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On the Components of the Complement of a Special Polygonal Curve

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Summary. By the special polygonal curve we meana simple closed curve, that is a polygone and moreover has edges parallel to axes. We continue the formalization of the Takeuti-Nakamura proof [12] of the Jordan curve theorem. In the paper we prove that the complement of the special polygonal curve consists of at least two components. With the theorem which has at most two components we completed the theorem that a special polygonal curve cuts the plane into exactly two components.

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The articles [13], [1], [3], [2], [16], [8], [11], [5], [6], [4], [15], [10], [14], [9], and [7] provide the notation and terminology for this paper.

In this paper j is a natural number.

Let T be T_2 non empty topological space. One can verify that every subset of T which is compact is also closed.

We now state several propositions:

(1) Let f be a S-sequence in $\mathbb{R}^2$ and Q be a closed subset of $\mathcal{E}^2_{\mathrm{T}}$. If $\widetilde{\mathcal{L}}(f)$ meets Q and $f_1 \notin Q$, then $\widetilde{\mathcal{L}}(\downharpoonleft f, \mathrm{FPoint}(\widetilde{\mathcal{L}}(f), f_1, f_{\mathrm{len}\, f}, Q)) \cap Q = \{\mathrm{FPoint}(\widetilde{\mathcal{L}}(f), f_1, f_{\mathrm{len}\, f}, Q)\}$.

(2) Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ and p be a point of $\mathcal{E}^2_{\mathrm{T}}$. If f is a special sequence and $p = f_{\mathrm{len}\, f}$, then $\widetilde{\mathcal{L}}(\downharpoonleft p, f) = \emptyset$.

(4)[1] Let f be a S-sequence in $\mathbb{R}^2$, p be a point of $\mathcal{E}^2_{\mathrm{T}}$, and given j. If $1 \leq j$ and $j < \mathrm{len}\, f$ and $p \in \widetilde{\mathcal{L}}(\mathrm{mid}(f, j, \mathrm{len}\, f))$, then LE f_j, p, $\widetilde{\mathcal{L}}(f)$, f_1, $f_{\mathrm{len}\, f}$.

(5) Let f be a S-sequence in $\mathbb{R}^2$, p, q be points of $\mathcal{E}^2_{\mathrm{T}}$, and given j. If $1 \leq j$ and $j < \mathrm{len}\, f$ and $p \in \mathcal{L}(f, j)$ and $q \in \mathcal{L}(p, f_{j+1})$, then LE p, q, $\widetilde{\mathcal{L}}(f)$, f_1, $f_{\mathrm{len}\, f}$.

(6) Let f be a S-sequence in $\mathbb{R}^2$ and Q be a closed subset of $\mathcal{E}^2_{\mathrm{T}}$. If $\widetilde{\mathcal{L}}(f)$ meets Q and $f_{\mathrm{len}\, f} \notin Q$, then $\widetilde{\mathcal{L}}(\downharpoonleft \mathrm{LPoint}(\widetilde{\mathcal{L}}(f), f_1, f_{\mathrm{len}\, f}, Q), f) \cap Q = \{\mathrm{LPoint}(\widetilde{\mathcal{L}}(f), f_1, f_{\mathrm{len}\, f}, Q)\}$.

(7) For every non constant standard special circular sequence f holds $\mathrm{LeftComp}(f) \neq \mathrm{RightComp}(f)$.

[1] The proposition (3) has been removed.

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