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# Relocability for $\mathbf{SCM}_{\mathrm{FSA}}$

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The articles [13], [18], [11], [1], [17], [2], [6], [19], [4], [3], [7], [9], [5], [8], [10], [14], [12], [16], and [15] provide the notation and terminology for this paper.

## 1. Relocability

In this paper $j$, $k$ denote natural numbers.

Let $p$ be a finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$ and let $k$ be a natural number. The functor $\mathrm{Relocated}(p,k)$ yields a finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$ and is defined by:

(Def. 1) $\mathrm{Relocated}(p,k) = \text{Start-At}(\mathbf{IC}_p + k) +\cdot \mathrm{IncAddr}(\mathrm{Shift}(\mathrm{ProgramPart}(p),k),k) +\cdot \mathrm{DataPart}(p)$.

The following propositions are true:

(1) For every finite partial state $p$ of $\mathbf{SCM}_{\mathrm{FSA}}$ and for every natural number $k$ holds $\mathrm{DataPart}(\mathrm{Relocated}(p,k)) = \mathrm{DataPart}(p)$.

(2) For every finite partial state $p$ of $\mathbf{SCM}_{\mathrm{FSA}}$ and for every natural number $k$ holds $\mathrm{ProgramPart}(\mathrm{Relocated}(p,k)) = \mathrm{IncAddr}(\mathrm{Shift}(\mathrm{ProgramPart}(p),k),k)$.

(3) For every finite partial state $p$ of $\mathbf{SCM}_{\mathrm{FSA}}$ holds $\mathrm{dom}\,\mathrm{ProgramPart}(\mathrm{Relocated}(p,k)) = \{\mathrm{insloc}(j+k) : \mathrm{insloc}(j) \in \mathrm{dom}\,\mathrm{ProgramPart}(p)\}$.

(4) Let $p$ be a finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$, $k$ be a natural number, and $l$ be an instruction-location of $\mathbf{SCM}_{\mathrm{FSA}}$. Then $l \in \mathrm{dom}\,p$ if and only if $l + k \in \mathrm{dom}\,\mathrm{Relocated}(p,k)$.

(5) For every finite partial state $p$ of $\mathbf{SCM}_{\mathrm{FSA}}$ and for every natural number $k$ holds $\mathbf{IC}_{\mathbf{SCM}_{\mathrm{FSA}}} \in \mathrm{dom}\,\mathrm{Relocated}(p,k)$.

(6) For every finite partial state $p$ of $\mathbf{SCM}_{\mathrm{FSA}}$ and for every natural number $k$ holds $\mathbf{IC}_{\mathrm{Relocated}(p,k)} = \mathbf{IC}_p + k$.

(7) Let $p$ be a finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$, $k$ be a natural number, $l_1$ be an instruction-location of $\mathbf{SCM}_{\mathrm{FSA}}$, and $I$ be an instruction of $\mathbf{SCM}_{\mathrm{FSA}}$. If $l_1 \in \mathrm{dom}\,\mathrm{ProgramPart}(p)$ and $I = p(l_1)$, then $\mathrm{IncAddr}(I,k) = (\mathrm{Relocated}(p,k))(l_1 + k)$.

(8) For every finite partial state $p$ of $\mathbf{SCM}_{\mathrm{FSA}}$ and for every natural number $k$ holds $\text{Start-At}(\mathbf{IC}_p + k) \subseteq \mathrm{Relocated}(p,k)$.

(9) Let $s$ be a data-only finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$, $p$ be a finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$, and $k$ be a natural number. If $\mathbf{IC}_{\mathbf{SCM}_{\mathrm{FSA}}} \in \mathrm{dom}\,p$, then $\mathrm{Relocated}(p +\cdot s, k) = \mathrm{Relocated}(p,k) +\cdot s$.

(10) Let $k$ be a natural number, $p$ be an autonomic finite partial state of $\mathbf{SCM}_{\rm FSA}$, and $s_1$, $s_2$ be states of $\mathbf{SCM}_{\rm FSA}$. If $p \subseteq s_1$ and $\mathrm{Relocated}(p,k) \subseteq s_2$, then $p \subseteq s_1 {+\cdot} s_2 \restriction (\text{Int-Locations} \cup \text{FinSeq-Locations})$.

## 2. Main Theorems of Relocability

One can prove the following propositions:

(11) Let $k$ be a natural number and $p$ be an autonomic finite partial state of $\mathbf{SCM}_{\rm FSA}$. Suppose $\mathbf{IC}_{\mathbf{SCM}_{\rm FSA}} \in \mathrm{dom}\, p$. Let $s$ be a state of $\mathbf{SCM}_{\rm FSA}$. Suppose $p \subseteq s$. Let $i$ be a natural number. Then $(\mathrm{Computation}(s {+\cdot} \mathrm{Relocated}(p,k)))(i) = (\mathrm{Computation}(s))(i) {+\cdot}\, \text{Start-At}(\mathbf{IC}_{(\mathrm{Computation}(s))(i)} + k) {+\cdot}\, \mathrm{ProgramPart}(\mathrm{Relocated}(p,k))$.

(12) Let $k$ be a natural number, $p$ be an autonomic finite partial state of $\mathbf{SCM}_{\rm FSA}$, and $s_1$, $s_2$, $s_3$ be states of $\mathbf{SCM}_{\rm FSA}$. Suppose $\mathbf{IC}_{\mathbf{SCM}_{\rm FSA}} \in \mathrm{dom}\, p$ and $p \subseteq s_1$ and $\mathrm{Relocated}(p,k) \subseteq s_2$ and $s_3 = s_1 {+\cdot} s_2 \restriction (\text{Int-Locations} \cup \text{FinSeq-Locations})$. Let $i$ be a natural number. Then $\mathbf{IC}_{(\mathrm{Computation}(s_1))(i)} + k = \mathbf{IC}_{(\mathrm{Computation}(s_2))(i)}$ and $\mathrm{IncAddr}(\mathrm{CurInstr}((\mathrm{Computation}(s_1))(i)),k) = \mathrm{CurInstr}((\mathrm{Computation}(s_2))(i))$ and $(\mathrm{Computation}(s_1))(i) \restriction \mathrm{dom}\, \mathrm{DataPart}(p$ $(\mathrm{Computation}(s_2))(i) \restriction \mathrm{dom}\, \mathrm{DataPart}(\mathrm{Relocated}(p,k))$ and $(\mathrm{Computation}(s_3))(i) \restriction (\text{Int-Locations} \cup \text{FinSeq-Locations}) = (\mathrm{Computation}(s_2))(i) \restriction (\text{Int-Locations} \cup \text{FinSeq-Locations})$.

(13) Let $p$ be an autonomic finite partial state of $\mathbf{SCM}_{\rm FSA}$ and $k$ be a natural number. If $\mathbf{IC}_{\mathbf{SCM}_{\rm FSA}} \in \mathrm{dom}\, p$, then $p$ is halting iff $\mathrm{Relocated}(p,k)$ is halting.

(14) Let $k$ be a natural number and $p$ be an autonomic finite partial state of $\mathbf{SCM}_{\rm FSA}$. Suppose $\mathbf{IC}_{\mathbf{SCM}_{\rm FSA}} \in \mathrm{dom}\, p$. Let $s$ be a state of $\mathbf{SCM}_{\rm FSA}$. Suppose $\mathrm{Relocated}(p,k) \subseteq s$. Let $i$ be a natural number. Then $(\mathrm{Computation}(s))(i) = (\mathrm{Computation}(s {+\cdot} p))(i) {+\cdot}\, \text{Start-At}(\mathbf{IC}_{(\mathrm{Computation}(s {+\cdot} p))(i)} + k) {+\cdot}\, s \restriction \mathrm{dom}\, \mathrm{ProgramPart}(p) {+\cdot}\, \mathrm{ProgramPart}(\mathrm{Relocated}(p,k))$.

(15) Let $k$ be a natural number and $p$ be a finite partial state of $\mathbf{SCM}_{\rm FSA}$. Suppose $\mathbf{IC}_{\mathbf{SCM}_{\rm FSA}} \in \mathrm{dom}\, p$. Let $s$ be a state of $\mathbf{SCM}_{\rm FSA}$. Suppose $p \subseteq s$ and $\mathrm{Relocated}(p,k)$ is autonomic. Let $i$ be a natural number. Then $(\mathrm{Computation}(s))(i) = (\mathrm{Computation}(s {+\cdot} \mathrm{Relocated}(p,k)))(i) {+\cdot}\, \text{Start-At}(\mathbf{IC}_{(\mathrm{Computation}(s {+\cdot} \mathrm{Relocated}(p,k)))(i)} -' k) {+\cdot}\, s \restriction \mathrm{dom}\, \mathrm{ProgramPart}(\mathrm{Relocated}($

(16) Let $p$ be a finite partial state of $\mathbf{SCM}_{\rm FSA}$. Suppose $\mathbf{IC}_{\mathbf{SCM}_{\rm FSA}} \in \mathrm{dom}\, p$. Let $k$ be a natural number. Then $p$ is autonomic if and only if $\mathrm{Relocated}(p,k)$ is autonomic.

(17) Let $p$ be a halting autonomic finite partial state of $\mathbf{SCM}_{\rm FSA}$. If $\mathbf{IC}_{\mathbf{SCM}_{\rm FSA}} \in \mathrm{dom}\, p$, then for every natural number $k$ holds $\mathrm{DataPart}(\mathrm{Result}(p)) = \mathrm{DataPart}(\mathrm{Result}(\mathrm{Relocated}(p,k)))$.

(18) Let $F$ be a partial function from $\mathrm{FinPartSt}(\mathbf{SCM}_{\rm FSA})$ to $\mathrm{FinPartSt}(\mathbf{SCM}_{\rm FSA})$ and $p$ be a finite partial state of $\mathbf{SCM}_{\rm FSA}$. Suppose $\mathbf{IC}_{\mathbf{SCM}_{\rm FSA}} \in \mathrm{dom}\, p$ and $F$ is data-only. Let $k$ be a natural number. Then $p$ computes $F$ if and only if $\mathrm{Relocated}(p,k)$ computes $F$.

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