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On the Concept of the Triangulation

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The articles [13], [7], [22], [17], [23], [24], [12], [2], [5], [6], [14], [9], [4], [8], [20], [18], [3], [21], [10], [11], [1], [15], [16], and [19] provide the notation and terminology for this paper.

1. Introduction

In this paper A denotes a set and k, m, n denote natural numbers.

The scheme *Regr1* deals with a natural number $\mathcal{A}$ and a unary predicate $\mathcal{P}$, and states that:

For every k such that $k \leq \mathcal{A}$ holds $\mathcal{P}[k]$

provided the parameters meet the following conditions:

- $\mathcal{P}[\mathcal{A}]$, and
- For every k such that $k < \mathcal{A}$ and $\mathcal{P}[k+1]$ holds $\mathcal{P}[k]$.

Let n be a natural number. Note that $\mathrm{Seg}(n+1)$ is non empty.

Let X be a non empty set and let R be an order in X. Note that $\langle X, R \rangle$ is non empty.

The following proposition is true

(1) $\emptyset \,|^2 A = \emptyset$.

Let X be a set. Observe that there exists a subset of $\mathrm{Fin}\, X$ which is non empty.

Let X be a non empty set. One can check that there exists a subset of $\mathrm{Fin}\, X$ which is non empty and has non empty elements.

Let X be a non empty set and let F be a non empty subset of $\mathrm{Fin}\, X$ with non empty elements. Note that there exists an element of F which is non empty.

Let I_1 be a set. We say that I_1 has a non-empty element if and only if:

(Def. 1) There exists a non empty set X such that $X \in I_1$.

One can verify that there exists a set which has a non-empty element.

Let X be a set with a non-empty element. Note that there exists an element of X which is non empty.

Let us observe that every set which has a non-empty element is also non empty.

Let X be a non empty set. Note that there exists a subset of $\mathrm{Fin}\, X$ which has a non-empty element.

Let X be a non empty set, let R be an order in X, and let A be a subset of X. Then $R \,|^2 A$ is an order in A.

The scheme *SubFinite* deals with a set $\mathcal{A}$, a subset $\mathcal{B}$ of $\mathcal{A}$, and a unary predicate $\mathcal{P}$, and states that:

$\mathcal{P}[\mathcal{B}]$

provided the following conditions are met:

- $\mathcal{B}$ is finite,
- $\mathcal{P}[\emptyset_{\mathcal{A}}]$, and
- For every element x of $\mathcal{A}$ and for every subset B of $\mathcal{A}$ such that $x \in \mathcal{B}$ and $B \subseteq \mathcal{B}$ and $\mathcal{P}[B]$ holds $\mathcal{P}[B \cup \{x\}]$.

One can prove the following proposition

(2) Let F be a non empty poset and A be a subset of F. Suppose A is finite and $A \neq \emptyset$ and for all elements B, C of F such that $B \in A$ and $C \in A$ holds $B \leq C$ or $C \leq B$. Then there exists an element m of F such that $m \in A$ and for every element C of F such that $C \in A$ holds $m \leq C$.

Let X be a non empty set and let F be a subset of $\mathrm{Fin}\,X$ with a non-empty element. Note that there exists an element of F which is finite and non empty.

Let P be a non empty poset, let A be a non empty finite subset of P, and let x be an element of P. One can check that $\mathrm{InitSegm}(A,x)$ is finite.

The following proposition is true

(3) For all finite sets A, B such that $A \subseteq B$ and $\mathrm{card}\,A = \mathrm{card}\,B$ holds $A = B$.

Let X be a set, let A be a finite subset of X, and let R be an order in X. Let us assume that R linearly orders A. The functor $\mathrm{SgmX}(R,A)$ yielding a finite sequence of elements of X is defined by the conditions (Def. 2).

(Def. 2)(i) $\mathrm{rng}\,\mathrm{SgmX}(R,A) = A$, and

(ii) for all natural numbers n, m such that $n \in \mathrm{dom}\,\mathrm{SgmX}(R,A)$ and $m \in \mathrm{dom}\,\mathrm{SgmX}(R,A)$ and $n < m$ holds $(\mathrm{SgmX}(R,A))_n \neq (\mathrm{SgmX}(R,A))_m$ and $\langle (\mathrm{SgmX}(R,A))_n, (\mathrm{SgmX}(R,A))_m \rangle \in R$.

We now state the proposition

(4) Let X be a set, A be a finite subset of X, R be an order in X, and f be a finite sequence of elements of X. Suppose $\mathrm{rng}\,f = A$ and for all natural numbers n, m such that $n \in \mathrm{dom}\,f$ and $m \in \mathrm{dom}\,f$ and $n < m$ holds $f_n \neq f_m$ and $\langle f_n, f_m \rangle \in R$. Then $f = \mathrm{SgmX}(R,A)$.

2. Abstract Complexes

Let C be a non empty poset. The functor $\mathrm{symplexes}(C)$ yields a subset of Fin (the carrier of C) and is defined as follows:

(Def. 3) $\mathrm{symplexes}(C) = \{A; A$ ranges over elements of Fin (the carrier of C): the internal relation of Clinearly orders $A\}$.

Let C be a non empty poset. Note that $\mathrm{symplexes}(C)$ has a non-empty element.

In the sequel C is a non empty poset.

One can prove the following propositions:

(5) For every element x of C holds $\{x\} \in \mathrm{symplexes}(C)$.

(6) $\emptyset \in \mathrm{symplexes}(C)$.

(7) For all sets x, s such that $x \subseteq s$ and $s \in \mathrm{symplexes}(C)$ holds $x \in \mathrm{symplexes}(C)$.

Let X be a set and let F be a non empty subset of $\mathrm{Fin}\,X$. Observe that every element of F is finite.

Let X be a set and let F be a non empty subset of $\mathrm{Fin}\,X$. We see that the element of F is a subset of X.

Next we state three propositions:

(8) Let X be a set, A be a finite subset of X, and R be an order in X. If R linearly orders A, then $\mathrm{SgmX}(R,A)$ is one-to-one.

(9) Let X be a set, A be a finite subset of X, and R be an order in X. If R linearly orders A, then $\operatorname{len}\operatorname{SgmX}(R,A) = \overline{\overline{A}}$.

(10) Let C be a non empty poset and A be a non empty element of symplexes(C). If $\overline{\overline{A}} = n$, then $\operatorname{dom}\operatorname{SgmX}(\text{the internal relation of } C, A) = \operatorname{Seg} n$.

Let C be a non empty poset. One can check that there exists an element of symplexes(C) which is non empty.

3. Triangulations

A set sequence is a many sorted set indexed by $\mathbb{N}$.

Let I_1 be a set sequence. We say that I_1 is lower non-empty if and only if:

(Def. 4) For every n such that $I_1(n)$ is non empty and for every m such that $m < n$ holds $I_1(m)$ is non empty.

Let us note that there exists a set sequence which is lower non-empty.

Let X be a set sequence. The functor FuncsSeq(X) yielding a set sequence is defined as follows:

(Def. 5) For every natural number n holds $(\operatorname{FuncsSeq}(X))(n) = X(n)^{X(n+1)}$.

Let X be a lower non-empty set sequence and let n be a natural number. One can check that $(\operatorname{FuncsSeq}(X))(n)$ is non empty.

Let us consider n and let f be an element of $(\operatorname{Seg}(n+1))^{\operatorname{Seg} n}$. The functor ${}^{@}f$ yields a finite sequence of elements of $\mathbb{R}$ and is defined as follows:

(Def. 6) ${}^{@}f = f$.

The set sequence NatEmbSeq is defined by:

(Def. 7) For every natural number n holds $(\operatorname{NatEmbSeq})(n) = \{f;f$ ranges over elements of $(\operatorname{Seg}(n+1))^{\operatorname{Seg} n}$: ${}^{@}f$ is increasing$\}$.

Let us consider n. One can verify that $(\operatorname{NatEmbSeq})(n)$ is non empty.

Let n be a natural number. Observe that every element of $(\operatorname{NatEmbSeq})(n)$ is function-like and relation-like.

Let X be a set sequence. A triangulation of X is a many sorted function from NatEmbSeq into FuncsSeq(X).

We introduce triangulation structures which are systems

⟨ a skeleton sequence, a faces assignment ⟩,

where the skeleton sequence is a set sequence and the faces assignment is a many sorted function from NatEmbSeq into FuncsSeq(the skeleton sequence).

Let T be a triangulation structure. We say that T is lower non-empty if and only if:

(Def. 9)[1] The skeleton sequence of T is lower non-empty.

One can check that there exists a triangulation structure which is lower non-empty and strict.

Let T be a lower non-empty triangulation structure. Note that the skeleton sequence of T is lower non-empty.

Let S be a lower non-empty set sequence and let F be a many sorted function from NatEmbSeq into FuncsSeq(S). Observe that $\langle S, F\rangle$ is lower non-empty.

[1] The definition (Def. 8) has been removed.

4. Relationship Between Abstract Complexes and Triangulations

Let T be a triangulation structure and let n be a natural number. A symplex of T and n is an element of (the skeleton sequence of T)(n).

Let n be a natural number. A face of n is an element of $(\mathrm{NatEmbSeq})(n)$.

Let T be a lower non-empty triangulation structure, let n be a natural number, let x be a symplex of T and $n+1$, and let f be a face of n. Let us assume that (the skeleton sequence of T)$(n+1) \neq \emptyset$. The functor $\mathrm{face}(x, f)$ yielding a symplex of T and n is defined by:

(Def. 10) For all functions F, G such that $F =$ (the faces assignment of T)(n) and $G = F(f)$ holds $\mathrm{face}(x, f) = G(x)$.

Let C be a non empty poset. The functor $\mathrm{Triang}(C)$ yields a lower non-empty strict triangulation structure and is defined by the conditions (Def. 11).

(Def. 11)(i) (The skeleton sequence of $\mathrm{Triang}(C)$)$(0) = \{\emptyset\}$,

(ii) for every natural number n such that $n > 0$ holds (the skeleton sequence of $\mathrm{Triang}(C)$)$(n) =$ $\{\mathrm{SgmX}($the internal relation of C, $A)$;A ranges over non empty elements of $\mathrm{symplexes}(C)$: $\overline{\overline{A}} = n\}$, and

(iii) for every natural number n and for every face f of n and for every element s of (the skeleton sequence of $\mathrm{Triang}(C)$)$(n+1)$ such that $s \in$ (the skeleton sequence of $\mathrm{Triang}(C)$)$(n+1)$ and for every non empty element A of $\mathrm{symplexes}(C)$ such that $\mathrm{SgmX}($the internal relation of C, $A) = s$ holds $\mathrm{face}(s, f) = \mathrm{SgmX}($the internal relation of $C, A) \cdot f$.

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