FORMALIZED MATHEMATICS
Volume 8, Number 1, 1999
University of Białystok

Correctness of Johnson Counter Circuits

Yuguang Yang
Shinshu University
Nagano

Katsumi Wasaki
Shinshu University
Nagano

Yasushi Fuwa
Shinshu University
Nagano

Yatsuka Nakamura
Shinshu University
Nagano

Summary. This article introduces the verification of the correctness for the operations and the specification of the Johnson counter. We formalize the concepts of 2-bit, 3-bit and 4-bit Johnson counter circuits with a reset input, and define the specification of the state transitions without the minor loop.

MML Identifier: GATE_3.

The notation and terminology used here are introduced in the paper [1].

The following propositions are true:

(1) Let s_0, s_1, s_2, s_3, n_0, n_1, n_2, n_3, q_1, q_2, n_4, n_5 be sets such that NE s_0 iff NE AND2(NOT1 q_2, NOT1 q_1) and NE s_1 iff NE AND2(NOT1 q_2, q_1) and NE s_2 iff NE AND2(q_2, NOT1 q_1) and NE s_3 iff NE AND2(q_2, q_1) and NE n_0 iff NE AND2(NOT1 n_5, NOT1 n_4) and NE n_1 iff NE AND2(NOT1 n_5, n_4) and NE n_2 iff NE AND2(n_5, NOT1 n_4) and NE n_3 iff NE AND2(n_5, n_4) and NE n_4 iff NE NOT1 q_2 and NE n_5 iff NE q_1. Then
- (i) NE n_1 iff NE s_0,
- (ii) NE n_3 iff NE s_1,
- (iii) NE n_2 iff NE s_3, and
- (iv) NE n_0 iff NE s_2.

(2) Let s_0, s_1, s_2, s_3, n_0, n_1, n_2, n_3, q_1, q_2, n_4, n_5, R be sets such that NE s_0 iff NE AND2(NOT1 q_2, NOT1 q_1) and NE s_1 iff NE AND2(NOT1 q_2, q_1) and NE s_2 iff NE AND2(q_2, NOT1 q_1) and NE s_3 iff NE AND2(q_2, q_1) and NE n_0 iff NE AND2(NOT1 n_5, NOT1 n_4) and NE n_1 iff NE AND2(NOT1 n_5, n_4) and NE n_2 iff NE AND2(n_5, NOT1 n_4) and NE n_3

ISSN 1426–2630

iff NE AND2(n_5, n_4) and NE n_4 iff NE AND2(NOT1 q_2, R) and NE n_5 iff NE AND2(q_1, R). Then

(i) NE n_1 iff NE AND2(s_0, R),
(ii) NE n_3 iff NE AND2(s_1, R),
(iii) NE n_2 iff NE AND2(s_3, R), and
(iv) NE n_0 iff NE OR2(AND2(s_2, R), NOT1 R).

(3) Let s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, n_0, n_1, n_2, n_3, n_6, n_7, n_8, n_9, q_1, q_2, q_3, n_4, n_5, n_{10} be sets such that NE s_0 iff NE AND3(NOT1 q_3, NOT1 q_2, NOT1 q_1) and NE s_1 iff NE AND3(NOT1 q_3, NOT1 q_2, q_1) and NE s_2 iff NE AND3(NOT1 q_3, q_2, NOT1 q_1) and NE s_3 iff NE AND3(NOT1 q_3, q_2, q_1) and NE s_4 iff NE AND3(q_3, NOT1 q_2, NOT1 q_1) and NE s_5 iff NE AND3(q_3, NOT1 q_2, q_1) and NE s_6 iff NE AND3(q_3, q_2, NOT1 q_1) and NE s_7 iff NE AND3(q_3, q_2, q_1) and NE n_0 iff NE AND3(NOT1 n_{10}, NOT1 n_5, NOT1 n_4) and NE n_1 iff NE AND3(NOT1 n_{10}, NOT1 n_5, n_4) and NE n_2 iff NE AND3(NOT1 n_{10}, n_5, NOT1 n_4) and NE n_3 iff NE AND3(NOT1 n_{10}, n_5, n_4) and NE n_6 iff NE AND3(n_{10}, NOT1 n_5, NOT1 n_4) and NE n_7 iff NE AND3(n_{10}, NOT1 n_5, n_4) and NE n_8 iff NE AND3(n_{10}, n_5, NOT1 n_4) and NE n_9 iff NE AND3(n_{10}, n_5, n_4) and NE n_4 iff NE NOT1 q_3 and NE n_5 iff NE q_1 and NE n_{10} iff NE q_2. Then

(i) NE n_1 iff NE s_0,
(ii) NE n_3 iff NE s_1,
(iii) NE n_9 iff NE s_3,
(iv) NE n_8 iff NE s_7,
(v) NE n_6 iff NE s_6,
(vi) NE n_0 iff NE s_4,
(vii) NE n_2 iff NE s_5, and
(viii) NE n_7 iff NE s_2.

(4) Let s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, n_0, n_1, n_2, n_3, n_6, n_7, n_8, n_9, q_1, q_2, q_3, n_4, n_5, n_{10}, R be sets such that NE s_0 iff NE AND3(NOT1 q_3, NOT1 q_2, NOT1 q_1) and NE s_1 iff NE AND3(NOT1 q_3, NOT1 q_2, q_1) and NE s_2 iff NE AND3(NOT1 q_3, q_2, NOT1 q_1) and NE s_3 iff NE AND3(NOT1 q_3, q_2, q_1) and NE s_4 iff NE AND3(q_3, NOT1 q_2, NOT1 q_1) and NE s_5 iff NE AND3(q_3, NOT1 q_2, q_1) and NE s_6 iff NE AND3(q_3, q_2, NOT1 q_1) and NE s_7 iff NE AND3(q_3, q_2, q_1) and NE n_0 iff NE AND3(NOT1 n_{10}, NOT1 n_5, NOT1 n_4) and NE n_1 iff NE AND3(NOT1 n_{10}, NOT1 n_5, n_4) and NE n_2 iff NE AND3(NOT1 n_{10}, n_5, NOT1 n_4) and NE n_3 iff NE AND3(NOT1 n_{10}, n_5, n_4) and NE n_6 iff NE AND3(n_{10}, NOT1 n_5, NOT1 n_4) and NE n_7 iff NE AND3(n_{10}, NOT1 n_5, n_4) and NE n_8 iff NE AND3(n_{10}, n_5, NOT1 n_4) and NE n_9 iff NE AND3(n_{10}, n_5, n_4) and NE n_4 iff NE AND2(NOT1 q_3, R) and NE n_5 iff NE AND2(q_1, R) and NE n_{10} iff NE AND2(q_2, R). Then

(i) NE n_1 iff NE AND2(s_0, R),
(ii) NE n_3 iff NE AND2(s_1, R),
(iii) NE n_9 iff NE AND2(s_3, R),
(iv) NE n_8 iff NE AND2(s_7, R),
(v) NE n_6 iff NE AND2(s_6, R),
(vi) NE n_0 iff NE OR2(AND2(s_4, R), NOT1 R),
(vii) NE n_2 iff NE AND2(s_5, R), and
(viii) NE n_7 iff NE AND2(s_2, R).

(5) Let s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9, s_{10}, s_{11}, s_{12}, s_{13}, s_{14}, s_{15}, n_0, n_1, n_2, n_3, n_6, n_7, n_8, n_9, n_{11}, n_{12}, n_{13}, n_{14}, n_{15}, n_{16}, n_{17}, n_{18}, q_1, q_2, q_3, q_4, n_4, n_5, n_{10}, n_{19} be sets such that NE s_0 iff NE AND4(NOT1 q_4, NOT1 q_3, NOT1 q_2, NOT1 q_1) and NE s_1 iff NE AND4(NOT1 q_4, NOT1 q_3, NOT1 q_2, q_1) and NE s_2 iff NE AND4(NOT1 q_4, NOT1 q_3, q_2, NOT1 q_1) and NE s_3 iff NE AND4(NOT1 q_4, NOT1 q_3, q_2, q_1) and NE s_4 iff NE AND4(NOT1 q_4, q_3, NOT1 q_2, NOT1 q_1) and NE s_5 iff NE AND4(NOT1 q_4, q_3, NOT1 q_2, q_1) and NE s_6 iff NE AND4(NOT1 q_4, q_3, q_2, NOT1 q_1) and NE s_7 iff NE AND4(NOT1 q_4, q_3, q_2, q_1) and NE s_8 iff NE AND4(q_4, NOT1 q_3, NOT1 q_2, NOT1 q_1) and NE s_9 iff NE AND4(q_4, NOT1 q_3, NOT1 q_2, q_1) and NE s_{10} iff NE AND4(q_4, NOT1 q_3, q_2, NOT1 q_1) and NE s_{11} iff NE AND4(q_4, NOT1 q_3, q_2, q_1) and NE s_{12} iff NE AND4(q_4, q_3, NOT1 q_2, NOT1 q_1) and NE s_{13} iff NE AND4(q_4, q_3, NOT1 q_2, q_1) and NE s_{14} iff NE AND4(q_4, q_3, q_2, NOT1 q_1) and NE s_{15} iff NE AND4(q_4, q_3, q_2, q_1) and NE n_0 iff NE AND4(NOT1 n_{19}, NOT1 n_{10}, NOT1 n_5, NOT1 n_4) and NE n_1 iff NE AND4(NOT1 n_{19}, NOT1 n_{10}, NOT1 n_5, n_4) and NE n_2 iff NE AND4(NOT1 n_{19}, NOT1 n_{10}, n_5, NOT1 n_4) and NE n_3 iff NE AND4(NOT1 n_{19}, NOT1 n_{10}, n_5, n_4) and NE n_6 iff NE AND4(NOT1 n_{19}, n_{10}, NOT1 n_5, NOT1 n_4) and NE n_7 iff NE AND4(NOT1 n_{19}, n_{10}, NOT1 n_5, n_4) and NE n_8 iff NE AND4(NOT1 n_{19}, n_{10}, n_5, NOT1 n_4) and NE n_9 iff NE AND4(NOT1 n_{19}, n_{10}, n_5, n_4) and NE n_{11} iff NE AND4(n_{19}, NOT1 n_{10}, NOT1 n_5, NOT1 n_4) and NE n_{12} iff NE AND4(n_{19}, NOT1 n_{10}, NOT1 n_5, n_4) and NE n_{13} iff NE AND4(n_{19}, NOT1 n_{10}, n_5, NOT1 n_4) and NE n_{14} iff NE AND4(n_{19}, NOT1 n_{10}, n_5, n_4) and NE n_{15} iff NE AND4(n_{19}, n_{10}, NOT1 n_5, NOT1 n_4) and NE n_{16} iff NE AND4(n_{19}, n_{10}, NOT1 n_5, n_4) and NE n_{17} iff NE AND4(n_{19}, n_{10}, n_5, NOT1 n_4) and NE n_{18} iff NE AND4(n_{19}, n_{10}, n_5, n_4) and NE n_4 iff NE NOT1 q_4 and NE n_5 iff NE q_1 and NE n_{10} iff NE q_2 and NE n_{19} iff NE q_3. Then
(i) NE n_1 iff NE s_0,
(ii) NE n_3 iff NE s_1,
(iii) NE n_9 iff NE s_3,
(iv) NE n_{18} iff NE s_7,
(v) NE n_{17} iff NE s_{15},

(vi) NE n_{15} iff NE s_{14},
(vii) NE n_{11} iff NE s_{12},
(viii) NE n_0 iff NE s_8,
(ix) NE n_7 iff NE s_2,
(x) NE n_{14} iff NE s_5,
(xi) NE n_8 iff NE s_{11},
(xii) NE n_{16} iff NE s_6,
(xiii) NE n_{13} iff NE s_{13},
(xiv) NE n_6 iff NE s_{10},
(xv) NE n_{12} iff NE s_4, and
(xvi) NE n_2 iff NE s_9.

(6) Let s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9, s_{10}, s_{11}, s_{12}, s_{13}, s_{14}, s_{15}, n_0, n_1, n_2, n_3, n_6, n_7, n_8, n_9, n_{11}, n_{12}, n_{13}, n_{14}, n_{15}, n_{16}, n_{17}, n_{18}, q_1, q_2, q_3, q_4, n_4, n_5, n_{10}, n_{19}, R be sets such that NE s_0 iff NE AND4(NOT1 q_4, NOT1 q_3, NOT1 q_2, NOT1 q_1) and NE s_1 iff NE AND4(NOT1 q_4, NOT1 q_3, NOT1 q_2, q_1) and NE s_2 iff NE AND4(NOT1 q_4, NOT1 q_3, q_2, NOT1 q_1) and NE s_3 iff NE AND4(NOT1 q_4, NOT1 q_3, q_2, q_1) and NE s_4 iff NE AND4(NOT1 q_4, q_3, NOT1 q_2, NOT1 q_1) and NE s_5 iff NE AND4(NOT1 q_4, q_3, NOT1 q_2, q_1) and NE s_6 iff NE AND4(NOT1 q_4, q_3, q_2, NOT1 q_1) and NE s_7 iff NE AND4(NOT1 q_4, q_3, q_2, q_1) and NE s_8 iff NE AND4(q_4, NOT1 q_3, NOT1 q_2, NOT1 q_1) and NE s_9 iff NE AND4(q_4, NOT1 q_3, NOT1 q_2, q_1) and NE s_{10} iff NE AND4(q_4, NOT1 q_3, q_2, NOT1 q_1) and NE s_{11} iff NE AND4(q_4, NOT1 q_3, q_2, q_1) and NE s_{12} iff NE AND4(q_4, q_3, NOT1 q_2, NOT1 q_1) and NE s_{13} iff NE AND4(q_4, q_3, NOT1 q_2, q_1) and NE s_{14} iff NE AND4(q_4, q_3, q_2, NOT1 q_1) and NE s_{15} iff NE AND4(q_4, q_3, q_2, q_1) and NE n_0 iff NE AND4(NOT1 n_{19}, NOT1 n_{10}, NOT1 n_5, NOT1 n_4) and NE n_1 iff NE AND4(NOT1 n_{19}, NOT1 n_{10}, NOT1 n_5, n_4) and NE n_2 iff NE AND4(NOT1 n_{19}, NOT1 n_{10}, n_5, NOT1 n_4) and NE n_3 iff NE AND4(NOT1 n_{19}, NOT1 n_{10}, n_5, n_4) and NE n_6 iff NE AND4(NOT1 n_{19}, n_{10}, NOT1 n_5, NOT1 n_4) and NE n_7 iff NE AND4(NOT1 n_{19}, n_{10}, NOT1 n_5, n_4) and NE n_8 iff NE AND4(NOT1 n_{19}, n_{10}, n_5, NOT1 n_4) and NE n_9 iff NE AND4(NOT1 n_{19}, n_{10}, n_5, n_4) and NE n_{11} iff NE AND4(n_{19}, NOT1 n_{10}, NOT1 n_5, NOT1 n_4) and NE n_{12} iff NE AND4(n_{19}, NOT1 n_{10}, NOT1 n_5, n_4) and NE n_{13} iff NE AND4(n_{19}, NOT1 n_{10}, n_5, NOT1 n_4) and NE n_{14} iff NE AND4(n_{19}, NOT1 n_{10}, n_5, n_4) and NE n_{15} iff NE AND4(n_{19}, n_{10}, NOT1 n_5, NOT1 n_4) and NE n_{16} iff NE AND4(n_{19}, n_{10}, NOT1 n_5, n_4) and NE n_{17} iff NE AND4(n_{19}, n_{10}, n_5, NOT1 n_4) and NE n_{18} iff NE AND4(n_{19}, n_{10}, n_5, n_4) and NE n_4 iff NE AND2(NOT1 q_4, R) and NE n_5 iff NE AND2(q_1, R) and NE n_{10} iff NE AND2(q_2, R) and NE n_{19} iff NE AND2(q_3, R). Then

(i) NE n_1 iff NE AND2(s_0, R),

(ii) NE n_3 iff NE AND2(s_1, R),
(iii) NE n_9 iff NE AND2(s_3, R),
(iv) NE n_{18} iff NE AND2(s_7, R),
(v) NE n_{17} iff NE AND2(s_{15}, R),
(vi) NE n_{15} iff NE AND2(s_{14}, R),
(vii) NE n_{11} iff NE AND2(s_{12}, R),
(viii) NE n_0 iff NE OR2(AND2(s_8, R), NOT1 R),
(ix) NE n_7 iff NE AND2(s_2, R),
(x) NE n_{14} iff NE AND2(s_5, R),
(xi) NE n_8 iff NE AND2(s_{11}, R),
(xii) NE n_{16} iff NE AND2(s_6, R),
(xiii) NE n_{13} iff NE AND2(s_{13}, R),
(xiv) NE n_6 iff NE AND2(s_{10}, R),
(xv) NE n_{12} iff NE AND2(s_4, R), and
(xvi) NE n_2 iff NE AND2(s_9, R).

References

[1] Yatsuka Nakamura. Logic gates and logical equivalence of adders. *Formalized Mathematics*, 8(**1**):35–45, 1999.

Received March 13, 1999