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Schemes[1]

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Summary. Some basic schemes of quantifier calculus are proved.

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WWW: http://mizar.org/JFM/Vol2/schems_1.html

In this paper a, b denote sets.

In this article we present several logical schemes. The scheme *Schemat0* concerns a unary predicate $\mathcal{P}$, and states that:

There exists a such that $\mathcal{P}[a]$

provided the following requirement is met:

- For every a holds $\mathcal{P}[a]$.

The scheme *Schemat1a* concerns a 0-ary predicate $\mathcal{Q}$ and a unary predicate $\mathcal{P}$, and states that:

For every a holds $\mathcal{P}[a]$ and $\mathcal{Q}$

provided the parameters have the following property:

- For every a holds $\mathcal{P}[a]$ and $\mathcal{Q}$.

The scheme *Schemat1b* concerns a 0-ary predicate $\mathcal{Q}$ and a unary predicate $\mathcal{P}$, and states that:

For every a holds $\mathcal{P}[a]$ and $\mathcal{Q}$

provided the parameters meet the following requirement:

- For every a holds $\mathcal{P}[a]$ and $\mathcal{Q}$.

The scheme *Schemat2a* concerns a 0-ary predicate $\mathcal{Q}$ and a unary predicate $\mathcal{P}$, and states that:

There exists a such that $\mathcal{P}[a]$ or $\mathcal{Q}$

provided the following condition is met:

- There exists a such that $\mathcal{P}[a]$ or $\mathcal{Q}$.

The scheme *Schemat2b* concerns a 0-ary predicate $\mathcal{Q}$ and a unary predicate $\mathcal{P}$, and states that:

There exists a such that $\mathcal{P}[a]$ or $\mathcal{Q}$

provided the parameters meet the following condition:

- There exists a such that $\mathcal{P}[a]$ or $\mathcal{Q}$.

The scheme *Schemat3* concerns a binary predicate $\mathcal{P}$, and states that:

For every b there exists a such that $\mathcal{P}[a,b]$

provided the following condition is met:

- There exists a such that for every b holds $\mathcal{P}[a,b]$.

The scheme *Schemat4a* concerns two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a such that $\mathcal{P}[a]$ or there exists a such that $\mathcal{Q}[a]$

provided the following condition is satisfied:

- There exists a such that $\mathcal{P}[a]$ or $\mathcal{Q}[a]$.

The scheme *Schemat4b* concerns two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a such that $\mathcal{P}[a]$ or $\mathcal{Q}[a]$

[1] Supported by RPBP.III-24.C10.

provided the following requirement is met:

- There exists a such that $\mathcal{P}[a]$ or there exists a such that $\mathcal{Q}[a]$.

The scheme *Schemat5* concerns two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a such that $\mathcal{P}[a]$ and there exists a such that $\mathcal{Q}[a]$

provided the parameters satisfy the following condition:

- There exists a such that $\mathcal{P}[a]$ and $\mathcal{Q}[a]$.

The scheme *Schemat6a* concerns two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

For every a holds $\mathcal{P}[a]$ and for every a holds $\mathcal{Q}[a]$

provided the parameters satisfy the following condition:

- For every a holds $\mathcal{P}[a]$ and $\mathcal{Q}[a]$.

The scheme *Schemat6b* concerns two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

For every a holds $\mathcal{P}[a]$ and $\mathcal{Q}[a]$

provided the following condition is satisfied:

- For every a holds $\mathcal{P}[a]$ and for every a holds $\mathcal{Q}[a]$.

The scheme *Schemat7* concerns two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

For every a holds $\mathcal{P}[a]$ or $\mathcal{Q}[a]$

provided the following condition is met:

- For every a holds $\mathcal{P}[a]$ or for every a holds $\mathcal{Q}[a]$.

The scheme *Schemat8* concerns two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

If for every a holds $\mathcal{P}[a]$, then for every a holds $\mathcal{Q}[a]$

provided the parameters satisfy the following condition:

- For every a such that $\mathcal{P}[a]$ holds $\mathcal{Q}[a]$.

The scheme *Schemat9* concerns two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

For every a holds $\mathcal{P}[a]$ iff for every a holds $\mathcal{Q}[a]$

provided the parameters meet the following requirement:

- For every a holds $\mathcal{P}[a]$ iff $\mathcal{Q}[a]$.

The scheme *Schemat10b* concerns a 0-ary predicate $\mathcal{P}$, and states that:

For every a holds $\mathcal{P}$

provided the following requirement is met:

- $\mathcal{P}$.

The scheme *Schemat11a* concerns a 0-ary predicate $\mathcal{Q}$ and a unary predicate $\mathcal{P}$, and states that:

For every a holds $\mathcal{P}[a]$ or $\mathcal{Q}$

provided the parameters satisfy the following condition:

- For every a holds $\mathcal{P}[a]$ or $\mathcal{Q}$.

The scheme *Schemat11b* concerns a 0-ary predicate $\mathcal{Q}$ and a unary predicate $\mathcal{P}$, and states that:

For every a holds $\mathcal{P}[a]$ or $\mathcal{Q}$

provided the parameters satisfy the following condition:

- For every a holds $\mathcal{P}[a]$ or $\mathcal{Q}$.

The scheme *Schemat12a* concerns a 0-ary predicate $\mathcal{Q}$ and a unary predicate $\mathcal{P}$, and states that:

There exists a such that $\mathcal{Q}$ and $\mathcal{P}[a]$

provided the parameters meet the following requirement:

- $\mathcal{Q}$ and there exists a such that $\mathcal{P}[a]$.

The scheme *Schemat12b* concerns a 0-ary predicate $\mathcal{Q}$ and a unary predicate $\mathcal{P}$, and states that:

$\mathcal{Q}$ and there exists a such that $\mathcal{P}[a]$

provided the parameters have the following property:

- There exists a such that $\mathcal{Q}$ and $\mathcal{P}[a]$.

The scheme *Schemat13a* concerns a 0-ary predicate $\mathcal{Q}$ and a unary predicate $\mathcal{P}$, and states that:

For every a such that $\mathcal{Q}$ holds $\mathcal{P}[a]$

provided the parameters meet the following requirement:

- If $\mathcal{Q}$, then for every a holds $\mathcal{P}[a]$.

The scheme *Schemat13b* concerns a 0-ary predicate $\mathcal{Q}$ and a unary predicate $\mathcal{P}$, and states that:

If $\mathcal{Q}$, then for every a holds $\mathcal{P}[a]$

provided the following requirement is met:

- For every a such that $\mathcal{Q}$ holds $\mathcal{P}[a]$.

The scheme *Schemat14* concerns a 0-ary predicate $\mathcal{Q}$ and a unary predicate $\mathcal{P}$, and states that:

There exists a such that if $\mathcal{Q}$, then $\mathcal{P}[a]$

provided the following requirement is met:

- If $\mathcal{Q}$, then there exists a such that $\mathcal{P}[a]$.

The scheme *Schemat17* concerns a 0-ary predicate $\mathcal{Q}$ and a unary predicate $\mathcal{P}$, and states that:

If for every a holds $\mathcal{P}[a]$, then $\mathcal{Q}$

provided the parameters meet the following requirement:

- For every a such that $\mathcal{P}[a]$ holds $\mathcal{Q}$.

The scheme *Schemat18a* concerns two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a such that for every b holds $\mathcal{P}[a]$ or $\mathcal{Q}[b]$

provided the following condition is met:

- There exists a such that $\mathcal{P}[a]$ or for every b holds $\mathcal{Q}[b]$.

The scheme *Schemat18b* concerns two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a such that $\mathcal{P}[a]$ or for every b holds $\mathcal{Q}[b]$

provided the parameters satisfy the following condition:

- There exists a such that for every b holds $\mathcal{P}[a]$ or $\mathcal{Q}[b]$.

The scheme *Schemat19a* concerns two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

For every b there exists a such that $\mathcal{P}[a]$ or $\mathcal{Q}[b]$

provided the following condition is met:

- There exists a such that $\mathcal{P}[a]$ or for every b holds $\mathcal{Q}[b]$.

The scheme *Schemat19b* concerns two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a such that $\mathcal{P}[a]$ or for every b holds $\mathcal{Q}[b]$

provided the following requirement is met:

- For every b there exists a such that $\mathcal{P}[a]$ or $\mathcal{Q}[b]$.

The scheme *Schemat20b* concerns two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a such that for every b holds $\mathcal{P}[a]$ or $\mathcal{Q}[b]$

provided the following requirement is met:

- For every b there exists a such that $\mathcal{P}[a]$ or $\mathcal{Q}[b]$.

The scheme *Schemat21a* concerns two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a such that for every b holds $\mathcal{P}[a]$ and $\mathcal{Q}[b]$

provided the parameters have the following property:

- There exists a such that $\mathcal{P}[a]$ and for every b holds $\mathcal{Q}[b]$.

The scheme *Schemat21b* concerns two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a such that $\mathcal{P}[a]$ and for every b holds $\mathcal{Q}[b]$

provided the following condition is met:

- There exists a such that for every b holds $\mathcal{P}[a]$ and $\mathcal{Q}[b]$.

The scheme *Schemat22a* concerns two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

For every b there exists a such that $\mathcal{P}[a]$ and $\mathcal{Q}[b]$

provided the following requirement is met:

- There exists a such that $\mathcal{P}[a]$ and for every b holds $\mathcal{Q}[b]$.

The scheme *Schemat22b* concerns two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a such that $\mathcal{P}[a]$ and for every b holds $\mathcal{Q}[b]$

provided the following requirement is met:

- For every b there exists a such that $\mathcal{P}[a]$ and $\mathcal{Q}[b]$.

The scheme *Schemat23b* concerns two unary predicates $\mathcal{P}$, $\mathcal{Q}$, and states that:

There exists a such that for every b holds $\mathcal{P}[a]$ and $\mathcal{Q}[b]$

provided the parameters meet the following requirement:

- For every b there exists a such that $\mathcal{P}[a]$ and $\mathcal{Q}[b]$.

The scheme *Schemat24a* concerns a unary predicate $\mathcal{Q}$ and a binary predicate $\mathcal{P}$, and states that:

For every a there exists b such that if $\mathcal{P}[a,b]$, then $\mathcal{Q}[a]$

provided the parameters meet the following requirement:

- For every a such that for every b holds $\mathcal{P}[a,b]$ holds $\mathcal{Q}[a]$.

The scheme *Schemat24b* concerns a unary predicate $\mathcal{Q}$ and a binary predicate $\mathcal{P}$, and states that:

For every a such that for every b holds $\mathcal{P}[a,b]$ holds $\mathcal{Q}[a]$

provided the following requirement is met:

- For every a there exists b such that if $\mathcal{P}[a,b]$, then $\mathcal{Q}[a]$.

The scheme *Schemat25a* concerns a unary predicate $\mathcal{Q}$ and a binary predicate $\mathcal{P}$, and states that:

For all a, b such that $\mathcal{P}[a,b]$ holds $\mathcal{Q}[a]$

provided the following condition is satisfied:

- For every a such that there exists b such that $\mathcal{P}[a,b]$ holds $\mathcal{Q}[a]$.

The scheme *Schemat25b* concerns a unary predicate $\mathcal{Q}$ and a binary predicate $\mathcal{P}$, and states that:

For every a such that there exists b such that $\mathcal{P}[a,b]$ holds $\mathcal{Q}[a]$

provided the parameters meet the following condition:

- For all a, b such that $\mathcal{P}[a,b]$ holds $\mathcal{Q}[a]$.

The scheme *Schemat27* concerns a binary predicate $\mathcal{P}$, and states that:

For every a holds $\mathcal{P}[a,a]$

provided the parameters satisfy the following condition:

- For all a, b holds $\mathcal{P}[a,b]$.

The scheme *Schemat28* concerns a binary predicate $\mathcal{P}$, and states that:

There exists b such that for every a holds $\mathcal{P}[a,b]$

provided the following condition is satisfied:

- For all a, b holds $\mathcal{P}[a,b]$.

The scheme *Schemat30* concerns a binary predicate $\mathcal{P}$, and states that:

There exists a such that $\mathcal{P}[a,a]$

provided the parameters satisfy the following condition:

- There exists a such that for every b holds $\mathcal{P}[a,b]$.

The scheme *Schemat31* concerns a binary predicate $\mathcal{P}$, and states that:

For every a there exists b such that $\mathcal{P}[b,a]$

provided the following requirement is met:

- For every a holds $\mathcal{P}[a,a]$.

The scheme *Schemat33* concerns a binary predicate $\mathcal{P}$, and states that:

For every a there exists b such that $\mathcal{P}[a,b]$

provided the following requirement is met:

- For every a holds $\mathcal{P}[a,a]$.

The scheme *Schemat36* concerns a binary predicate $\mathcal{P}$, and states that:

There exist a, b such that $\mathcal{P}[a,b]$

provided the following condition is met:

- For every b there exists a such that $\mathcal{P}[a,b]$.

The scheme *Schemat37* concerns a binary predicate $\mathcal{P}$, and states that:

There exist a, b such that $\mathcal{P}[a,b]$

provided the parameters satisfy the following condition:

- There exists a such that $\mathcal{P}[a,a]$.

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