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Inst. of Computer Science, Univ. of Białystok

The Basic Properties of SCM over Ring

Artur Korniłowicz
University of Białystok

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The articles [13], [12], [17], [18], [3], [4], [11], [16], [8], [14], [15], [1], [2], [5], [6], [9], [10], and [7] provide the notation and terminology for this paper.

1. SCM over Ring

In this paper I is an element of $\mathbb{Z}_8$, S is a non empty 1-sorted structure, t is an element of S, and x is a set.

Let R be a good ring. The functor $\mathbf{SCM}(R)$ yields a strict AMI over $\{$the carrier of $R\}$ and is defined by the conditions (Def. 1).

(Def. 1) The carrier of $\mathbf{SCM}(R) = \mathbb{N}$ and the instruction counter of $\mathbf{SCM}(R) = 0$ and the instruction locations of $\mathbf{SCM}(R) = \text{Instr-Loc}_{\text{SCM}}$ and the instruction codes of $\mathbf{SCM}(R) = \mathbb{Z}_8$ and the instructions of $\mathbf{SCM}(R) = \text{Instr}_{\text{SCM}}(R)$ and the object kind of $\mathbf{SCM}(R) = \text{OK}_{\text{SCM}}(R)$ and the execution of $\mathbf{SCM}(R) = \text{Exec}_{\text{SCM}}(R)$.

Let R be a good ring. One can check that $\mathbf{SCM}(R)$ is non empty and non void.

Let R be a good ring, let s be a state of $\mathbf{SCM}(R)$, and let a be an element of $\text{Data-Loc}_{\text{SCM}}$. Then $s(a)$ is an element of R.

Let R be a good ring. An object of $\mathbf{SCM}(R)$ is called a Data-Location of R if:

(Def. 2) It $\in$ (the carrier of $\mathbf{SCM}(R)$) $\setminus$ $(\text{Instr-Loc}_{\text{SCM}} \cup \{0\})$.

For simplicity, we follow the rules: R denotes a good ring, r denotes an element of R, a, b, c, d_1, d_2 denote Data-Locations of R, and i_1 denotes an instruction-location of $\mathbf{SCM}(R)$.

We now state the proposition

(1) x is a Data-Location of R iff $x \in \text{Data-Loc}_{\text{SCM}}$.

Let R be a good ring, let s be a state of $\mathbf{SCM}(R)$, and let a be a Data-Location of R. Then $s(a)$ is an element of R.

The following propositions are true:

(2) $\langle 0, \emptyset \rangle \in \text{Instr}_{\text{SCM}}(S)$.

(3) $\langle 0, \emptyset \rangle$ is an instruction of $\mathbf{SCM}(R)$.

(4) If $x \in \{1, 2, 3, 4\}$, then $\langle x, \langle d_1, d_2 \rangle \rangle \in \text{Instr}_{\text{SCM}}(S)$.

(5) $\langle 5, \langle d_1, t \rangle \rangle \in \text{Instr}_{\text{SCM}}(S)$.

(6) $\langle 6, \langle i_1 \rangle \rangle \in \mathrm{Instr}_{\mathrm{SCM}}(S)$.

(7) $\langle 7, \langle i_1, d_1 \rangle \rangle \in \mathrm{Instr}_{\mathrm{SCM}}(S)$.

Let R be a good ring and let a, b be Data-Locations of R. The functor $a{:=}b$ yielding an instruction of $\mathbf{SCM}(R)$ is defined by:

(Def. 3) $a{:=}b = \langle 1, \langle a, b \rangle \rangle$.

The functor $\mathrm{AddTo}(a,b)$ yields an instruction of $\mathbf{SCM}(R)$ and is defined as follows:

(Def. 4) $\mathrm{AddTo}(a,b) = \langle 2, \langle a, b \rangle \rangle$.

The functor $\mathrm{SubFrom}(a,b)$ yields an instruction of $\mathbf{SCM}(R)$ and is defined as follows:

(Def. 5) $\mathrm{SubFrom}(a,b) = \langle 3, \langle a, b \rangle \rangle$.

The functor $\mathrm{MultBy}(a,b)$ yielding an instruction of $\mathbf{SCM}(R)$ is defined by:

(Def. 6) $\mathrm{MultBy}(a,b) = \langle 4, \langle a, b \rangle \rangle$.

Let R be a good ring, let a be a Data-Location of R, and let r be an element of R. The functor $a{:=}r$ yielding an instruction of $\mathbf{SCM}(R)$ is defined by:

(Def. 7) $a{:=}r = \langle 5, \langle a, r \rangle \rangle$.

Let R be a good ring and let l be an instruction-location of $\mathbf{SCM}(R)$. The functor goto l yields an instruction of $\mathbf{SCM}(R)$ and is defined as follows:

(Def. 8) $\mathrm{goto}\ l = \langle 6, \langle l \rangle \rangle$.

Let R be a good ring, let l be an instruction-location of $\mathbf{SCM}(R)$, and let a be a Data-Location of R. The functor **if** $a = 0$ **goto** l yields an instruction of $\mathbf{SCM}(R)$ and is defined as follows:

(Def. 9) **if** $a = 0$ **goto** $l = \langle 7, \langle l, a \rangle \rangle$.

One can prove the following proposition

(8) Let I be a set. Then I is an instruction of $\mathbf{SCM}(R)$ if and only if one of the following conditions is satisfied:

$I = \langle 0, \emptyset \rangle$ or there exist a, b such that $I = a{:=}b$ or there exist a, b such that $I = \mathrm{AddTo}(a,b)$ or there exist a, b such that $I = \mathrm{SubFrom}(a,b)$ or there exist a, b such that $I = \mathrm{MultBy}(a,b)$ or there exists i_1 such that $I = \mathrm{goto}\ i_1$ or there exist a, i_1 such that $I =$ **if** $a = 0$ **goto** i_1 or there exist a, r such that $I = a{:=}r$.

In the sequel s denotes a state of $\mathbf{SCM}(R)$.

Let us consider R. One can check that $\mathbf{SCM}(R)$ is IC-Ins-separated.

The following two propositions are true:

(9) $\mathbf{IC}_{\mathbf{SCM}(R)} = 0$.

(10) For every $\mathbf{SCM}$-state S over R such that $S = s$ holds $\mathbf{IC}_s = \mathbf{IC}_S$.

Let R be a good ring and let i_1 be an instruction-location of $\mathbf{SCM}(R)$. The functor $\mathrm{Next}(i_1)$ yielding an instruction-location of $\mathbf{SCM}(R)$ is defined as follows:

(Def. 10) There exists an element m_1 of $\mathrm{Instr\text{-}Loc}_{\mathrm{SCM}}$ such that $m_1 = i_1$ and $\mathrm{Next}(i_1) = \mathrm{Next}(m_1)$.

We now state two propositions:

(11) For every instruction-location i_1 of $\mathbf{SCM}(R)$ and for every element m_1 of $\mathrm{Instr\text{-}Loc}_{\mathrm{SCM}}$ such that $m_1 = i_1$ holds $\mathrm{Next}(m_1) = \mathrm{Next}(i_1)$.

(12) Let I be an instruction of $\mathbf{SCM}(R)$ and i be an element of $\mathrm{Instr}_{\mathrm{SCM}}(R)$. If $i = I$, then for every $\mathbf{SCM}$-state S over R such that $S = s$ holds $\mathrm{Exec}(I,s) = \mathrm{Exec\text{-}Res}_{\mathrm{SCM}}(i,S)$.

2. Users Guide

We now state several propositions:

(13) $(\mathrm{Exec}(a{:=}b,s))(\mathbf{IC}_{\mathbf{SCM}(R)}) = \mathrm{Next}(\mathbf{IC}_s)$ and $(\mathrm{Exec}(a{:=}b,s))(a) = s(b)$ and for every c such that $c \neq a$ holds $(\mathrm{Exec}(a{:=}b,s))(c) = s(c)$.

(14) $(\mathrm{Exec}(\mathrm{AddTo}(a,b),s))(\mathbf{IC}_{\mathbf{SCM}(R)}) = \mathrm{Next}(\mathbf{IC}_s)$ and $(\mathrm{Exec}(\mathrm{AddTo}(a,b),s))(a) = s(a) + s(b)$ and for every c such that $c \neq a$ holds $(\mathrm{Exec}(\mathrm{AddTo}(a,b),s))(c) = s(c)$.

(15) $(\mathrm{Exec}(\mathrm{SubFrom}(a,b),s))(\mathbf{IC}_{\mathbf{SCM}(R)}) = \mathrm{Next}(\mathbf{IC}_s)$ and $(\mathrm{Exec}(\mathrm{SubFrom}(a,b),s))(a) = s(a) - s(b)$ and for every c such that $c \neq a$ holds $(\mathrm{Exec}(\mathrm{SubFrom}(a,b),s))(c) = s(c)$.

(16) $(\mathrm{Exec}(\mathrm{MultBy}(a,b),s))(\mathbf{IC}_{\mathbf{SCM}(R)}) = \mathrm{Next}(\mathbf{IC}_s)$ and $(\mathrm{Exec}(\mathrm{MultBy}(a,b),s))(a) = s(a) \cdot s(b)$ and for every c such that $c \neq a$ holds $(\mathrm{Exec}(\mathrm{MultBy}(a,b),s))(c) = s(c)$.

(17) $(\mathrm{Exec}(\mathrm{goto}\ i_1,s))(\mathbf{IC}_{\mathbf{SCM}(R)}) = i_1$ and $(\mathrm{Exec}(\mathrm{goto}\ i_1,s))(c) = s(c)$.

(18) If $s(a) = 0_R$, then $(\mathrm{Exec}(\mathbf{if}\ a = 0\ \mathbf{goto}\ i_1,s))(\mathbf{IC}_{\mathbf{SCM}(R)}) = i_1$ and if $s(a) \neq 0_R$, then $(\mathrm{Exec}(\mathbf{if}\ a = 0\ \mathbf{goto}\ i_1,s))(\mathbf{IC}_{\mathbf{SCM}(R)}) = \mathrm{Next}(\mathbf{IC}_s)$ and $(\mathrm{Exec}(\mathbf{if}\ a = 0\ \mathbf{goto}\ i_1,s))(c) = s(c)$.

(19) $(\mathrm{Exec}(a{:=}r,s))(\mathbf{IC}_{\mathbf{SCM}(R)}) = \mathrm{Next}(\mathbf{IC}_s)$ and $(\mathrm{Exec}(a{:=}r,s))(a) = r$ and for every c such that $c \neq a$ holds $(\mathrm{Exec}(a{:=}r,s))(c) = s(c)$.

3. Halt Instruction

The following two propositions are true:

(20) For every instruction I of $\mathbf{SCM}(R)$ such that there exists s such that $(\mathrm{Exec}(I,s))(\mathbf{IC}_{\mathbf{SCM}(R)}) = \mathrm{Next}(\mathbf{IC}_s)$ holds I is non halting.

(21) For every instruction I of $\mathbf{SCM}(R)$ such that $I = \langle 0, \emptyset \rangle$ holds I is halting.

Let us consider R, a, b. One can check the following observations:

* $a{:=}b$ is non halting,
* $\mathrm{AddTo}(a,b)$ is non halting,
* $\mathrm{SubFrom}(a,b)$ is non halting, and
* $\mathrm{MultBy}(a,b)$ is non halting.

Let us consider R, i_1. One can check that goto i_1 is non halting.
Let us consider R, a, i_1. Observe that **if** $a = 0$ **goto** i_1 is non halting.
Let us consider R, a, r. Note that $a{:=}r$ is non halting.
Let us consider R. Observe that $\mathbf{SCM}(R)$ is halting, definite, data-oriented, steady-programmed, and realistic.

The following two propositions are true:

(29)[1] For every instruction I of $\mathbf{SCM}(R)$ such that I is halting holds $I = \mathbf{halt}_{\mathbf{SCM}(R)}$.

(30) $\mathbf{halt}_{\mathbf{SCM}(R)} = \langle 0, \emptyset \rangle$.

[1] The propositions (22)–(28) have been removed.

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