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Inst. of Computer Science, Univ. of Białystok

Properties of Subsets

Zinaida Trybulec
Warsaw University
Białystok

Summary. The text includes theorems concerning properties of subsets, and some operations on sets. The functions yielding improper subsets of a set, i.e. the empty set and the set itself are introduced. Functions and predicates introduced for sets are redefined. Some theorems about enumerated sets are proved.

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The articles [3], [2], and [1] provide the notation and terminology for this paper.

In this paper E, X, x, y are sets.

Let X be a set. Observe that 2^X is non empty.

Let us consider x. Note that $\{x\}$ is non empty. Let us consider y. Note that $\{x,y\}$ is non empty.

Let us consider X. Element of X is defined as follows:

(Def. 2)[1] (i) It $\in X$ if X is non empty,

(ii) it is empty, otherwise.

Let us consider X. A subset of X is an element of 2^X.

Let X be a non empty set. Observe that there exists a subset of X which is non empty.

Let X_1, X_2 be non empty sets. Observe that $[:X_1, X_2:]$ is non empty.

Let X_1, X_2, X_3 be non empty sets. One can verify that $[:X_1, X_2, X_3:]$ is non empty.

Let X_1, X_2, X_3, X_4 be non empty sets. Observe that $[:X_1, X_2, X_3, X_4:]$ is non empty.

Let D be a non empty set and let X be a non empty subset of D. We see that the element of X is an element of D.

Let us consider E. One can check that there exists a subset of E which is empty.

Let us consider E. The functor $\emptyset_E$ yielding an empty subset of E is defined by:

(Def. 3) $\emptyset_E = \emptyset$.

The functor Ω_E yields a subset of E and is defined by:

(Def. 4) $\Omega_E = E$.

Next we state the proposition

(4)[2] $\emptyset$ is a subset of X.

In the sequel A, B, C are subsets of E.

Next we state three propositions:

[1] The definition (Def. 1) has been removed.
[2] The propositions (1)–(3) have been removed.

(7)[3] If for every element x of E such that $x \in A$ holds $x \in B$, then $A \subseteq B$.

(8) If for every element x of E holds $x \in A$ iff $x \in B$, then $A = B$.

(10)[4] If $A \neq \emptyset$, then there exists an element x of E such that $x \in A$.

Let us consider E, A. The functor $A^{\rm c}$ yielding a subset of E is defined by:

(Def. 5) $A^{\rm c} = E \setminus A$.

Let us notice that the functor $A^{\rm c}$ is involutive. Let us consider B. Then $A \cup B$ is a subset of E. Then $A \cap B$ is a subset of E. Then $A \setminus B$ is a subset of E. Then $A \dot{-} B$ is a subset of E.

We now state a number of propositions:

(15)[5] If for every element x of E holds $x \in A$ iff $x \in B$ or $x \in C$, then $A = B \cup C$.

(16) If for every element x of E holds $x \in A$ iff $x \in B$ and $x \in C$, then $A = B \cap C$.

(17) If for every element x of E holds $x \in A$ iff $x \in B$ and $x \notin C$, then $A = B \setminus C$.

(18) If for every element x of E holds $x \in A$ iff $x \in B$ iff $x \notin C$, then $A = B \dot{-} C$.

(21)[6] $\emptyset_E = (\Omega_E)^{\rm c}$.

(22) $\Omega_E = (\emptyset_E)^{\rm c}$.

(25)[7] $A \cup A^{\rm c} = \Omega_E$.

(26) A misses $A^{\rm c}$.

(28)[8] $A \cup \Omega_E = \Omega_E$.

(29) $(A \cup B)^{\rm c} = A^{\rm c} \cap B^{\rm c}$.

(30) $(A \cap B)^{\rm c} = A^{\rm c} \cup B^{\rm c}$.

(31) $A \subseteq B$ iff $B^{\rm c} \subseteq A^{\rm c}$.

(32) $A \setminus B = A \cap B^{\rm c}$.

(33) $(A \setminus B)^{\rm c} = A^{\rm c} \cup B$.

(34) $(A \dot{-} B)^{\rm c} = A \cap B \cup A^{\rm c} \cap B^{\rm c}$.

(35) If $A \subseteq B^{\rm c}$, then $B \subseteq A^{\rm c}$.

(36) If $A^{\rm c} \subseteq B$, then $B^{\rm c} \subseteq A$.

(38)[9] $A \subseteq A^{\rm c}$ iff $A = \emptyset_E$.

(39) $A^{\rm c} \subseteq A$ iff $A = \Omega_E$.

(40) If $X \subseteq A$ and $X \subseteq A^{\rm c}$, then $X = \emptyset$.

(41) $(A \cup B)^{\rm c} \subseteq A^{\rm c}$.

(42) $A^{\rm c} \subseteq (A \cap B)^{\rm c}$.

(43) A misses B iff $A \subseteq B^{\rm c}$.

[3] The propositions (5) and (6) have been removed.
[4] The proposition (9) has been removed.
[5] The propositions (11)–(14) have been removed.
[6] The propositions (19) and (20) have been removed.
[7] The propositions (23) and (24) have been removed.
[8] The proposition (27) has been removed.
[9] The proposition (37) has been removed.

(44) A misses $B^{\rm c}$ iff $A \subseteq B$.

(46)[10] If A misses B and $A^{\rm c}$ misses $B^{\rm c}$, then $A = B^{\rm c}$.

(47) If $A \subseteq B$ and C misses B, then $A \subseteq C^{\rm c}$.

(48) If for every element a of A holds $a \in B$, then $A \subseteq B$.

(49) If for every element x of E holds $x \in A$, then $E = A$.

(50) If $E \neq \emptyset$, then for every B and for every element x of E such that $x \notin B$ holds $x \in B^{\rm c}$.

(51) For all A, B such that for every element x of E holds $x \in A$ iff $x \notin B$ holds $A = B^{\rm c}$.

(52) For all A, B such that for every element x of E holds $x \notin A$ iff $x \in B$ holds $A = B^{\rm c}$.

(53) For all A, B such that for every element x of E holds $x \in A$ iff $x \notin B$ holds $A = B^{\rm c}$.

(54) If $x \in A^{\rm c}$, then $x \notin A$.

In the sequel x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 are elements of X.
One can prove the following propositions:

(55) If $X \neq \emptyset$, then $\{x_1\}$ is a subset of X.

(56) If $X \neq \emptyset$, then $\{x_1, x_2\}$ is a subset of X.

(57) If $X \neq \emptyset$, then $\{x_1, x_2, x_3\}$ is a subset of X.

(58) If $X \neq \emptyset$, then $\{x_1, x_2, x_3, x_4\}$ is a subset of X.

(59) If $X \neq \emptyset$, then $\{x_1, x_2, x_3, x_4, x_5\}$ is a subset of X.

(60) If $X \neq \emptyset$, then $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ is a subset of X.

(61) If $X \neq \emptyset$, then $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ is a subset of X.

(62) If $X \neq \emptyset$, then $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ is a subset of X.

(63) If $x \in X$, then $\{x\}$ is a subset of X.

In this article we present several logical schemes. The scheme *Subset Ex* deals with a set $\mathcal{A}$ and a unary predicate $\mathcal{P}$, and states that:

There exists a subset X of $\mathcal{A}$ such that for every x holds $x \in X$ iff $x \in \mathcal{A}$ and $\mathcal{P}[x]$

for all values of the parameters.

The scheme *Subset Eq* deals with a set $\mathcal{A}$ and a unary predicate $\mathcal{P}$, and states that:

Let X_1, X_2 be subsets of $\mathcal{A}$. Suppose for every element y of $\mathcal{A}$ holds $y \in X_1$ iff $\mathcal{P}[y]$ and for every element y of $\mathcal{A}$ holds $y \in X_2$ iff $\mathcal{P}[y]$. Then $X_1 = X_2$

for all values of the parameters.

Let X, Y be non empty sets. Let us note that the predicate X misses Y is irreflexive. We introduce X meets Y as an antonym of X misses Y.

Let S be a set. Let us assume that *contradiction*.[11]

(Def. 6) $\mathrm{choose}(S)$ is an element of S.

[10] The proposition (45) has been removed.

[11] This definition is absolutely permissive, i.e. we assume a *contradiction*, but we are interested only in the type of the functor ‘choose’.

REFERENCES

[1] Czesław Byliński. Some basic properties of sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/zfmisc_1.html.

[2] Andrzej Trybulec. Enumerated sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/enumset1.html.

[3] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

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