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Homeomorphism between [:$\mathcal{E}^i_{\mathrm{T}}$, $\mathcal{E}^j_{\mathrm{T}}$:] and $\mathcal{E}^{i+j}_{\mathrm{T}}$

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Summary. In this paper we introduce the cartesian product of two metric spaces. As the distance between two points in the product we take maximal distance between coordinates of these points. In the main theorem we show the homeomorphism between [:$\mathcal{E}^i_{\mathrm{T}}$, $\mathcal{E}^j_{\mathrm{T}}$:] and $\mathcal{E}^{i+j}_{\mathrm{T}}$.

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The articles [15], [7], [20], [21], [5], [6], [4], [16], [14], [18], [11], [19], [1], [2], [9], [13], [3], [17], [12], [10], [8], and [22] provide the notation and terminology for this paper.

We adopt the following convention: i, j, n denote natural numbers, f, g, h, k denote finite sequences of elements of $\mathbb{R}$, and M, N denote non empty metric spaces.

The following propositions are true:

(1) For all real numbers a, b such that $\max(a,b) \leq a$ holds $\max(a,b) = a$.

(2) For all real numbers a, b, c, d holds $\max(a+c, b+d) \leq \max(a,b) + \max(c,d)$.

(3) For all real numbers a, b, c, d, e, f such that $a \leq b+c$ and $d \leq e+f$ holds $\max(a,d) \leq \max(b,e) + \max(c,f)$.

(4) For all finite sequences f, g holds $\mathrm{dom}\, g \subseteq \mathrm{dom}(f \frown g)$.

(5) For all finite sequences f, g such that $\mathrm{len}\, f < i$ and $i \leq \mathrm{len}\, f + \mathrm{len}\, g$ holds $i - \mathrm{len}\, f \in \mathrm{dom}\, g$.

(6) For all finite sequences f, g, h, k such that $f \frown g = h \frown k$ and $\mathrm{len}\, f = \mathrm{len}\, h$ and $\mathrm{len}\, g = \mathrm{len}\, k$ holds $f = h$ and $g = k$.

(7) If $\mathrm{len}\, f = \mathrm{len}\, g$ or $\mathrm{dom}\, f = \mathrm{dom}\, g$, then $\mathrm{len}(f+g) = \mathrm{len}\, f$ and $\mathrm{dom}(f+g) = \mathrm{dom}\, f$.

(8) If $\mathrm{len}\, f = \mathrm{len}\, g$ or $\mathrm{dom}\, f = \mathrm{dom}\, g$, then $\mathrm{len}(f-g) = \mathrm{len}\, f$ and $\mathrm{dom}(f-g) = \mathrm{dom}\, f$.

(9) $\mathrm{len}\, f = \mathrm{len}\, ^2 f$ and $\mathrm{dom}\, f = \mathrm{dom}\, ^2 f$.

(10) $\mathrm{len}\, f = \mathrm{len}|f|$ and $\mathrm{dom}\, f = \mathrm{dom}|f|$.

(11) $^2(f \frown g) = (^2 f) \frown (^2 g)$.

(12) $|f \frown g| = |f| \frown |g|$.

(13) If $\mathrm{len}\, f = \mathrm{len}\, h$ and $\mathrm{len}\, g = \mathrm{len}\, k$, then $^2(f \frown g + h \frown k) = (^2(f+h)) \frown (^2(g+k))$.

(14) If $\mathrm{len}\, f = \mathrm{len}\, h$ and $\mathrm{len}\, g = \mathrm{len}\, k$, then $|f \frown g + h \frown k| = |f+h| \frown |g+k|$.

(15) If $\mathrm{len}\, f = \mathrm{len}\, h$ and $\mathrm{len}\, g = \mathrm{len}\, k$, then $^2(f \frown g - h \frown k) = (^2(f-h)) \frown (^2(g-k))$.

(16) If $\mathrm{len}\, f = \mathrm{len}\, h$ and $\mathrm{len}\, g = \mathrm{len}\, k$, then $|f \frown g - h \frown k| = |f - h| \frown |g - k|$.

(17) If $\mathrm{len}\, f = n$, then $f \in$ the carrier of $\mathcal{E}^n$.

(18) If $\mathrm{len}\, f = n$, then $f \in$ the carrier of $\mathcal{E}^n_{\mathrm{T}}$.

(19) For every finite sequence f such that $f \in$ the carrier of $\mathcal{E}^n$ holds $\mathrm{len}\, f = n$.

Let M, N be non empty metric structures. The functor max-Prod2(M,N) yielding a strict metric structure is defined by the conditions (Def. 1).

(Def. 1)(i) The carrier of max-Prod2$(M,N) = [\!:$the carrier of M, the carrier of $N\!:\!]$, and

(ii) for all points x, y of max-Prod2(M,N) there exist points x_1, y_1 of M and there exist points x_2, y_2 of N such that $x = \langle x_1, x_2 \rangle$ and $y = \langle y_1, y_2 \rangle$ and (the distance of max-Prod2(M,N))$(x, y) = \max($(the distance of $M)(x_1, y_1)$, (the distance of $N)(x_2, y_2))$.

Let M, N be non empty metric structures. Observe that max-Prod2(M,N) is non empty.

Let M, N be non empty metric structures, let x be a point of M, and let y be a point of N. Then $\langle x, y \rangle$ is an element of max-Prod2(M,N).

Let M, N be non empty metric structures and let x be a point of max-Prod2(M,N). Then $x_{\mathbf{1}}$ is an element of M. Then $x_{\mathbf{2}}$ is an element of N.

The following three propositions are true:

(20) Let M, N be non empty metric structures, m_1, m_2 be points of M, and n_1, n_2 be points of N. Then $\rho(\langle m_1, n_1 \rangle, \langle m_2, n_2 \rangle) = \max(\rho(m_1, m_2), \rho(n_1, n_2))$.

(21) For all non empty metric structures M, N and for all points m, n of max-Prod2(M,N) holds $\rho(m,n) = \max(\rho(m_{\mathbf{1}}, n_{\mathbf{1}}), \rho(m_{\mathbf{2}}, n_{\mathbf{2}}))$.

(22) For all Reflexive non empty metric structures M, N holds max-Prod2(M,N) is Reflexive.

Let M, N be Reflexive non empty metric structures. Note that max-Prod2(M,N) is Reflexive.

The following proposition is true

(23) For all symmetric non empty metric structures M, N holds max-Prod2(M,N) is symmetric.

Let M, N be symmetric non empty metric structures. One can verify that max-Prod2(M,N) is symmetric.

The following proposition is true

(24) For all triangle non empty metric structures M, N holds max-Prod2(M,N) is triangle.

Let M, N be triangle non empty metric structures. Note that max-Prod2(M,N) is triangle.

Let M, N be non empty metric spaces. One can check that max-Prod2(M,N) is discernible.

We now state three propositions:

(25) $[\!: M_{\mathrm{top}}, N_{\mathrm{top}} :\!] = (\text{max-Prod2}(M,N))_{\mathrm{top}}$.

(26) Suppose that

(i) the carrier of $M =$ the carrier of N,

(ii) for every point m of M and for every point n of N and for every real number r such that $r > 0$ and $m = n$ there exists a real number r_1 such that $r_1 > 0$ and $\mathrm{Ball}(n, r_1) \subseteq \mathrm{Ball}(m, r)$, and

(iii) for every point m of M and for every point n of N and for every real number r such that $r > 0$ and $m = n$ there exists a real number r_1 such that $r_1 > 0$ and $\mathrm{Ball}(m, r_1) \subseteq \mathrm{Ball}(n, r)$.

Then $M_{\mathrm{top}} = N_{\mathrm{top}}$.

(27) $[\!: \mathcal{E}^i_{\mathrm{T}}, \mathcal{E}^j_{\mathrm{T}} :\!]$ and $\mathcal{E}^{i+j}_{\mathrm{T}}$ are homeomorphic.

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