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Insert Sort on $\mathbf{SCM}_{\mathrm{FSA}}$[1]

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Summary. This article describes the insert sorting algorithm using macro instructions such as if-Macro (conditional branch macro instructions), for-loop macro instructions and While-Macro instructions etc. From the viewpoint of initialization, we generalize the halting and computing problem of the While-Macro. Generally speaking, it is difficult to judge whether the While-Macro is halting or not by way of loop inspection. For this reason, we introduce a practical and simple method, called body-inspection. That is, in many cases, we can prove the halting problem of the While-Macro by only verifying the nature of the body of the While-Macro, rather than the While-Macro itself. In fact, we have used this method in justifying the halting of the insert sorting algorithm. Finally, we prove that the insert sorting algorithm given in the article is autonomic and its computing result is correct.

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The articles [28], [39], [20], [8], [13], [40], [14], [38], [15], [16], [12], [7], [10], [9], [23], [30], [11], [26], [34], [31], [32], [33], [25], [5], [6], [3], [1], [17], [2], [35], [37], [18], [27], [29], [24], [4], [22], [19], [21], and [36] provide the terminology and notation for this paper.

1. Preliminaries

Let i be a good instruction of $\mathbf{SCM}_{\mathrm{FSA}}$. Observe that Macro($i$) is good.

Let a be a read-write integer location and let b be an integer location. Note that AddTo(a, b) is good.

We now state several propositions:

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(1) For every function f and for all sets d, r such that $d \in \operatorname{dom} f$ holds $\operatorname{dom} f = \operatorname{dom}(f+\cdot(d \mapsto r))$.

(2) Let p be a programmed finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$, l be an instruction-location of $\mathbf{SCM}_{\mathrm{FSA}}$, and i_1 be an instruction of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $l \in \operatorname{dom} p$ and there exists an instruction p_1 of $\mathbf{SCM}_{\mathrm{FSA}}$ such that $p_1 = p(l)$ and $\operatorname{UsedIntLoc}(p_1) = \operatorname{UsedIntLoc}(i_1)$. Then $\operatorname{UsedIntLoc}(p) = \operatorname{UsedIntLoc}(p+\cdot(l \mapsto i_1))$.

(3) For every integer location a and for every macro instruction I holds (**if** $a > 0$ **then** I; $\operatorname{Goto}(\operatorname{insloc}(0))$ **else** $(\operatorname{Stop}_{\mathrm{SCM}_{\mathrm{FSA}}})$)$(\operatorname{insloc}(\operatorname{card} I + 4)) = \operatorname{goto} \operatorname{insloc}(\operatorname{card} I + 4)$.

(4) Let p be a programmed finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$, l be an instruction-location of $\mathbf{SCM}_{\mathrm{FSA}}$, and i_1 be an instruction of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $l \in \operatorname{dom} p$ and there exists an instruction p_1 of $\mathbf{SCM}_{\mathrm{FSA}}$ such that $p_1 = p(l)$ and $\operatorname{UsedInt}^* \operatorname{Loc}(p_1) = \operatorname{UsedInt}^* \operatorname{Loc}(i_1)$. Then $\operatorname{UsedInt}^* \operatorname{Loc}(p) = \operatorname{UsedInt}^* \operatorname{Loc}(p+\cdot(l \mapsto i_1))$.

(5) For every natural number k holds $k + 1 > 0$.

For simplicity, we adopt the following convention: s is a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I is a macro instruction, a is a read-write integer location, and j, k, n are natural numbers.

Next we state a number of propositions:

(6) For every state s of $\mathbf{SCM}_{\mathrm{FSA}}$ and for every macro instruction I such that $s(\operatorname{intloc}(0)) = 1$ and $\mathbf{IC}_s = \operatorname{insloc}(0)$ holds $s+\cdot I = s+\cdot \operatorname{Initialized}(I)$.

(7) Let I be a macro instruction and a, b be integer locations. If I does not destroy b, then **while** $a > 0$ **do** I does not destroy b.

(8) If $n \leqslant 11$, then $n = 0$ or $n = 1$ or $n = 2$ or $n = 3$ or $n = 4$ or $n = 5$ or $n = 6$ or $n = 7$ or $n = 8$ or $n = 9$ or $n = 10$ or $n = 11$.

(9) Let f, g be finite sequences of elements of $\mathbb{Z}$ and m, n be natural numbers. Suppose $1 \leqslant n$ and $n \leqslant \operatorname{len} f$ and $1 \leqslant m$ and $m \leqslant \operatorname{len} f$ and $g = f +\cdot (m, \pi_n f) +\cdot (n, \pi_m f)$. Then

(i) $f(m) = g(n)$,

(ii) $f(n) = g(m)$,

(iii) for every set k such that $k \neq m$ and $k \neq n$ and $k \in \operatorname{dom} f$ holds $f(k) = g(k)$, and

(iv) f and g are fiberwise equipotent.

(10) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$ and I be a macro instruction. Suppose I is halting on $\operatorname{Initialize}(s)$. Let a be an integer location. Then $(\operatorname{IExec}(I, s))(a) = (\operatorname{Computation}(\operatorname{Initialize}(s)+\cdot(I+\cdot \operatorname{Start-At}(\operatorname{insloc}(0)))))$ $(\operatorname{LifeSpan}(\operatorname{Initialize}(s)+\cdot(I+\cdot \operatorname{Start-At}(\operatorname{insloc}(0)))))(a)$.

(11) Let s_1, s_2 be states of $\mathbf{SCM}_{\mathrm{FSA}}$ and I be a InitHalting macro instruction. Suppose $\operatorname{Initialized}(I) \subseteq s_1$ and $\operatorname{Initialized}(I) \subseteq$

s_2 and s_1 and s_2 are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$. Let k be a natural number. Then (Computation(s_1))(k) and (Computation(s_2))(k) are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$ and CurInstr((Computation(s_1))(k)) = CurInstr((Computation (s_2))(k)).

(12) Let s_1, s_2 be states of $\mathbf{SCM}_{\mathrm{FSA}}$ and I be a InitHalting macro instruction. Suppose Initialized(I) $\subseteq$ s_1 and Initialized(I) $\subseteq$ s_2 and s_1 and s_2 are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$. Then LifeSpan($s_1$) = LifeSpan($s_2$) and Result($s_1$) and Result($s_2$) are equal outside the instruction locations of $\mathbf{SCM}_{\mathrm{FSA}}$.

(13) For every macro instruction I and for every finite sequence location f holds $f \notin \operatorname{dom} I$.

(14) For every macro instruction I and for every integer location a holds $a \notin \operatorname{dom} I$.

(15) Let N be a non empty set with non empty elements, S be a halting von Neumann definite AMI over N, and s be a state of S. If LifeSpan(s) $\leqslant$ j and s is halting, then (Computation(s))(j) = (Computation(s))(LifeSpan(s)).

2. Basic Property of **while** Macro

We now state several propositions:

(16) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I be a macro instruction, and a be a read-write integer location. Suppose $s(a) \leqslant 0$. Then **while** $a > 0$ **do** I is halting onInit s and **while** $a > 0$ **do** I is closed onInit s.

(17) Let a be an integer location, I be a macro instruction, s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, and k be a natural number. Suppose that

(i) I is closed onInit s,

(ii) I is halting onInit s,

(iii) $k <$ LifeSpan(s+$\cdot$ Initialized(I)),

(iv) $\mathbf{IC}_{(\text{Computation}(s+\cdot \text{Initialized}(\textbf{while } a>0 \textbf{ do } I)))(1+k)} = \mathbf{IC}_{(\text{Computation}(s+\cdot \text{Initialized}(I)))(k)} + 4$, and

(v) (Computation(s+$\cdot$ Initialized(**while** $a > 0$ **do** I)))$(1 + k)\restriction D$ = (Computation(s+$\cdot$ Initialized(I)))(k)$\restriction D$.

Then $\mathbf{IC}_{(\text{Computation}(s+\cdot \text{Initialized}(\textbf{while } a>0 \textbf{ do } I)))(1+k+1)} = \mathbf{IC}_{(\text{Computation}(s+\cdot \text{Initialized}(I)))(k+1)} + 4$ and (Computation(s+$\cdot$ Initialized (**while** $a > 0$ **do** I)))$(1+k+1)\restriction D$ = (Computation(s+$\cdot$ Initialized(I)))(k+ 1)$\restriction D$, where D = Int-Locations $\cup$ FinSeq-Locations.

(18) Let a be an integer location, I be a macro instruction, and s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose I is closed onInit s and I is halting onInit s and $\mathbf{IC}_{(\mathrm{Computation}(s+\cdot \mathrm{Initialized}(\textbf{while } a>0 \textbf{ do } I)))(1+\mathrm{LifeSpan}(s+\cdot \mathrm{Initialized}(I)))} = \mathbf{IC}_{(\mathrm{Computation}(s+\cdot \mathrm{Initialized}(I)))(\mathrm{LifeSpan}(s+\cdot \mathrm{Initialized}(I)))} + 4$.
Then $\mathrm{CurInstr}((\mathrm{Computation}(s+\cdot \mathrm{Initialized}(\textbf{while } a > 0 \textbf{ do } I)))(1 + \mathrm{LifeSpan}(s+\cdot \mathrm{Initialized}(I)))) = \mathrm{goto}\ \mathrm{insloc}(\mathrm{card}\, I + 4)$.

(19) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I be a macro instruction, and a be a read-write integer location. Suppose I is closed onInit s and I is halting onInit s and $s(a) > 0$. Then $\mathbf{IC}_{(\mathrm{Computation}(s+\cdot \mathrm{Initialized}(\textbf{while } a>0 \textbf{ do } I)))(\mathrm{LifeSpan}(s+\cdot \mathrm{Initialized}(I))+3)} = \mathrm{insloc}(0)$ and for every natural number k such that $k \leqslant \mathrm{LifeSpan}(s+\cdot \mathrm{Initialized}(I)) + 3$ holds $\mathbf{IC}_{(\mathrm{Computation}(s+\cdot \mathrm{Initialized}(\textbf{while } a>0 \textbf{ do } I)))(k)} \in \mathrm{dom}(\textbf{while } a > 0 \textbf{ do } I)$.

(20) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I be a macro instruction, and a be a read-write integer location. Suppose I is closed onInit s and I is halting onInit s and $s(a) > 0$. Let k be a natural number. If $k \leqslant \mathrm{LifeSpan}(s+\cdot \mathrm{Initialized}(I)) + 3$, then $\mathbf{IC}_{(\mathrm{Computation}(s+\cdot \mathrm{Initialized}(\textbf{while } a>0 \textbf{ do } I)))(k)} \in \mathrm{dom}(\textbf{while } a > 0 \textbf{ do } I)$.

(21) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I be a macro instruction, and a be a read-write integer location. Suppose I is closed onInit s and I is halting onInit s and $s(a) > 0$. Then $\mathbf{IC}_{(\mathrm{Computation}(s+\cdot \mathrm{Initialized}(\textbf{while } a>0 \textbf{ do } I)))(\mathrm{LifeSpan}(s+\cdot \mathrm{Initialized}(I))+3)} = \mathrm{insloc}(0)$ and $(\mathrm{Computation}(s+\cdot \mathrm{Initialized}(\textbf{while } a > 0 \textbf{ do } I)))(\mathrm{LifeSpan}(s+\cdot \mathrm{Initialized}(I)) + 3) \restriction D = (\mathrm{Computation}(s+\cdot \mathrm{Initialized}(I)))(\mathrm{LifeSpan}(s+\cdot \mathrm{Initialized}(I))) \restriction D$, where $D = \mathrm{Int\text{-}Locations} \cup \mathrm{FinSeq\text{-}Locations}$.

(22) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I be a InitHalting macro instruction, and a be a read-write integer location. Suppose $s(a) > 0$. Then there exists a state s_2 of $\mathbf{SCM}_{\mathrm{FSA}}$ and there exists a natural number k such that

(i) $s_2 = s+\cdot \mathrm{Initialized}(\textbf{while } a > 0 \textbf{ do } I)$,

(ii) $k = \mathrm{LifeSpan}(s+\cdot \mathrm{Initialized}(I)) + 3$,

(iii) $\mathbf{IC}_{(\mathrm{Computation}(s_2))(k)} = \mathrm{insloc}(0)$,

(iv) for every integer location b holds $(\mathrm{Computation}(s_2))(k)(b) = (\mathrm{IExec}(I, s))(b)$, and

(v) for every finite sequence location f holds $(\mathrm{Computation}(s_2))(k)(f) = (\mathrm{IExec}(I, s))(f)$.

Let us consider s, I, a. The functor *StepWhile>0*(a, s, I) yields a function from $\mathbb{N}$ into $\prod$(the object kind of $\mathbf{SCM}_{\mathrm{FSA}}$) and is defined by the conditions (Def. 1).

(Def. 1)(i) (*StepWhile>0*$(a, s, I))(0) = s$ **qua** element of $\prod$(the object kind of $\mathbf{SCM}_{\mathrm{FSA}}$) **qua** non empty set, and

(ii) for every natural number i and for every element x of $\prod$(the object kind of $\mathbf{SCM}_{\mathrm{FSA}}$) **qua** non empty set such that $x = (\mathit{StepWhile}{>}0(a,s,I))(i)$ holds $(\mathit{StepWhile}{>}0(a,s,I))(i+1) = (\mathrm{Computation}(x{+}\cdot \mathrm{Initialized}(\textbf{while } a > 0 \textbf{ do } I)))(\mathrm{LifeSpan}(x{+}\cdot \mathrm{Initialized}(I)) + 3)$.

We now state several propositions:

(23) $(\mathit{StepWhile}{>}0(a,s,I))(0) = s$.

(24) $(\mathit{StepWhile}{>}0(a,s,I))(k{+}1) = (\mathrm{Computation}((\mathit{StepWhile}{>}0(a,s,I))(k) {+}\cdot \mathrm{Initialized}(\textbf{while } a > 0 \textbf{ do } I)))(\mathrm{LifeSpan}((\mathit{StepWhile}{>}0(a,s,I))(k){+}\cdot \mathrm{Initialized}(I)) + 3)$.

(25) $(\mathit{StepWhile}{>}0(a,s,I))(k{+}1) = (\mathit{StepWhile}{>}0(a,(\mathit{StepWhile}{>}0(a,s,I))(k),I))(1)$.

(26) Let I be a macro instruction, a be a read-write integer location, and s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Then $(\mathit{StepWhile}{>}0(a,s,I))(0+1) = (\mathrm{Computation}(s{+}\cdot \mathrm{Initialized}(\textbf{while } a > 0 \textbf{ do } I)))(\mathrm{LifeSpan}(s{+}\cdot \mathrm{Initialized}(I)) + 3)$.

(27) Let I be a macro instruction, a be a read-write integer location, s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, and k, n be natural numbers. Suppose $\mathbf{IC}_{(\mathit{StepWhile}{>}0(a,s,I))(k)} = \mathrm{insloc}(0)$ and $(\mathit{StepWhile}{>}0(a,s,I))(k) = (\mathrm{Computation}(s{+}\cdot \mathrm{Initialized}(\textbf{while } a > 0 \textbf{ do } I)))(n)$ and $(\mathit{StepWhile}{>}0(a,s,I))(k)(\mathrm{intloc}(0)) = 1$.
Then $(\mathit{StepWhile}{>}0(a,s,I))(k) = (\mathit{StepWhile}{>}0(a,s,I))(k){+}\cdot \mathrm{Initialized}(\textbf{while } a > 0 \textbf{ do } I)$ and $(\mathit{StepWhile}{>}0(a,s,I))(k{+}1) = (\mathrm{Computation}(s{+}\cdot \mathrm{Initialized}(\textbf{while } a > 0 \textbf{ do } I)))(n{+}(\mathrm{LifeSpan}((\mathit{StepWhile}{>}0(a,s,I))(k){+}\cdot \mathrm{Initialized}(I)) + 3))$.

(28) Let I be a macro instruction, a be a read-write integer location, and s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Given a function f from $\prod$(the object kind of $\mathbf{SCM}_{\mathrm{FSA}}$) into $\mathbb{N}$ such that let k be a natural number. Then

(i) if $f((\mathit{StepWhile}{>}0(a,s,I))(k)) \neq 0$, then $f((\mathit{StepWhile}{>}0(a,s,I))(k+1)) < f((\mathit{StepWhile}{>}0(a,s,I))(k))$ and I is closed onInit $(\mathit{StepWhile}{>}0(a,s,I))(k)$ and I is halting onInit $(\mathit{StepWhile}{>}0(a,s,I))(k)$,

(ii) $(\mathit{StepWhile}{>}0(a,s,I))(k+1)(\mathrm{intloc}(0)) = 1$, and

(iii) $f((\mathit{StepWhile}{>}0(a,s,I))(k)) = 0$ iff $(\mathit{StepWhile}{>}0(a,s,I))(k)(a) \leqslant 0$.

Then **while** $a > 0$ **do** I is halting onInit s and **while** $a > 0$ **do** I is closed onInit s.

(29) Let I be a good InitHalting macro instruction and a be a read-write integer location. Suppose that for every state s of $\mathbf{SCM}_{\mathrm{FSA}}$ such that $s(a) > 0$ holds $(\mathrm{IExec}(I,s))(a) < s(a)$. Then **while** $a > 0$ **do** I is InitHalting.

(30) Let I be a good InitHalting macro instruction and a be a read-write integer location. Suppose that for every state s of $\mathbf{SCM}_{\mathrm{FSA}}$ holds

$(\mathrm{IExec}(I,s))(a) < s(a)$ or $(\mathrm{IExec}(I,s))(a) \leqslant 0$. Then **while** $a > 0$ **do** I is InitHalting.

Let D be a set, let f be a function from D into $\mathbb{Z}$, and let d be an element of D. Then $f(d)$ is an integer.

One can prove the following propositions:

(31) Let I be a good InitHalting macro instruction and a be a read-write integer location. Given a function f from $\prod$ (the object kind of $\mathbf{SCM}_{\mathrm{FSA}}$) into $\mathbb{Z}$ such that let s, t be states of $\mathbf{SCM}_{\mathrm{FSA}}$. Then

(i) if $f(s) > 0$, then $f(\mathrm{IExec}(I,s)) < f(s)$,

(ii) if $s{\restriction}D = t{\restriction}D$, then $f(s) = f(t)$, and

(iii) $f(s) \leqslant 0$ iff $s(a) \leqslant 0$.

Then **while** $a > 0$ **do** I is InitHalting, where $D = \mathrm{Int\text{-}Locations} \cup \mathrm{FinSeq\text{-}Locations}$.

(32) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I be a macro instruction, and a be a read-write integer location. If $s(a) \leqslant 0$, then $\mathrm{IExec}(\textbf{while } a > 0 \textbf{ do } I, s){\restriction}(\mathrm{Int\text{-}Locations} \cup \mathrm{FinSeq\text{-}Locations}) = \mathrm{Initialize}(s){\restriction}(\mathrm{Int\text{-}Locations} \cup \mathrm{FinSeq\text{-}Locations})$.

(33) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I be a good InitHalting macro instruction, and a be a read-write integer location. If $s(a) > 0$ and **while** $a > 0$ **do** I is InitHalting, then $\mathrm{IExec}(\textbf{while } a > 0 \textbf{ do } I, s){\restriction}(\mathrm{Int\text{-}Locations} \cup \mathrm{FinSeq\text{-}Locations}) = \mathrm{IExec}(\textbf{while } a > 0 \textbf{ do } I, \mathrm{IExec}(I,s)){\restriction}(\mathrm{Int\text{-}Locations} \cup \mathrm{FinSeq\text{-}Locations})$.

(34) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I be a macro instruction, f be a finite sequence location, and a be a read-write integer location. If $s(a) \leqslant 0$, then $(\mathrm{IExec}(\textbf{while } a > 0 \textbf{ do } I, s))(f) = s(f)$.

(35) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I be a macro instruction, b be an integer location, and a be a read-write integer location. If $s(a) \leqslant 0$, then $(\mathrm{IExec}(\textbf{while } a > 0 \textbf{ do } I, s))(b) = (\mathrm{Initialize}(s))(b)$.

(36) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I be a good InitHalting macro instruction, f be a finite sequence location, and a be a read-write integer location. If $s(a) > 0$ and **while** $a > 0$ **do** I is InitHalting, then $(\mathrm{IExec}(\textbf{while } a > 0 \textbf{ do } I, s))(f) = (\mathrm{IExec}(\textbf{while } a > 0 \textbf{ do } I, \mathrm{IExec}(I,s)))(f)$.

(37) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, I be a good InitHalting macro instruction, b be an integer location, and a be a read-write integer location. If $s(a) > 0$ and **while** $a > 0$ **do** I is InitHalting, then $(\mathrm{IExec}(\textbf{while } a > 0 \textbf{ do } I, s))(b) = (\mathrm{IExec}(\textbf{while } a > 0 \textbf{ do } I, \mathrm{IExec}(I,s)))(b)$.

3. Insert Sort Algorithm

Let f be a finite sequence location. The functor insert $-$ sort f yields a macro instruction and is defined as follows:

(Def. 2) insert $-$ sort f = i_2;(a_1:=len$f$);SubFrom(a_1, a_0); Times(a_1, (a_2:=len$f$); SubFrom(a_2, a_1);(a_3:=a_2);AddTo(a_3, a_0);(a_6:=f_{a_3});SubFrom(a_4, a_4); (**while** $a_2 > 0$ **do** ((a_5:=f_{a_2});SubFrom(a_5, a_6);(**if** $a_5 > 0$ **then** Macro (SubFrom(a_2, a_2)) **else** (AddTo(a_4, a_0);SubFrom(a_2, a_0))))); Times(a_4, (a_2:=a_3);SubFrom(a_3, a_0);(a_5:=f_{a_2});(a_6:=f_{a_3});(f_{a_2}:=a_6);(f_{a_3}:=a_5))), where i_2 = (a_2:=a_0);(a_3:=a_0);(a_4:=a_0);(a_5:=a_0);(a_6:=a_0), a_2 = intloc(2), a_0 = intloc(0), a_3 = intloc(3), a_4 = intloc(4), a_5 = intloc(5), a_6 = intloc(6), and a_1 = intloc(1).

The macro instruction Insert $-$ Sort $-$ Algorithm is defined by:

(Def. 3) Insert $-$ Sort $-$ Algorithm = insert $-$ sort fsloc(0).

We now state a number of propositions:

(38) For every finite sequence location f holds UsedIntLoc(insert $-$ sort f) = $\{a_0, a_1, a_2, a_3, a_4, a_5, a_6\}$, where a_0 = intloc(0), a_1 = intloc(1), a_2 = intloc(2), a_3 = intloc(3), a_4 = intloc(4), a_5 = intloc(5), and a_6 = intloc(6).

(39) For every finite sequence location f holds UsedInt* Loc(insert $-$ sort f) = $\{f\}$.

(40) For all instructions k_1, k_2, k_3, k_4 of $\mathbf{SCM}_{\mathrm{FSA}}$ holds card(k_1;k_2;k_3;k_4) = 8.

(41) For all instructions k_1, k_2, k_3, k_4, k_5 of $\mathbf{SCM}_{\mathrm{FSA}}$ holds card(k_1;k_2;k_3;k_4;k_5) = 10.

(42) For every finite sequence location f holds card insert $-$ sort f = 82.

(43) For every finite sequence location f and for every natural number k such that $k < 82$ holds insloc(k) $\in$ dom insert $-$ sort f.

(44) insert $-$ sort fsloc(0) is keepInt0 1 and InitHalting.

(45) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Then

(i) $s(f_0)$ and (IExec(insert $-$ sort f_0, s))(f_0) are fiberwise equipotent, and

(ii) for all natural numbers i, j such that $i \geqslant 1$ and $j \leqslant$ len $s(f_0)$ and $i < j$ and for all integers x_1, x_2 such that x_1 = (IExec(insert $-$ sort f_0, s))(f_0)(i) and x_2 = (IExec(insert $-$ sort f_0, s))(f_0)(j) holds $x_1 \geqslant x_2$,

where f_0 = fsloc(0).

(46) Let i be a natural number, s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$, and w be a finite sequence of elements of $\mathbb{Z}$. If Initialized(Insert $-$ Sort $-$ Algorithm)+$\cdot$(fsloc(0) $\mapsto$ w) $\subseteq$ s, then $\mathbf{IC}_{(\mathrm{Computation}(s))(i)} \in$ dom Insert $-$ Sort $-$ Algorithm.

(47) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$ and t be a finite sequence of elements of $\mathbb{Z}$. Suppose Initialized(Insert $-$ Sort $-$ Algorithm)+$\cdot$(fsloc(0)$\mapsto$ t) $\subseteq$ s. Then there exists a finite sequence u of elements of $\mathbb{R}$ such that

(i) t and u are fiberwise equipotent,

(ii) u is non-increasing and a finite sequence of elements of $\mathbb{Z}$, and

(iii) $(\mathrm{Result}(s))(\mathrm{fsloc}(0)) = u$.

(48) For every finite sequence w of elements of $\mathbb{Z}$ holds $\mathrm{Initialized}(\mathrm{Insert}-\mathrm{Sort}-\mathrm{Algorithm})+\cdot(\mathrm{fsloc}(0)\mapsto w)$ is autonomic.

(49) $\mathrm{Initialized}(\mathrm{Insert}-\mathrm{Sort}-\mathrm{Algorithm})$ computes Sorting-Function.

References

[1] Noriko Asamoto. Conditional branch macro instructions of $\mathbf{SCM}_{\mathrm{FSA}}$. Part I. *Formalized Mathematics*, 6(**1**):65–72, 1997.

[2] Noriko Asamoto. Conditional branch macro instructions of $\mathbf{SCM}_{\mathrm{FSA}}$. Part II. *Formalized Mathematics*, 6(**1**):73–80, 1997.

[3] Noriko Asamoto. Constant assignment macro instructions of $\mathbf{SCM}_{\mathrm{FSA}}$. Part II. *Formalized Mathematics*, 6(**1**):59–63, 1997.

[4] Noriko Asamoto. The `loop` and `Times` macroinstruction for $\mathbf{SCM}_{\mathrm{FSA}}$. *Formalized Mathematics*, 6(**4**):483–497, 1997.

[5] Noriko Asamoto, Yatsuka Nakamura, Piotr Rudnicki, and Andrzej Trybulec. On the composition of macro instructions. Part II. *Formalized Mathematics*, 6(**1**):41–47, 1997.

[6] Noriko Asamoto, Yatsuka Nakamura, Piotr Rudnicki, and Andrzej Trybulec. On the composition of macro instructions. Part III. *Formalized Mathematics*, 6(**1**):53–57, 1997.

[7] Grzegorz Bancerek. Cardinal numbers. *Formalized Mathematics*, 1(**2**):377–382, 1990.

[8] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.

[9] Grzegorz Bancerek. König's theorem. *Formalized Mathematics*, 1(**3**):589–593, 1990.

[10] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.

[11] Grzegorz Bancerek and Piotr Rudnicki. Development of terminology for **scm**. *Formalized Mathematics*, 4(**1**):61–67, 1993.

[12] Grzegorz Bancerek and Andrzej Trybulec. Miscellaneous facts about functions. *Formalized Mathematics*, 5(**4**):485–492, 1996.

[13] Czesław Byliński. A classical first order language. *Formalized Mathematics*, 1(**4**):669–676, 1990.

[14] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

[15] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.

[16] Czesław Byliński. The modification of a function by a function and the iteration of the composition of a function. *Formalized Mathematics*, 1(**3**):521–527, 1990.

[17] Jing-Chao Chen. While macro instructions of $\mathbf{SCM}_{\mathrm{FSA}}$. *Formalized Mathematics*, 6(**4**):553–561, 1997.

[18] Jing-Chao Chen and Yatsuka Nakamura. Bubble sort on $\mathbf{SCM}_{\mathrm{FSA}}$. *Formalized Mathematics*, 7(**1**):153–161, 1998.

[19] Jing-Chao Chen and Yatsuka Nakamura. Initialization halting concepts and their basic properties of $\mathbf{SCM}_{\mathrm{FSA}}$. *Formalized Mathematics*, 7(**1**):139–151, 1998.

[20] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.

[21] Krzysztof Hryniewiecki. Recursive definitions. *Formalized Mathematics*, 1(**2**):321–328, 1990.

[22] Jarosław Kotowicz. Functions and finite sequences of real numbers. *Formalized Mathematics*, 3(**2**):275–278, 1992.

[23] Yatsuka Nakamura and Andrzej Trybulec. A mathematical model of CPU. *Formalized Mathematics*, 3(**2**):151–160, 1992.

[24] Andrzej Nędzusiak. σ-fields and probability. *Formalized Mathematics*, 1(**2**):401–407, 1990.

[25] Piotr Rudnicki and Andrzej Trybulec. Memory handling for $\mathbf{SCM}_{\mathrm{FSA}}$. *Formalized Mathematics*, 6(**1**):29–36, 1997.

[26] Yasushi Tanaka. On the decomposition of the states of SCM. *Formalized Mathematics*, 5(**1**):1–8, 1996.
[27] Andrzej Trybulec. Enumerated sets. *Formalized Mathematics*, 1(**1**):25–34, 1990.
[28] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[29] Andrzej Trybulec and Agata Darmochwał. Boolean domains. *Formalized Mathematics*, 1(**1**):187–190, 1990.
[30] Andrzej Trybulec and Yatsuka Nakamura. Some remarks on the simple concrete model of computer. *Formalized Mathematics*, 4(**1**):51–56, 1993.
[31] Andrzej Trybulec and Yatsuka Nakamura. Modifying addresses of instructions of $\mathbf{SCM}_{\mathrm{FSA}}$. *Formalized Mathematics*, 5(**4**):571–576, 1996.
[32] Andrzej Trybulec and Yatsuka Nakamura. Relocability for $\mathbf{SCM}_{\mathrm{FSA}}$. *Formalized Mathematics*, 5(**4**):583–586, 1996.
[33] Andrzej Trybulec, Yatsuka Nakamura, and Noriko Asamoto. On the compositions of macro instructions. Part I. *Formalized Mathematics*, 6(**1**):21–27, 1997.
[34] Andrzej Trybulec, Yatsuka Nakamura, and Piotr Rudnicki. The $\mathbf{SCM}_{\mathrm{FSA}}$ computer. *Formalized Mathematics*, 5(**4**):519–528, 1996.
[35] Michał J. Trybulec. Integers. *Formalized Mathematics*, 1(**3**):501–505, 1990.
[36] Wojciech A. Trybulec. Groups. *Formalized Mathematics*, 1(**5**):821–827, 1990.
[37] Wojciech A. Trybulec. Pigeon hole principle. *Formalized Mathematics*, 1(**3**):575–579, 1990.
[38] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[39] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.
[40] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.

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