JOURNAL OF FORMALIZED MATHEMATICS
Volume 8, Released 1996, Published 2003
Inst. of Computer Science, Univ. of Białystok

# Some Multi Instructions Defined by Sequence of Instructions of $\mathbf{SCM}_{\rm FSA}$

Noriko Asamoto
Ochanomizu University
Tokyo

MML Identifier: SCMFSA_7.

WWW: http://mizar.org/JFM/Vol8/scmfsa_7.html

The articles [12], [18], [3], [13], [2], [11], [10], [4], [15], [19], [1], [5], [8], [17], [6], [16], [7], [9], and [14] provide the notation and terminology for this paper.

In this paper $m$ is a natural number.

Let us observe that every finite partial state of $\mathbf{SCM}_{\rm FSA}$ is finite.

Let $p$ be a finite sequence and let $x$, $y$ be sets. Note that $p +\cdot (x,y)$ is finite sequence-like.

We now state four propositions:

(1) For every natural number $k$ holds $|k| = k$.

(2) For all natural numbers $a$, $b$, $c$ such that $a \geq c$ and $b \geq c$ and $a -' c = b -' c$ holds $a = b$.

(3) For all natural numbers $a$, $b$ such that $a \geq b$ holds $a -' b = a - b$.

(4) For all integers $a$, $b$ such that $a < b$ holds $a \leq b - 1$.

The scheme *CardMono”* deals with a set $\mathcal{A}$, a non empty set $\mathcal{B}$, and a unary functor $\mathcal{F}$ yielding a set, and states that:

$\mathcal{A} \approx \{\mathcal{F}(d); d$ ranges over elements of $\mathcal{B} : d \in \mathcal{A}\}$

provided the parameters meet the following requirements:

- $\mathcal{A} \subseteq \mathcal{B}$, and
- For all elements $d_1$, $d_2$ of $\mathcal{B}$ such that $d_1 \in \mathcal{A}$ and $d_2 \in \mathcal{A}$ and $\mathcal{F}(d_1) = \mathcal{F}(d_2)$ holds $d_1 = d_2$.

One can prove the following propositions:

(5) For all finite sequences $p_1$, $p_2$, $q$ such that $p_1 \subseteq q$ and $p_2 \subseteq q$ and $\mathrm{len}\, p_1 = \mathrm{len}\, p_2$ holds $p_1 = p_2$.

(8)[1] For all finite sequences $p$, $q$ such that $p \subseteq q$ holds $\mathrm{len}\, p \leq \mathrm{len}\, q$.

(9) For all finite sequences $p$, $q$ and for every natural number $i$ such that $1 \leq i$ and $i \leq \mathrm{len}\, p$ holds $(p \frown q)(i) = p(i)$.

(10) For all finite sequences $p$, $q$ and for every natural number $i$ such that $1 \leq i$ and $i \leq \mathrm{len}\, q$ holds $(p \frown q)(\mathrm{len}\, p + i) = q(i)$.

[1] The propositions (6) and (7) have been removed.

(12)[2] For every finite sequence $p$ such that $p \neq \emptyset$ holds $\operatorname{len} p \in \operatorname{dom} p$.

(13) For every set $D$ holds $\operatorname{Flat}(\varepsilon_{D^*}) = \varepsilon_D$.

(14) For every set $D$ and for all finite sequences $F$, $G$ of elements of $D^*$ holds $\operatorname{Flat}(F \frown G) = \operatorname{Flat}(F) \frown \operatorname{Flat}(G)$.

(15) For every set $D$ and for all elements $p$, $q$ of $D^*$ holds $\operatorname{Flat}(\langle p, q\rangle) = p \frown q$.

(16) For every set $D$ and for all elements $p$, $q$, $r$ of $D^*$ holds $\operatorname{Flat}(\langle p, q, r\rangle) = p \frown q \frown r$.

(17) Let $D$ be a non empty set and $p$, $q$ be finite sequences of elements of $D$. If $p \subseteq q$, then there exists a finite sequence $p'$ of elements of $D$ such that $p \frown p' = q$.

(18) Let $D$ be a non empty set, $p$, $q$ be finite sequences of elements of $D$, and $i$ be a natural number. If $p \subseteq q$ and $1 \leq i$ and $i \leq \operatorname{len} p$, then $q(i) = p(i)$.

(19) For every set $D$ and for all finite sequences $F$, $G$ of elements of $D^*$ such that $F \subseteq G$ holds $\operatorname{Flat}(F) \subseteq \operatorname{Flat}(G)$.

(20) For every finite sequence $p$ holds $p \restriction \operatorname{Seg} 0 = \emptyset$.

(21) For all finite sequences $f$, $g$ holds $f \restriction \operatorname{Seg} 0 = g \restriction \operatorname{Seg} 0$.

(22) For every non empty set $D$ and for every element $x$ of $D$ holds $\langle x\rangle$ is a finite sequence of elements of $D$.

(23) Let $D$ be a set and $p$, $q$ be finite sequences of elements of $D$. Then $p \frown q$ is a finite sequence of elements of $D$.

Let $f$ be a finite sequence of elements of the instructions of $\mathbf{SCM}_{\mathrm{FSA}}$. The functor $\operatorname{Load}(f)$ yields a finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$ and is defined as follows:

(Def. 1) $\operatorname{dom}\operatorname{Load}(f) = \{\operatorname{insloc}(m -' 1) : m \in \operatorname{dom} f\}$ and for every natural number $k$ such that $\operatorname{insloc}(k) \in \operatorname{dom}\operatorname{Load}(f)$ holds $(\operatorname{Load}(f))(\operatorname{insloc}(k)) = f_{k+1}$.

We now state several propositions:

(25)[3] For every finite sequence $f$ of elements of the instructions of $\mathbf{SCM}_{\mathrm{FSA}}$ holds $\operatorname{card}\operatorname{Load}(f) = \operatorname{len} f$.

(26) Let $p$ be a finite sequence of elements of the instructions of $\mathbf{SCM}_{\mathrm{FSA}}$ and $k$ be a natural number. Then $\operatorname{insloc}(k) \in \operatorname{dom}\operatorname{Load}(p)$ if and only if $k+1 \in \operatorname{dom} p$.

(27) For all natural numbers $k$, $n$ holds $k < n$ iff $0 < k+1$ and $k+1 \leq n$.

(28) For all natural numbers $k$, $n$ holds $k < n$ iff $1 \leq k+1$ and $k+1 \leq n$.

(29) Let $p$ be a finite sequence of elements of the instructions of $\mathbf{SCM}_{\mathrm{FSA}}$ and $k$ be a natural number. Then $\operatorname{insloc}(k) \in \operatorname{dom}\operatorname{Load}(p)$ if and only if $k < \operatorname{len} p$.

(30) For every non empty finite sequence $f$ of elements of the instructions of $\mathbf{SCM}_{\mathrm{FSA}}$ holds $1 \in \operatorname{dom} f$ and $\operatorname{insloc}(0) \in \operatorname{dom}\operatorname{Load}(f)$.

(31) For all finite sequences $p$, $q$ of elements of the instructions of $\mathbf{SCM}_{\mathrm{FSA}}$ holds $\operatorname{Load}(p) \subseteq \operatorname{Load}(p \frown q)$.

(32) For all finite sequences $p$, $q$ of elements of the instructions of $\mathbf{SCM}_{\mathrm{FSA}}$ such that $p \subseteq q$ holds $\operatorname{Load}(p) \subseteq \operatorname{Load}(q)$.

Let $a$ be an integer location and let $k$ be an integer. The functor $a{:=}k$ yields a finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$ and is defined as follows:

[2] The proposition (11) has been removed.
[3] The proposition (24) has been removed.

(Def. 2)(i) There exists a natural number $k_1$ such that $k_1+1=k$ and $a{:=}k = \mathrm{Load}(\langle a{:=}\mathrm{intloc}(0)\rangle \mathbin{^\frown} (k_1 \longmapsto \mathrm{AddTo}(a,\mathrm{intloc}(0))) \mathbin{^\frown} \langle \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}\rangle)$ if $k>0$,

(ii) there exists a natural number $k_1$ such that $k_1+k=1$ and $a{:=}k = \mathrm{Load}(\langle a{:=}\mathrm{intloc}(0)\rangle \mathbin{^\frown} (k_1 \longmapsto \mathrm{SubFrom}(a,\mathrm{intloc}(0))) \mathbin{^\frown} \langle \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}\rangle)$, otherwise.

Let $a$ be an integer location and let $k$ be an integer. The functor $\mathrm{aSeq}(a,k)$ yields a finite sequence of elements of the instructions of $\mathbf{SCM}_{\mathrm{FSA}}$ and is defined by:

(Def. 3)(i) There exists a natural number $k_1$ such that $k_1+1=k$ and $\mathrm{aSeq}(a,k) = \langle a{:=}\mathrm{intloc}(0)\rangle \mathbin{^\frown} (k_1 \longmapsto \mathrm{AddTo}(a,\mathrm{intloc}(0)))$ if $k>0$,

(ii) there exists a natural number $k_1$ such that $k_1+k=1$ and $\mathrm{aSeq}(a,k) = \langle a{:=}\mathrm{intloc}(0)\rangle \mathbin{^\frown} (k_1 \longmapsto \mathrm{SubFrom}(a,\mathrm{intloc}(0)))$, otherwise.

The following proposition is true

(33) For every integer location $a$ and for every integer $k$ holds $a{:=}k = \mathrm{Load}((\mathrm{aSeq}(a,k)) \mathbin{^\frown} \langle \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}\rangle)$.

Let $f$ be a finite sequence location and let $p$ be a finite sequence of elements of $\mathbb{Z}$. The functor $\mathrm{aSeq}(f,p)$ yields a finite sequence of elements of the instructions of $\mathbf{SCM}_{\mathrm{FSA}}$ and is defined by the condition (Def. 4).

(Def. 4) There exists a finite sequence $p_3$ of elements of (the instructions of $\mathbf{SCM}_{\mathrm{FSA}})^*$ such that

(i) $\mathrm{len}\, p_3 = \mathrm{len}\, p$,

(ii) for every natural number $k$ such that $1 \leq k$ and $k \leq \mathrm{len}\, p$ there exists an integer $i$ such that $i = p(k)$ and $p_3(k) = (\mathrm{aSeq}(\mathrm{intloc}(1),k)) \mathbin{^\frown} \mathrm{aSeq}(\mathrm{intloc}(2),i) \mathbin{^\frown} \langle f_{\mathrm{intloc}(1)}{:=}\mathrm{intloc}(2)\rangle$, and

(iii) $\mathrm{aSeq}(f,p) = \mathrm{Flat}(p_3)$.

Let $f$ be a finite sequence location and let $p$ be a finite sequence of elements of $\mathbb{Z}$. The functor $f{:=}p$ yielding a finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$ is defined by:

(Def. 5) $f{:=}p = \mathrm{Load}((\mathrm{aSeq}(\mathrm{intloc}(1),\mathrm{len}\, p)) \mathbin{^\frown} \langle f{:=}\langle\underbrace{0,\ldots,0}_{\mathrm{intloc}(1)}\rangle\rangle \mathbin{^\frown} \mathrm{aSeq}(f,p) \mathbin{^\frown} \langle \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}\rangle)$.

We now state several propositions:

(34) For every integer location $a$ holds $a{:=}1 = \mathrm{Load}(\langle a{:=}\mathrm{intloc}(0)\rangle \mathbin{^\frown} \langle \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}\rangle)$.

(35) For every integer location $a$ holds $a{:=}0 = \mathrm{Load}(\langle a{:=}\mathrm{intloc}(0)\rangle \mathbin{^\frown} \langle \mathrm{SubFrom}(a,\mathrm{intloc}(0))\rangle \mathbin{^\frown} \langle \mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}\rangle)$.

(36) Let $s$ be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $s(\mathrm{intloc}(0)) = 1$. Let $c_0$ be a natural number. Suppose $\mathbf{IC}_s = \mathrm{insloc}(c_0)$. Let $a$ be an integer location and $k$ be an integer. Suppose $a \neq \mathrm{intloc}(0)$ and for every natural number $c$ such that $c \in \mathrm{dom}\,\mathrm{aSeq}(a,k)$ holds $(\mathrm{aSeq}(a,k))(c) = s(\mathrm{insloc}((c_0+c) -' 1))$. Then

(i) for every natural number $i$ such that $i \leq \mathrm{len}\,\mathrm{aSeq}(a,k)$ holds $\mathbf{IC}_{(\mathrm{Computation}(s))(i)} = \mathrm{insloc}(c_0+i)$ and for every integer location $b$ such that $b \neq a$ holds $(\mathrm{Computation}(s))(i)(b) = s(b)$ and for every finite sequence location $f$ holds $(\mathrm{Computation}(s))(i)(f) = s(f)$, and

(ii) $(\mathrm{Computation}(s))(\mathrm{len}\,\mathrm{aSeq}(a,k))(a) = k$.

(37) Let $s$ be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $\mathbf{IC}_s = \mathrm{insloc}(0)$ and $s(\mathrm{intloc}(0)) = 1$. Let $a$ be an integer location and $k$ be an integer. Suppose $\mathrm{Load}(\mathrm{aSeq}(a,k)) \subseteq s$ and $a \neq \mathrm{intloc}(0)$. Then

(i) for every natural number $i$ such that $i \leq \mathrm{len}\,\mathrm{aSeq}(a,k)$ holds $\mathbf{IC}_{(\mathrm{Computation}(s))(i)} = \mathrm{insloc}(i)$ and for every integer location $b$ such that $b \neq a$ holds $(\mathrm{Computation}(s))(i)(b) = s(b)$ and for every finite sequence location $f$ holds $(\mathrm{Computation}(s))(i)(f) = s(f)$, and

(ii) $(\mathrm{Computation}(s))(\mathrm{len}\,\mathrm{aSeq}(a,k))(a) = k$.

(38) Let $s$ be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $\mathbf{IC}_s = \mathrm{insloc}(0)$ and $s(\mathrm{intloc}(0)) = 1$. Let $a$ be an integer location and $k$ be an integer. Suppose $a{:=}k \subseteq s$ and $a \neq \mathrm{intloc}(0)$. Then

(i) $s$ is halting,

(ii) $(\mathrm{Result}(s))(a) = k$,

(iii) for every integer location $b$ such that $b \neq a$ holds $(\mathrm{Result}(s))(b) = s(b)$, and

(iv) for every finite sequence location $f$ holds $(\mathrm{Result}(s))(f) = s(f)$.

(39) Let $s$ be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $\mathbf{IC}_s = \mathrm{insloc}(0)$ and $s(\mathrm{intloc}(0)) = 1$. Let $f$ be a finite sequence location and $p$ be a finite sequence of elements of $\mathbb{Z}$. Suppose $f{:=}p \subseteq s$. Then

(i) $s$ is halting,

(ii) $(\mathrm{Result}(s))(f) = p$,

(iii) for every integer location $b$ such that $b \neq \mathrm{intloc}(1)$ and $b \neq \mathrm{intloc}(2)$ holds $(\mathrm{Result}(s))(b) = s(b)$, and

(iv) for every finite sequence location $g$ such that $g \neq f$ holds $(\mathrm{Result}(s))(g) = s(g)$.

## REFERENCES


[1] Grzegorz Bancerek. Cardinal numbers. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/card_1.html`.

[2] Grzegorz Bancerek. The fundamental properties of natural numbers. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/nat_1.html`.

[3] Grzegorz Bancerek. The ordinal numbers. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/ordinal1.html`.

[4] Grzegorz Bancerek. Sequences of ordinal numbers. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/ordinal2.html`.

[5] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/finseq_1.html`.

[6] Grzegorz Bancerek and Piotr Rudnicki. On defining functions on trees. *Journal of Formalized Mathematics*, 5, 1993. `http://mizar.org/JFM/Vol5/dtconstr.html`.

[7] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/funct_1.html`.

[8] Agata Darmochwał. Finite sets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/finset_1.html`.

[9] Yatsuka Nakamura and Andrzej Trybulec. A mathematical model of CPU. *Journal of Formalized Mathematics*, 4, 1992. `http://mizar.org/JFM/Vol4/ami_1.html`.

[10] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. *Journal of Formalized Mathematics*, 5, 1993. `http://mizar.org/JFM/Vol5/binarith.html`.

[11] Jan Popiołek. Some properties of functions modul and signum. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/absvalue.html`.

[12] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. `http://mizar.org/JFM/Axiomatics/tarski.html`.

[13] Andrzej Trybulec. Subsets of real numbers. *Journal of Formalized Mathematics*, Addenda, 2003. `http://mizar.org/JFM/Addenda/numbers.html`.

[14] Andrzej Trybulec, Yatsuka Nakamura, and Piotr Rudnicki. The $\mathbf{SCM}_{\mathrm{FSA}}$ computer. *Journal of Formalized Mathematics*, 8, 1996. `http://mizar.org/JFM/Vol8/scmfsa_2.html`.

[15] Michał J. Trybulec. Integers. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/int_1.html`.

[16] Wojciech A. Trybulec. Binary operations on finite sequences. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/finsop_1.html`.

[17] Wojciech A. Trybulec. Pigeon hole principle. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/finseq_4.html`.

[18] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/subset_1.html`.

[19] Edmund Woronowicz. Relations and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/relat_1.html`.

*Received April 24, 1996*

*Published January 2, 2004*