JOURNAL OF FORMALIZED MATHEMATICS
Volume 7, Released 1995, Published 2003
Inst. of Computer Science, Univ. of Białystok

Replacement of Subtrees in a Tree

Oleg Okhotnikov
Ural University
Ekaterinburg

Summary. This paper is based on previous works [1], [2] in which the operation replacement of subtree in a tree has been defined. We extend this notion for arbitrary non empty antichain.

MML Identifier: `TREES_A`.

WWW: `http://mizar.org/JFM/Vol7/trees_a.html`

The articles [5], [7], [6], [8], [4], [3], [1], and [2] provide the notation and terminology for this paper.

For simplicity, we follow the rules: T, T_1 denote trees, P denotes an antichain of prefixes of T, p, q, r denote finite sequences of elements of $\mathbb{N}$, and t denotes an element of T.

The following proposition is true

(1) For all finite sequences p, q, r, s such that $p \frown q = s \frown r$ holds p and s are $\subseteq$-comparable.

Let us consider T, T_1 and let us consider P. Let us assume that $P \neq \emptyset$. The functor $\overbrace{T,P,T_1}$ yielding a tree is defined as follows:

(Def. 1) $q \in \overbrace{T,P,T_1}$ iff $q \in T$ and for every p such that $p \in P$ holds $p \not\prec q$ or there exist p, r such that $p \in P$ and $r \in T_1$ and $q = p \frown r$.

We now state several propositions:

(2) Suppose $P \neq \emptyset$. Then $\overbrace{T,P,T_1} = \{t_1; t_1$ ranges over elements of T: $\bigwedge_p (p \in P \Rightarrow p \not\prec t_1)\} \cup \{p \frown s; p$ ranges over elements of T, s ranges over elements of T_1: $p \in P\}$.

(3) $\{t_1; t_1$ ranges over elements of T: $\bigwedge_p (p \in P \Rightarrow p \not\preceq t_1)\} \subseteq \{t_1; t_1$ ranges over elements of T: $\bigwedge_p (p \in P \Rightarrow p \not\prec t_1)\}$.

(4) $P \subseteq \{t_1; t_1$ ranges over elements of T: $\bigwedge_p (p \in P \Rightarrow p \not\prec t_1)\}$.

(5) $\{t_1; t_1$ ranges over elements of T: $\bigwedge_p (p \in P \Rightarrow p \not\prec t_1)\} \setminus \{t_1; t_1$ ranges over elements of T: $\bigwedge_p (p \in P \Rightarrow p \not\preceq t_1)\} = P$.

(6) For all T, T_1, P holds $P \subseteq \{p \frown s; p$ ranges over elements of T, s ranges over elements of T_1: $p \in P\}$.

(7) Suppose $P \neq \emptyset$. Then $\overbrace{T,P,T_1} = \{t_1; t_1$ ranges over elements of T: $\bigwedge_p (p \in P \Rightarrow p \not\preceq t_1)\} \cup \{p \frown s; p$ ranges over elements of T, s ranges over elements of T_1: $p \in P\}$.

(9)[1] If $p \in P$, then $T_1 = \overbrace{T, P, T_1} \restriction p$.

Let us consider T. Observe that there exists an antichain of prefixes of T which is non empty.

Let us consider T and let t be an element of T. Then $\{t\}$ is a non empty antichain of prefixes of T.

One can prove the following proposition

(10) $\overbrace{T, \{t\}, T_1} = T$ with-replacement(t, T_1).

In the sequel T, T_1 are decorated trees, P is an antichain of prefixes of $\operatorname{dom} T$, and t is an element of $\operatorname{dom} T$.

Let us consider T, P, T_1. Let us assume that $P \neq \emptyset$. The functor $\overbrace{T, P, T_1}$ yielding a decorated tree is defined by the conditions (Def. 2).

(Def. 2)(i) $\operatorname{dom} \overbrace{T, P, T_1} = \overbrace{\operatorname{dom} T, P, \operatorname{dom} T_1}$, and

(ii) for every q such that $q \in \overbrace{\operatorname{dom} T, P, \operatorname{dom} T_1}$ holds for every p such that $p \in P$ holds $p \not\preceq q$ and $\overbrace{T, P, T_1}(q) = T(q)$ or there exist p, r such that $p \in P$ and $r \in \operatorname{dom} T_1$ and $q = p \frown r$ and $\overbrace{T, P, T_1}(q) = T_1(r)$.

One can prove the following propositions:

(13)[2] Suppose $P \neq \emptyset$. Let given q. Suppose $q \in \operatorname{dom} \overbrace{T, P, T_1}$. Then for every p such that $p \in P$ holds $p \not\preceq q$ and $\overbrace{T, P, T_1}(q) = T(q)$ or there exist p, r such that $p \in P$ and $r \in \operatorname{dom} T_1$ and $q = p \frown r$ and $\overbrace{T, P, T_1}(q) = T_1(r)$.

(14) Suppose $p \in \operatorname{dom} T$. Let given q. Suppose $q \in \operatorname{dom}(T$ with-replacement$(p, T_1))$. Then $p \not\preceq q$ and $(T$ with-replacement$(p, T_1))(q) = T(q)$ or there exists r such that $r \in \operatorname{dom} T_1$ and $q = p \frown r$ and $(T$ with-replacement$(p, T_1))(q) = T_1(r)$.

(15) Suppose $P \neq \emptyset$. Let given q. Suppose $q \in \operatorname{dom} \overbrace{T, P, T_1}$ and $q \in \{t_1; t_1$ ranges over elements of $\operatorname{dom} T : \bigwedge_p (p \in P \Rightarrow p \not\preceq t_1)\}$. Then $\overbrace{T, P, T_1}(q) = T(q)$.

(16) If $p \in \operatorname{dom} T$, then for every q such that $q \in \operatorname{dom}(T$ with-replacement$(p, T_1))$ and $q \in \{t_1; t_1$ ranges over elements of $\operatorname{dom} T : p \not\preceq t_1\}$ holds $(T$ with-replacement$(p, T_1))(q) = T(q)$.

(17) Let given q. Suppose $q \in \operatorname{dom} \overbrace{T, P, T_1}$ and $q \in \{p \frown s; p$ ranges over elements of $\operatorname{dom} T, s$ ranges over elements of $\operatorname{dom} T_1 : p \in P\}$. Then there exists an element p' of $\operatorname{dom} T$ and there exists an element r of $\operatorname{dom} T_1$ such that $q = p' \frown r$ and $p' \in P$ and $\overbrace{T, P, T_1}(q) = T_1(r)$.

(18) Suppose $p \in \operatorname{dom} T$. Let given q. Suppose $q \in \operatorname{dom}(T$ with-replacement$(p, T_1))$ and $q \in \{p \frown s; s$ ranges over elements of $\operatorname{dom} T_1 : s = s\}$. Then there exists an element r of $\operatorname{dom} T_1$ such that $q = p \frown r$ and $(T$ with-replacement$(p, T_1))(q) = T_1(r)$.

(19) $\overbrace{T, \{t\}, T_1} = T$ with-replacement(t, T_1).

In the sequel D denotes a non empty set, T, T_1 denote trees decorated with elements of D, and P denotes an antichain of prefixes of $\operatorname{dom} T$.

Let us consider D, T, P, T_1. Let us assume that $P \neq \emptyset$. The functor $\overbrace{T, P, T_1}$ yields a tree decorated with elements of D and is defined by:

(Def. 3) $\overbrace{T, P, T_1} = \overbrace{T, P, T_1}$.

[1] The proposition (8) has been removed.

[2] The propositions (11) and (12) have been removed.

ACKNOWLEDGMENTS

The author wishes to thank to G. Bancerek for his assistance during the preparation of this paper.

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Received October 1, 1995

Published January 2, 2004