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# Translations in Affine Planes [1]

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**Summary.** Connections between Minor Desargues Axiom and the transitivity of translation groups are investigated. A formal proof of the theorem which establishes the equivalence of these two properties of affine planes is given. We also prove that, under additional requirement, the plane in question satisfies Fano Axiom; its translation group is uniquely two-divisible.

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The articles [2], [1], [3], [5], [6], [4], and [7] provide the notation and terminology for this paper.

We follow the rules: $A_1$ is an affine space and $a$, $b$, $c$, $d$, $p$, $q$, $r$ are elements of $A_1$.

Let us consider $A_1$. We say that $A_1$ is Fanoian if and only if:

(Def. 1) For all $a$, $b$, $c$, $d$ such that $a,b \nparallel c,d$ and $a,c \nparallel b,d$ and $a,d \nparallel b,c$ holds $\mathbf{L}(a,b,c)$.

We introduce $A_1$ satisfies Fano Axiom as a synonym of $A_1$ is Fanoian.

We now state the proposition

(2)[1] Given $a$, $b$, $c$ such that $\mathbf{L}(a,b,c)$ and $a \neq b$ and $a \neq c$ and $b \neq c$. Let given $p$, $q$. If $p \neq q$, then there exists $r$ such that $\mathbf{L}(p,q,r)$ and $p \neq r$ and $q \neq r$.

We adopt the following rules: $A_2$ is an affine plane, $a$, $a'$, $b$, $b'$, $c$, $c'$, $d$, $p$, $q$, $r$, $x$, $y$ are elements of $A_2$, and $f$, $g$, $f_1$, $f_2$ are permutations of the carrier of $A_2$.

We now state a number of propositions:

(4)[2] If $A_2$ satisfies Fano Axiom and $a,b \nparallel c,d$ and $a,c \nparallel b,d$ and not $\mathbf{L}(a,b,c)$, then there exists $p$ such that $\mathbf{L}(b,c,p)$ and $\mathbf{L}(a,d,p)$.

(5) If $f$ is a translation and not $\mathbf{L}(a,f(a),x)$ and $a,f(a) \nparallel x,y$ and $a,x \nparallel f(a),y$, then $y = f(x)$.

(6) $A_2$ satisfies **des** if and only if for all $a$, $a'$, $b$, $c$, $b'$, $c'$ such that not $\mathbf{L}(a,a',b)$ and not $\mathbf{L}(a,a',c)$ and $a,a' \nparallel b,b'$ and $a,a' \nparallel c,c'$ and $a,b \nparallel a',b'$ and $a,c \nparallel a',c'$ holds $b,c \nparallel b',c'$.

(7) There exists $f$ such that $f$ is a translation and $f(a) = a$.

(8) If for all $p$, $q$, $r$ such that $p \neq q$ and $\mathbf{L}(p,q,r)$ holds $r = p$ or $r = q$ and $a,b \nparallel p,q$ and $a,p \nparallel b,q$ and not $\mathbf{L}(a,b,p)$, then $a,q \nparallel b,p$.

(9) If $A_2$ satisfies **des**, then there exists $f$ such that $f$ is a translation and $f(a) = b$.

[1] Supported by RPBP.III-24.C2.

[1] The proposition (1) has been removed.

[2] The proposition (3) has been removed.



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(10) If for all $a$, $b$ there exists $f$ such that $f$ is a translation and $f(a) = b$, then $A_2$ satisfies **des**.

(11) If $f$ is a translation and $g$ is a translation and not $\mathbf{L}(a, f(a), g(a))$, then $f \cdot g = g \cdot f$.

(12) If $A_2$ satisfies **des** and $f$ is a translation and $g$ is a translation, then $f \cdot g = g \cdot f$.

(13) If $f$ is a translation and $g$ is a translation and $p, f(p) \nparallel p, g(p)$, then $p, f(p) \nparallel p, (f \cdot g)(p)$.

(14) Suppose $A_2$ satisfies Fano Axiom and $A_2$ satisfies **des** and $f$ is a translation. Then there exists $g$ such that $g$ is a translation and $g \cdot g = f$.

(15) If $A_2$ satisfies Fano Axiom and $f$ is a translation and $f \cdot f = \mathrm{id}_{\text{the carrier of } A_2}$, then $f = \mathrm{id}_{\text{the carrier of } A_2}$.

(16) Suppose that $A_2$ satisfies **des** and $A_2$ satisfies Fano Axiom and $g$ is a translation and $f_1$ is a translation and $f_2$ is a translation and $g = f_1 \cdot f_1$ and $g = f_2 \cdot f_2$. Then $f_1 = f_2$.

## REFERENCES


[1] Czesław Byliński. Functions from a set to a set. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_2.html.

[2] Czesław Byliński. Partial functions. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/partfun1.html.

[3] Henryk Oryszczyszyn and Krzysztof Prażmowski. Analytical ordered affine spaces. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/analoaf.html.

[4] Henryk Oryszczyszyn and Krzysztof Prażmowski. Classical configurations in affine planes. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/aff_2.html.

[5] Henryk Oryszczyszyn and Krzysztof Prażmowski. Ordered affine spaces defined in terms of directed parallelity — part I. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/diraf.html.

[6] Henryk Oryszczyszyn and Krzysztof Prażmowski. Parallelity and lines in affine spaces. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/aff_1.html.

[7] Henryk Oryszczyszyn and Krzysztof Prażmowski. Transformations in affine spaces. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/transgeo.html.


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