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Some Properties of Isomorphism between Relational Structures. On the Product of Topological Spaces[1]

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The articles [26], [10], [31], [32], [33], [16], [15], [7], [9], [12], [11], [1], [27], [22], [24], [25], [23], [14], [35], [6], [20], [30], [2], [19], [3], [4], [13], [8], [18], [34], [5], [28], [29], [21], and [17] provide the notation and terminology for this paper.

1. Preliminaries

One can prove the following propositions:

(1) $2^1 = \{0,1\}$.

(2) For every set X and for every subset Y of X holds $\mathrm{rng}(\mathrm{id}_X \restriction Y) = Y$.

(3) For every function f and for all sets a, b holds $(f+\cdot(a \mapsto b))(a) = b$.

Let us note that there exists a relational structure which is strict and empty.

Next we state four propositions:

(4) Let S be an empty 1-sorted structure, T be a 1-sorted structure, and f be a map from S into T. If $\mathrm{rng}\, f = \Omega_T$, then T is empty.

(5) Let S be a 1-sorted structure, T be an empty 1-sorted structure, and f be a map from S into T. If $\mathrm{dom}\, f = \Omega_S$, then S is empty.

(6) Let S be a non empty 1-sorted structure, T be a 1-sorted structure, and f be a map from S into T. If $\mathrm{dom}\, f = \Omega_S$, then T is non empty.

(7) Let S be a 1-sorted structure, T be a non empty 1-sorted structure, and f be a map from S into T. If $\mathrm{rng}\, f = \Omega_T$, then S is non empty.

Let S be a non empty reflexive relational structure, let T be a non empty relational structure, and let f be a map from S into T. Let us observe that f is directed-sups-preserving if and only if:

(Def. 1) For every non empty directed subset X of S holds f preserves sup of X.

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Let R be a 1-sorted structure and let N be a net structure over R. We say that N is function yielding if and only if:

(Def. 2) The mapping of N is function yielding.

Let us observe that there exists a 1-sorted structure which is strict, non empty, and constituted functions.

Let us observe that there exists a relational structure which is strict, non empty, and constituted functions.

Let R be a constituted functions 1-sorted structure. Observe that every net structure over R is function yielding.

Let R be a constituted functions 1-sorted structure. Note that there exists a net structure over R which is strict and function yielding.

Let R be a non empty constituted functions 1-sorted structure. Observe that there exists a net structure over R which is strict, non empty, and function yielding.

Let R be a constituted functions 1-sorted structure and let N be a function yielding net structure over R. One can check that the mapping of N is function yielding.

Let R be a non empty constituted functions 1-sorted structure. Observe that there exists a net in R which is strict and function yielding.

Let S be a non empty 1-sorted structure and let N be a non empty net structure over S. Note that rng (the mapping of N) is non empty.

Let S be a non empty 1-sorted structure and let N be a non empty net structure over S. One can check that rng netmap(N,S) is non empty.

We now state two propositions:

(8) Let A, B, C be non empty relational structures, f be a map from B into C, and g, h be maps from A into B. If $g \leq h$ and f is monotone, then $f \cdot g \leq f \cdot h$.

(9) Let S be a non empty topological space, T be a non empty topological space-like FR-structure, f, g be maps from S into T, and x, y be elements of $[S \rightarrow T]$. If $x = f$ and $y = g$, then $x \leq y$ iff $f \leq g$.

Let I be a set and let R be a non empty relational structure. Observe that every element of R^I is function-like and relation-like.

Let I be a non empty set, let R be a non empty relational structure, let f be an element of R^I, and let i be an element of I. Then $f(i)$ is an element of R.

2. Some Properties of Isomorphism between Relational Structures

The following proposition is true

(10) For all relational structures S, T and for every map f from S into T such that f is isomorphic holds f is onto.

Let S, T be relational structures. One can check that every map from S into T which is isomorphic is also onto.

One can prove the following four propositions:

(11) Let S, T be non empty relational structures and f be a map from S into T. If f is isomorphic, then UNKNOWN(f) is isomorphic.

(12) For all non empty relational structures S, T such that S and T are isomorphic and S has g.l.b.'s holds T has g.l.b.'s.

(13) For all non empty relational structures S, T such that S and T are isomorphic and S has l.u.b.'s holds T has l.u.b.'s.

(14) For every relational structure L such that L is empty holds L is bounded.

Let us note that every relational structure which is empty is also bounded.

One can prove the following propositions:

(15) Let S, T be relational structures. Suppose S and T are isomorphic and S is lower-bounded. Then T is lower-bounded.

(16) Let S, T be relational structures. Suppose S and T are isomorphic and S is upper-bounded. Then T is upper-bounded.

(17) Let S, T be non empty relational structures, A be a subset of S, and f be a map from S into T. Suppose f is isomorphic and $\sup A$ exists in S. Then $\sup f^\circ A$ exists in T.

(18) Let S, T be non empty relational structures, A be a subset of S, and f be a map from S into T. Suppose f is isomorphic and $\inf A$ exists in S. Then $\inf f^\circ A$ exists in T.

3. On the Product of Topological Spaces

We now state two propositions:

(19) Let S, T be topological structures. Suppose S and T are homeomorphic or there exists a map f from S into T such that $\operatorname{dom} f = \Omega_S$ and $\operatorname{rng} f = \Omega_T$. Then S is empty if and only if T is empty.

(20) For every non empty topological space T holds T and the topological structure of T are homeomorphic.

Let T be a Scott reflexive non empty FR-structure. Observe that every subset of T which is open is also inaccessible and upper and every subset of T which is inaccessible and upper is also open.

We now state several propositions:

(21) Let T be a topological structure, x, y be points of T, and X, Y be subsets of T. If $X = \{x\}$ and $\overline{X} \subseteq \overline{Y}$, then $x \in \overline{Y}$.

(22) Let T be a topological structure, x, y be points of T, and Y, V be subsets of T. If $Y = \{y\}$ and $x \in \overline{Y}$ and V is open and $x \in V$, then $y \in V$.

(23) Let T be a topological structure, x, y be points of T, and X, Y be subsets of T. Suppose $X = \{x\}$ and $Y = \{y\}$. Suppose that for every subset V of T such that V is open holds if $x \in V$, then $y \in V$. Then $\overline{X} \subseteq \overline{Y}$.

(24) Let S, T be non empty topological spaces, A be an irreducible subset of S, and B be a subset of T. Suppose $A = B$ and the topological structure of $S =$ the topological structure of T. Then B is irreducible.

(25) Let S, T be non empty topological spaces, a be a point of S, b be a point of T, A be a subset of S, and B be a subset of T. Suppose $a = b$ and $A = B$ and the topological structure of $S =$ the topological structure of T and a is dense point of A. Then b is dense point of B.

(26) Let S, T be topological structures, A be a subset of S, and B be a subset of T. Suppose $A = B$ and the topological structure of $S =$ the topological structure of T and A is compact. Then B is compact.

(27) Let S, T be non empty topological spaces. Suppose the topological structure of $S =$ the topological structure of T and S is sober. Then T is sober.

(28) Let S, T be non empty topological spaces. Suppose the topological structure of $S =$ the topological structure of T and S is locally-compact. Then T is locally-compact.

(29) Let S, T be topological structures. Suppose the topological structure of $S =$ the topological structure of T and S is compact. Then T is compact.

Let I be a non empty set, let T be a non empty topological space, let x be a point of $\prod(I \longmapsto T)$, and let i be an element of I. Then $x(i)$ is an element of T.

The following propositions are true:

(30) Let M be a non empty set, J be a topological space yielding nonempty many sorted set indexed by M, and x, y be points of $\prod J$. Then $x \in \overline{\{y\}}$ if and only if for every element i of M holds $x(i) \in \overline{\{y(i)\}}$.

(31) Let M be a non empty set, T be a non empty topological space, and x, y be points of $\prod(M \longmapsto T)$. Then $x \in \overline{\{y\}}$ if and only if for every element i of M holds $x(i) \in \overline{\{y(i)\}}$.

(32) Let M be a non empty set, i be an element of M, J be a topological space yielding nonempty many sorted set indexed by M, and x be a point of $\prod J$. Then $\pi_i \overline{\{x\}} = \overline{\{x(i)\}}$.

(33) Let M be a non empty set, i be an element of M, T be a non empty topological space, and x be a point of $\prod(M \longmapsto T)$. Then $\pi_i \overline{\{x\}} = \overline{\{x(i)\}}$.

(34) Let X, Y be non empty topological structures, f be a map from X into Y, and g be a map from Y into X. Suppose $f = \mathrm{id}_X$ and $g = \mathrm{id}_X$ and f is continuous and g is continuous. Then the topological structure of $X =$ the topological structure of Y.

(35) Let X, Y be non empty topological spaces and f be a map from X into Y. If f° is continuous, then f is continuous.

Let X be a non empty topological space and let Y be a non empty subspace of X. Observe that $\overset{Y}{\hookrightarrow}$ is continuous.

Next we state three propositions:

(36) For every non empty topological space T and for every map f from T into T such that $f \cdot f = f$ holds $f^{\circ} \cdot (\overset{\mathrm{Im} f}{\hookrightarrow}) = \mathrm{id}_{\mathrm{Im} f}$.

(37) For every non empty topological space Y and for every non empty subspace W of Y holds $(\overset{W}{\hookrightarrow})^{\circ}$ is a homeomorphism.

(38) Let M be a non empty set and J be a topological space yielding nonempty many sorted set indexed by M. Suppose that for every element i of M holds $J(i)$ is a T_0 topological space. Then $\prod J$ is T_0.

Let I be a non empty set and let T be a non empty T_0 topological space. Observe that $\prod(I \longmapsto T)$ is T_0.

We now state the proposition

(39) Let M be a non empty set and J be a topological space yielding nonempty many sorted set indexed by M. Suppose that for every element i of M holds $J(i)$ is T_1 and topological space-like. Then $\prod J$ is a T_1 space.

Let I be a non empty set and let T be a non empty T_1 topological space. Note that $\prod(I \longmapsto T)$ is T_1.

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