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More on the Finite Sequences on the Plane[1]

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Summary. We continue proving lemmas needed for the proof of the Jordan curve theorem. The main goal was to prove the last theorem being a mutation of the first theorem in [12].

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The articles [18], [1], [15], [9], [16], [4], [2], [5], [19], [10], [13], [8], [21], [3], [17], [6], [7], [11], [14], and [20] provide the notation and terminology for this paper.

1. PRELIMINARIES

One can prove the following proposition

(1) For all sets A, x, y such that $A \subseteq \{x, y\}$ and $x \in A$ and $y \notin A$ holds $A = \{x\}$.

Let us observe that there exists a function which is trivial.

2. FINITE SEQUENCES

We use the following convention: G is a Go-board and i, j, k, m, n are natural numbers.

Let us observe that there exists a finite sequence which is non constant.

Next we state a number of propositions:

(2) For every non trivial finite sequence f holds $1 < \operatorname{len} f$.

(3) For every non trivial set D and for every non constant circular finite sequence f of elements of D holds $\operatorname{len} f > 2$.

(4) For every finite sequence f and for every set x holds $x \in \operatorname{rng} f$ or $x \hookleftarrow f = 0$.

(5) Let p be a set, D be a non empty set, f be a non empty finite sequence of elements of D, and g be a finite sequence of elements of D. If $p \hookleftarrow f = \operatorname{len} f$, then $f \frown g \rightarrow p = g$.

(6) For every non empty set D and for every non empty one-to-one finite sequence f of elements of D holds $f_{\operatorname{len} f} \hookleftarrow f = \operatorname{len} f$.

(7) For all finite sequences f, g holds $\operatorname{len} f \leq \operatorname{len}(f \frown\frown g)$.

(8) For all finite sequences f, g and for every set x such that $x \in \operatorname{rng} f$ holds $x \hookleftarrow f = x \hookleftarrow (f \frown \frown g)$.

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(9) For every non empty finite sequence f and for every finite sequence g holds $\operatorname{len} g \leq \operatorname{len}(f \frown g)$.

(10) For all finite sequences f, g holds $\operatorname{rng} f \subseteq \operatorname{rng}(f \frown\frown g)$.

(11) Let D be a non empty set, f be a non empty finite sequence of elements of D, and g be a non trivial finite sequence of elements of D. If $g_{\operatorname{len} g} = f_1$, then $f \frown\frown g$ is circular.

(12) Let D be a non empty set, M be a matrix over D, f be a finite sequence of elements of D, and g be a non empty finite sequence of elements of D. Suppose $f_{\operatorname{len} f} = g_1$ and f is a sequence which elements belong to M and g is a sequence which elements belong to M. Then $f \frown\frown g$ is a sequence which elements belong to M.

(13) For every set D and for every finite sequence f of elements of D such that $1 \leq k$ holds $\langle f(k+1), \ldots, f(\operatorname{len} f)\rangle = f_{\lfloor k}$.

(14) For every set D and for every finite sequence f of elements of D such that $k \leq \operatorname{len} f$ holds $\langle f(1), \ldots, f(k)\rangle = f \restriction k$.

(15) Let p be a set, D be a non empty set, f be a non empty finite sequence of elements of D, and g be a finite sequence of elements of D. If $p \leftarrow\!\!\!\!\!\!\,^{\circ} f = \operatorname{len} f$, then $f \frown g \leftarrow p = \langle f(1), \ldots, f(\operatorname{len} f -' 1)\rangle$.

(16) Let D be a non empty set and f, g be non empty finite sequences of elements of D. If $g_1 \leftarrow\!\!\!\!\!\!\,^{\circ} f = \operatorname{len} f$, then $(f \frown\frown g) :- g_1 = g$.

(17) Let D be a non empty set and f, g be non empty finite sequences of elements of D. If $g_1 \leftarrow\!\!\!\!\!\!\,^{\circ} f = \operatorname{len} f$, then $(f \frown\frown g) -: g_1 = f$.

(18) Let D be a non trivial set, f be a non empty finite sequence of elements of D, and g be a non trivial finite sequence of elements of D. Suppose $g_1 = f_{\operatorname{len} f}$ and for every i such that $1 \leq i$ and $i < \operatorname{len} f$ holds $f_i \neq g_1$. Then $f \frown\frown g \circlearrowleft g_1 = g \frown\frown f$.

3. On the Plane

Next we state several propositions:

(19) For every non trivial finite sequence f of elements of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathcal{L}(f,1) = \widetilde{\mathcal{L}}(f \restriction 2)$.

(20) For every s.c.c. finite sequence f of elements of $\mathcal{E}^2_{\mathrm{T}}$ and for every n such that $n < \operatorname{len} f$ holds $f \restriction n$ is s.n.c..

(21) For every s.c.c. finite sequence f of elements of $\mathcal{E}^2_{\mathrm{T}}$ and for every n such that $1 \leq n$ holds $f_{\lfloor n}$ is s.n.c..

(22) Let f be a circular s.c.c. finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ and given n. If $n < \operatorname{len} f$ and $\operatorname{len} f > 4$, then $f \restriction n$ is one-to-one.

(23) Let f be a circular s.c.c. finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $\operatorname{len} f > 4$. Let i, j be natural numbers. If $1 < i$ and $i < j$ and $j \leq \operatorname{len} f$, then $f_i \neq f_j$.

(24) Let f be a circular s.c.c. finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ and given n. If $1 \leq n$ and $\operatorname{len} f > 4$, then $f_{\lfloor n}$ is one-to-one.

(25) For every special non empty finite sequence f of elements of $\mathcal{E}^2_{\mathrm{T}}$ holds $\langle f(m), \ldots, f(n)\rangle$ is special.

(26) Let f be a special non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ and g be a special non trivial finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. If $f_{\operatorname{len} f} = g_1$, then $f \frown\frown g$ is special.

(27) For every circular unfolded s.c.c. finite sequence f of elements of $\mathcal{E}^2_{\mathrm{T}}$ such that $\operatorname{len} f > 4$ holds $\mathcal{L}(f,1) \cap \widetilde{\mathcal{L}}(f_{\lfloor 1}) = \{f_1, f_2\}$.

Let us note that there exists a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ which is one-to-one, special, unfolded, s.n.c., and non empty.

We now state several propositions:

(28) For all finite sequences f, g of elements of $\mathcal{E}^2_{\mathrm{T}}$ such that $j < \operatorname{len} f$ holds $\mathcal{L}(f \frown g, j) = \mathcal{L}(f, j)$.

(29) For all non empty finite sequences f, g of elements of $\mathcal{E}^2_{\mathrm{T}}$ such that $1 \leq j$ and $j+1 < \operatorname{len} g$ holds $\mathcal{L}(f \frown g, \operatorname{len} f + j) = \mathcal{L}(g, j+1)$.

(30) Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ and g be a non trivial finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. If $f_{\operatorname{len} f} = g_1$, then $\mathcal{L}(f \frown g, \operatorname{len} f) = \mathcal{L}(g, 1)$.

(31) Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ and g be a non trivial finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. If $j+1 < \operatorname{len} g$ and $f_{\operatorname{len} f} = g_1$, then $\mathcal{L}(f \frown g, \operatorname{len} f + j) = \mathcal{L}(g, j+1)$.

(32) Let f be a non empty s.n.c. unfolded finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ and given i. If $1 \leq i$ and $i < \operatorname{len} f$, then $\mathcal{L}(f, i) \cap \operatorname{rng} f = \{f_i, f_{i+1}\}$.

(33) Let f, g be non trivial s.n.c. one-to-one unfolded finite sequences of elements of $\mathcal{E}^2_{\mathrm{T}}$. If $\widetilde{\mathcal{L}}(f) \cap \widetilde{\mathcal{L}}(g) = \{f_1, g_1\}$ and $f_1 = g_{\operatorname{len} g}$ and $g_1 = f_{\operatorname{len} f}$, then $f \frown g$ is s.c.c..

In the sequel f, g are finite sequences of elements of $\mathcal{E}^2_{\mathrm{T}}$.

The following three propositions are true:

(34) If f is unfolded and g is unfolded and $f_{\operatorname{len} f} = g_1$ and $\mathcal{L}(f, \operatorname{len} f -' 1) \cap \mathcal{L}(g, 1) = \{f_{\operatorname{len} f}\}$, then $f \frown g$ is unfolded.

(35) If f is non empty and g is non trivial and $f_{\operatorname{len} f} = g_1$, then $\widetilde{\mathcal{L}}(f \frown g) = \widetilde{\mathcal{L}}(f) \cup \widetilde{\mathcal{L}}(g)$.

(36) Suppose that

(i) for every n such that $n \in \operatorname{dom} f$ there exist i, j such that $\langle i, j \rangle \in$ the indices of G and $f_n = G \circ (i, j)$,

(ii) f is non constant, circular, unfolded, s.c.c., and special, and

(iii) $\operatorname{len} f > 4$.

Then there exists g such that

(iv) g is a sequence which elements belong to G, unfolded, s.c.c., and special,

(v) $\widetilde{\mathcal{L}}(f) = \widetilde{\mathcal{L}}(g)$,

(vi) $f_1 = g_1$,

(vii) $f_{\operatorname{len} f} = g_{\operatorname{len} g}$, and

(viii) $\operatorname{len} f \leq \operatorname{len} g$.

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