \ S \) and \( \text{RelSymbolsOf} \ S \subseteq \text{LowerCompoundersOf} \ S \) and \( \text{OpSymbolsOf} \ S \subseteq \text{TermSymbolsOf} \ S \) and \( \text{LettersOf} \ S \subseteq \text{TermSymbolsOf} \ S \setminus \text{OwnSymbolsOf} \ S \) and \( \text{OpSymbolsOf} \ S \subseteq \text{LowerCompoundersOf} \ S \subseteq \text{AtomicFormulaSymbolsOf} \ S \).

Let \( S \) be a language. One can verify the following observations:

\* Every element of \( S \) which is own is also of-atomic-formula,
\* Every element of \( S \) which is relational is also low-compounding,
\* Every element of \( S \) which is operational is also termal,
\* Every element of \( S \) which is literal is also termal,
\* Every element of \( S \) which is termal is also own,
\* Every element of \( S \) which is operational is also low-compounding,
\* Every element of \( S \) which is low-compounding is also of-atomic-formula,
\* Every element of \( S \) which is termal is also non relational,
\* Every element of \( S \) which is literal is also non relational, and
\* Every element of \( S \) which is literal is also non operational.

Let \( S \) be a language. Note that there exists an element of \( S \) which is relational and there exists an element of \( S \) which is literal. Observe that every low-compounding element of \( S \) which is termal is also operational. One can check that there exists an element of \( S \) which is of-atomic-formula.

Let \( s \) be an of-atomic-formula element of \( S \). The functor \( \text{ar} \ s \) yielding an element of \( \mathbb{Z} \) is defined by:

\[ (\text{Def. 24}) \quad \text{ar} \ s = (\text{the adicity of} \ S)(s). \]

Let \( S \) be a language and let \( s \) be a literal element of \( S \). Note that \( \text{ar} \ s \) is zero. The functor \( S\text{-cons} \) yielding a binary operation on \( (\text{AllSymbolsOf} \ S)^* \) is defined as follows:

\[ (\text{Def. 25}) \quad S\text{-cons} = \text{the concatenation of} \ \text{AllSymbolsOf} \ S. \]

Let \( S \) be a language.
The functor $S$-multiCat yields a function from $((\text{AllSymbolsOf } S)^*)^*$ into $(\text{AllSymbolsOf } S)^*$ and is defined by:

(Def. 26) \[ S\text{-multiCat} = (\text{AllSymbolsOf } S)\text{-multiCat}. \]

Let $S$ be a language. The functor $S$-firstChar yielding a function from $(\text{AllSymbolsOf } S)^* \setminus \{\emptyset\}$ into $\text{AllSymbolsOf } S$ is defined as follows:

(Def. 27) \[ S\text{-firstChar} = (\text{AllSymbolsOf } S)\text{-firstChar}. \]

Let $S$ be a language and let $X$ be a set. We say that $X$ is $S$-prefix if and only if:

(Def. 28) \[ X \text{ is AllSymbolsOf } S\text{-prefix}. \]

Let $S$ be a language and let $X$ be a set. We say that $X$ is $S$-prefix if and only if:

(Def. 29) \[ \text{Compound}(s, S_3) = \{\langle s \rangle \triangleright S\text{-multiCat}(S_4); S_4 \text{ ranges over } \text{elements of } ((\text{AllSymbolsOf } S)^*)^*; \text{rng } S_4 \subseteq S_3 \land S_4 \text{ is } \{\text{ar } s\}\text{-element}\}. \]

Let $S$ be a language, let $s$ be an of-atomic-formula element of $S$, and let $S_3$ be a set. The functor $S$-termsOfMaxDepth yields a function and is defined by the conditions (Def. 30).

(Def. 30)(i) \[ \text{dom}(S\text{-termsOfMaxDepth}) = \mathbb{N}, \]

(ii) \[ S\text{-termsOfMaxDepth}(0) = \text{AtomicTermsOf } S, \]

(iii) for every natural number $n$ holds $S\text{-termsOfMaxDepth}(n + 1) = \bigcup\{\text{Compound}(s, S\text{-termsOfMaxDepth}(n)); s \text{ ranges over of-atomic-formula elements of } S; s \text{ is operational}\} \cup S\text{-termsOfMaxDepth}(n)$.

Let us consider $S$. Then $\text{AtomicTermsOf } S$ is a subset of $(\text{AllSymbolsOf } S)^*$. Let $S$ be a language. The functor $\text{AllTermsOf } S$ is defined as follows:

(Def. 31) \[ \text{AllTermsOf } S = \bigcup \text{rng}(S\text{-termsOfMaxDepth}). \]

One can prove the following proposition

(2) \[ S\text{-termsOfMaxDepth}(m_1) \subseteq \text{AllTermsOf } S. \]

Let $S$ be a language and let $w$ be a string of $S$. We say that $w$ is termal if and only if:

(Def. 32) \[ w \in \text{AllTermsOf } S. \]

Let $m$ be a natural number, let $S$ be a language, and let $w$ be a string of $S$. We say that $w$ is $m$-termal if and only if:
(Def. 33) \( w \in S\text{-termsOfMaxDepth}(m) \).

Let \( m \) be a natural number and let \( S \) be a language. Note that every string of \( S \) which is \( m \)-termal is also termal.

Let us consider \( S \). Then \( S\text{-termsOfMaxDepth} \) is a function from \( \mathbb{N} \) into \( 2^{(AllSymbolsOfS)^*} \). Then \( AllTermsOfS \) is a non empty subset of \( (AllSymbolsOfS)^* \). Note that \( AllTermsOfS \) is non empty.

Let us consider \( m \). One can verify that \( S\text{-termsOfMaxDepth}(m) \) is non empty. Observe that every element of \( S\text{-termsOfMaxDepth}(m) \) is non empty. Observe that every element of \( AllTermsOfS \) is non empty.

Let \( m \) be a natural number and let \( S \) be a language. Note that there exists a string of \( S \) which is \( m \)-termal. Observe that every string of \( S \) which is 0-termal is also 1-element.

Let \( S \) be a language and let \( w \) be a 0-termal string of \( S \). Observe that \( S\text{-firstChar}(w) \) is literal.

Let us consider \( S \) and let \( w \) be a termal string of \( S \). Observe that \( S\text{-firstChar}(w) \) is termal.

Let us consider \( S \) and let \( t \) be a termal string of \( S \). The functor \( ar_t \) yielding an element of \( Z \) is defined as follows:

(Def. 34) \( ar_t = ar_S\text{-firstChar}(t) \).

Next we state the proposition

(3) For every \( m_1 + 1 \)-termal string \( w \) of \( S \) there exists an element \( T \) of \( S\text{-termsOfMaxDepth}(m_1)^* \) such that \( T \) is \( |ar_S\text{-firstChar}(w)| \)-element and \( w = \langle S\text{-firstChar}(w) \rangle \triangleleft S\text{-multiCat}(T) \).

Let us consider \( S, m \). Note that \( S\text{-termsOfMaxDepth}(m) \) is \( S \)-prefix.

Let us consider \( S \) and let \( V \) be an element of \( (AllTermsOfS)^* \). Observe that \( S\text{-multiCat}(V) \) is relation-like.

Let us consider \( S \) and let \( V \) be an element of \( (AllTermsOfS)^* \). One can verify that \( S\text{-multiCat}(V) \) is function-like.

Let us consider \( S \) and let \( p_1 \) be a string of \( S \). We say that \( p_1 \) is 0-w.f.f. if and only if:

(Def. 35) There exists a relational element \( s \) of \( S \) and there exists an \( |ar s| \)-element element \( V \) of \( (AllTermsOfS)^* \) such that \( p_1 = \langle s \rangle \triangleleft S\text{-multiCat}(V) \).

Let us consider \( S \). Note that there exists a string of \( S \) which is 0-w.f.f.. Let \( p_1 \) be a 0-w.f.f. string of \( S \). Observe that \( S\text{-firstChar}(p_1) \) is relational.

The functor \( AtomicFormulasOfS \) is defined as follows:

(Def. 36) \( AtomicFormulasOfS = \{ p_1; p_1 \text{ ranges over strings of } S; p_1 \text{ is 0-w.f.f.} \} \).

Let us consider \( S \). Then \( AtomicFormulasOfS \) is a subset of \( (AllSymbolsOfS)^* \setminus \{ \emptyset \} \). Note that \( AtomicFormulasOfS \) is non empty. Observe that every element of \( AtomicFormulasOfS \) is 0-w.f.f.. Observe that \( AllTermsOfS \) is \( S \)-prefix.
Let us consider $S$ and let $t$ be a termal string of $S$. The functor $\text{SubTerms}_t$ yields an element of $(\text{AllTermsOf } S)^*$ and is defined by:

(Def. 37) $\text{SubTerms}_t$ is $| \text{ar } S\text{-firstChar}(t) |$-element and $t = \langle S\text{-firstChar}(t) \rangle \bowtie S\text{-multiCat}(\text{SubTerms}_t)$.

Let us consider $S$ and let $t$ be a termal string of $S$. One can verify that $\text{SubTerms}_t$ is $| \text{ar } t |$-element.

Let $t_0$ be a 0-termal string of $S$. Note that $\text{SubTerms}_t_0$ is empty.

Let us consider $m_1$, $S$ and let $t$ be an $m_1 + 1$-termal string of $S$. One can verify that $\text{SubTerms}_t$ is $S\text{-termsOfMaxDepth}(m_1)$-valued.

Let us consider $S$ and let $p_1$ be a 0-w.f.f. string of $S$. The functor $\text{SubTerms}_{p_1}$ yields an $| \text{ar } S\text{-firstChar}(p_1) |$-element element of $(\text{AllTermsOf } S)^*$ and is defined as follows:

(Def. 38) $p_1 = \langle S\text{-firstChar}(p_1) \rangle \bowtie S\text{-multiCat}(\text{SubTerms}_{p_1})$.

Let us consider $S$ and let $p_1$ be a 0-w.f.f. string of $S$. Note that $\text{SubTerms}_{p_1}$ is $| \text{ar } S\text{-firstChar}(p_1) |$-element.

Then $\text{AllTermsOf } S$ is an element of $2^{(\text{AllSymbolsOf } S)^* \setminus \{\emptyset\}}$. Note that every element of $\text{AllTermsOf } S$ is termal. The functor $S\text{-subTerms}$ yielding a function from $\text{AllTermsOf } S$ into $(\text{AllTermsOf } S)^*$ is defined by:

(Def. 39) For every element $t$ of $\text{AllTermsOf } S$ holds $S\text{-subTerms}(t) = \text{SubTerms}_t$.

We now state several propositions:

(4) $S\text{-termsOfMaxDepth}(m) \subseteq S\text{-termsOfMaxDepth}(m + n)$.

(5) If $x \in \text{AllTermsOf } S$, then there exists $n_1$ such that $x \in S\text{-termsOfMaxDepth}(n_1)$.

(6) $\text{AllTermsOf } S \subseteq (\text{AllSymbolsOf } S)^* \setminus \{\emptyset\}$.

(7) $\text{AllTermsOf } S$ is $S$-prefix.

(8) If $x \in \text{AllTermsOf } S$, then $x$ is a string of $S$.

(9) $\text{AtomicFormulaSymbolsOf } S \setminus \text{OwnSymbolsOf } S = \{\text{TheEqSymbOf } S\}$.

(10) $\text{TermSymbolsOf } S \setminus \text{LettersOf } S = \text{OpSymbolsOf } S$.

(11) $\text{AtomicFormulaSymbolsOf } S \setminus \text{RelSymbolsOf } S = \text{TermSymbolsOf } S$.

Let us consider $S$. Observe that every of-atomic-formula element of $S$ which is non relational is also termal.

Then $\text{ OwnSymbolsOf } S$ is a subset of $\text{ AllSymbolsOf } S$. Observe that every termal element of $S$ which is non literal is also operational.

Next we state three propositions:

(12) $x$ is a string of $S$ if and only if $x$ is a non empty element of $(\text{AllSymbolsOf } S)^*$.

(13) $x$ is a string of $S$ if and only if $x$ is a non empty finite sequence of elements of $\text{AllSymbolsOf } S$.

(14) $S\text{-termsOfMaxDepth}$ is a function from $\mathbb{N}$ into $2^{(\text{AllSymbolsOf } S)^*}$.
Let us consider $S$. Note that every element of LettersOf $S$ is literal. One can check that TheNorSymbOf $S$ is non low-compounding.

Observe that TheNorSymbOf $S$ is non own.

Next we state the proposition

(15) If $s \neq \text{TheNorSymbOf } S$ and $s \neq \text{TheEqSymbOf } S$, then $s \in \text{OwnSymbolsOf } S$.

For simplicity, we use the following convention: $l, l_1, l_2$ denote literal elements of $S$, $a$ denotes an of-atomic-formula element of $S$, $r$ denotes a relational element of $S$, $w, w_1$ denote strings of $S$, and $t_2$ denotes an element of AllTermsOf $S$.

Let us consider $S, t$. The functor Depth $t$ yielding a natural number is defined by:

(Def. 40) $t$ is Depth $t$-termal and for every $n$ such that $t$ is $n$-termal holds Depth $t \leq n$.

Let us consider $S$, let $m_0$ be a zero number, and let $t$ be an $m_0$-termal string of $S$. Note that Depth $t$ is zero.

Let us consider $S$ and let $s$ be a low-compounding element of $S$. Note that ar $s$ is non zero.

Let us consider $S$ and let $s$ be a termal element of $S$. Observe that ar $s$ is non negative and extended real.

Let us consider $S$ and let $s$ be a relational element of $S$. Note that ar $s$ is negative and extended real.

Next we state the proposition

(16) If $t$ is non 0-termal, then S-firstChar($t$) is operational and SubTerms $t \neq \emptyset$.

Let us consider $S$. Observe that $S$-multiCat is finite sequence-yielding.

Let us consider $S$ and let $W$ be a non empty AllSymbolsOf $S^* \setminus \{\emptyset\}$-valued finite sequence. One can verify that $S$-multiCat($W$) is non empty.

Let us consider $S, l$. Note that $\langle l \rangle$ is 0-termal.

Let us consider $S, m, n$. One can check that every string of $S$ which is $m + 0 \cdot n$-termal is also $m + n$-termal.

Let us consider $S$. One can check that every own element of $S$ which is non low-compounding is also literal.

Let us consider $S, t$. One can check that SubTerms $t$ is rng $t^*$-valued.

Let $p_0$ be a 0-w.f.f. string of $S$. Observe that SubTerms $p_0$ is rng $p_0^*$-valued. Then $S$-termsOfMaxDepth is a function from $N$ into $2^{(\text{AllSymbolsOf } S^* \setminus \{\emptyset\}$.

Let us consider $S, m_1$. Observe that $S$-termsOfMaxDepth($m_1$) has non empty elements.

Let us consider $S, m$ and let $t$ be a termal string of $S$. One can verify that $t \text{null } m$ is Depth $t + m$-termal. One can check that every string of $S$ which is termal is also TermSymbolsOf $S$-valued. Observe that AllTermsOf $S \setminus (\text{TermSymbolsOf } S)^*$ is empty.
Let $p_0$ be a 0-w.f.f. string of $S$. Observe that $\SubTerms p_0$ is $\TermSymbolsOf S^e$-valued. One can verify that every string of $S$ which is 0-w.f.f. is also $\AtomicFormulaSymbolsOf S$-valued. One can check that $\OwnSymbolsOf S$ is non empty.

In the sequel $p_0$ is a 0-w.f.f. string of $S$.

The following proposition is true
(17) If $S$-firstChar($p_0$) $\neq$ TheEqSymbOf $S$, then $p_0$ is $\OwnSymbolsOf S$-valued.

Let us observe that there exists a language-like which is strict and non empty.
Let $S_1$, $S_2$ be languages-like. We say that $S_2$ is $S_1$-extending if and only if:
(Def. 41) The adicity of $S_1$ $\subseteq$ the adicity of $S_2$ and TheEqSymbOf $S_1 = \TheEqSymbOf S_2$ and TheNorSymbOf $S_1 = \TheNorSymbOf S_2$.

Let us consider $S$. One can verify that $S$ null is $S$-extending. Observe that there exists a language which is $S$-extending.
Let us consider $S_1$ and let $S_2$ be an $S_1$-extending language. Observe that $\OwnSymbolsOf S_1 \setminus \OwnSymbolsOf S_2$ is empty.

Let $f$ be a $Z$-valued function and let $L$ be a non empty language-like. The functor $L$ extendWith $f$ yields a strict non empty language-like and is defined by the conditions (Def. 42).
(Def. 42)(i) The adicity of $L$ extendWith $f = f\|(\dom f \setminus \{\text{the one of } L\}) + \cdot$ the adicity of $L$,
(ii) the zero of $L$ extendWith $f$ = the zero of $L$, and
(iii) the one of $L$ extendWith $f$ = the one of $L$.

Let $S$ be a non empty language-like and let $f$ be a $Z$-valued function. Note that $S$ extendWith $f$ is $S$-extending.
Let $S$ be a non degenerated language-like. Observe that every language-like which is $S$-extending is also non degenerated.
Let $S$ be an eligible language-like. One can check that every language-like which is $S$-extending is also eligible.
Let $E$ be an empty binary relation and let us consider $X$. Note that $X|E$ is empty.
Let us consider $X$ and let $m$ be an integer number. Note that $X \mapsto m$ is $Z$-valued.
Let us consider $S$ and let $X$ be a functional set.
The functor $S$ addLettersNotIn $X$ yields an $S$-extending language and is defined as follows:
(Def. 43) $S$ addLettersNotIn $X$ =
$S$ extendWith($\{(\AllSymbolsOf S \cup \SymbolsOf X)$-freeCountableSet $\mapsto 0$ qua $Z$-valued function).
Let us consider $S_1$ and let $X$ be a functional set.
Note that $\text{LettersOf}(S_1 \text{addLettersNotIn} X) \setminus \text{SymbolsOf} X$ is infinite.
Let us note that there exists a language which is countable.
Let $S$ be a countable language. Observe that $\text{AllSymbolsOf} S$ is countable.
One can verify that $\langle \text{AllSymbolsOf} S \rangle^* \setminus \{\emptyset\}$ is countable.
Let $L$ be a non empty language-like and let $f$ be a $\mathbb{Z}$-valued function. Note that $\text{AllSymbolsOf}(L \text{extendWith} f) \setminus (\text{dom} f \cup \text{AllSymbolsOf} L)$ is empty.
Let $S$ be a countable language and let $X$ be a functional set. One can check that $S \text{addLettersNotIn} X$ is countable.

REFERENCES


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