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Subspaces of Real Linear Space Generated by One, Two, or Three Vectors and Their Cosets

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The articles [5], [12], [7], [1], [2], [3], [4], [8], [9], [11], [10], and [6] provide the notation and terminology for this paper.

For simplicity, we adopt the following convention: x is a set, a, b, c are real numbers, V is a real linear space, u, v, v_1, v_2, v_3, w, w_1, w_2, w_3 are vectors of V, and W, W_1, W_2 are subspaces of V.

In this article we present several logical schemes. The scheme *LambdaSep3* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, elements $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$ of $\mathcal{A}$, elements $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$ of $\mathcal{B}$, and a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$, and states that:

> There exists a function f from $\mathcal{A}$ into $\mathcal{B}$ such that $f(\mathcal{C}) = \mathcal{F}$ and $f(\mathcal{D}) = \mathcal{G}$ and $f(\mathcal{E}) = \mathcal{H}$ and for every element C of $\mathcal{A}$ such that $C \neq \mathcal{C}$ and $C \neq \mathcal{D}$ and $C \neq \mathcal{E}$ holds $f(C) = \mathcal{F}(C)$

provided the following conditions are met:

- $\mathcal{C} \neq \mathcal{D}$,
- $\mathcal{C} \neq \mathcal{E}$, and
- $\mathcal{D} \neq \mathcal{E}$.

The scheme *LinCEx1* deals with a real linear space $\mathcal{A}$, a vector $\mathcal{B}$ of $\mathcal{A}$, and a real number $\mathcal{C}$, and states that:

> There exists a linear combination l of $\{\mathcal{B}\}$ such that $l(\mathcal{B}) = \mathcal{C}$

for all values of the parameters.

The scheme *LinCEx2* deals with a real linear space $\mathcal{A}$, vectors $\mathcal{B}$, $\mathcal{C}$ of $\mathcal{A}$, and real numbers $\mathcal{D}$, $\mathcal{E}$, and states that:

> There exists a linear combination l of $\{\mathcal{B}, \mathcal{C}\}$ such that $l(\mathcal{B}) = \mathcal{D}$ and $l(\mathcal{C}) = \mathcal{E}$

provided the parameters meet the following condition:

- $\mathcal{B} \neq \mathcal{C}$.

The scheme *LinCEx3* deals with a real linear space $\mathcal{A}$, vectors $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$ of $\mathcal{A}$, and real numbers $\mathcal{E}$, $\mathcal{F}$, $\mathcal{G}$, and states that:

> There exists a linear combination l of $\{\mathcal{B}, \mathcal{C}, \mathcal{D}\}$ such that $l(\mathcal{B}) = \mathcal{E}$ and $l(\mathcal{C}) = \mathcal{F}$ and $l(\mathcal{D}) = \mathcal{G}$

provided the following conditions are satisfied:

- $\mathcal{B} \neq \mathcal{C}$,
- $\mathcal{B} \neq \mathcal{D}$, and
- $\mathcal{C} \neq \mathcal{D}$.

One can prove the following propositions:

(1) $(v+w)-v=w$ and $(w+v)-v=w$ and $(v-v)+w=w$ and $(w-v)+v=w$ and $v+(w-v)=w$ and $w+(v-v)=w$ and $v-(v-w)=w$.

(2) $(v+u)-w=(v-w)+u$.

(4)[1] If $v_1-w=v_2-w$, then $v_1=v_2$.

(6)[2] $-a\cdot v=(-a)\cdot v$.

(7) If W_1 is a subspace of W_2, then $v+W_1\subseteq v+W_2$.

(8) If $u\in v+W$, then $v+W=u+W$.

(9) For every linear combination l of $\{u,v,w\}$ such that $u\neq v$ and $u\neq w$ and $v\neq w$ holds $\sum l=l(u)\cdot u+l(v)\cdot v+l(w)\cdot w$.

(10) $u\neq v$ and $u\neq w$ and $v\neq w$ and $\{u,v,w\}$ is linearly independent if and only if for all a, b, c such that $a\cdot u+b\cdot v+c\cdot w=0_V$ holds $a=0$ and $b=0$ and $c=0$.

(11) $x\in\mathrm{Lin}(\{v\})$ iff there exists a such that $x=a\cdot v$.

(12) $v\in\mathrm{Lin}(\{v\})$.

(13) $x\in v+\mathrm{Lin}(\{w\})$ iff there exists a such that $x=v+a\cdot w$.

(14) $x\in\mathrm{Lin}(\{w_1,w_2\})$ iff there exist a, b such that $x=a\cdot w_1+b\cdot w_2$.

(15) $w_1\in\mathrm{Lin}(\{w_1,w_2\})$ and $w_2\in\mathrm{Lin}(\{w_1,w_2\})$.

(16) $x\in v+\mathrm{Lin}(\{w_1,w_2\})$ iff there exist a, b such that $x=v+a\cdot w_1+b\cdot w_2$.

(17) $x\in\mathrm{Lin}(\{v_1,v_2,v_3\})$ iff there exist a, b, c such that $x=a\cdot v_1+b\cdot v_2+c\cdot v_3$.

(18) $w_1\in\mathrm{Lin}(\{w_1,w_2,w_3\})$ and $w_2\in\mathrm{Lin}(\{w_1,w_2,w_3\})$ and $w_3\in\mathrm{Lin}(\{w_1,w_2,w_3\})$.

(19) $x\in v+\mathrm{Lin}(\{w_1,w_2,w_3\})$ iff there exist a, b, c such that $x=v+a\cdot w_1+b\cdot w_2+c\cdot w_3$.

(20) If $\{u,v\}$ is linearly independent and $u\neq v$, then $\{u,v-u\}$ is linearly independent.

(21) If $\{u,v\}$ is linearly independent and $u\neq v$, then $\{u,v+u\}$ is linearly independent.

(22) If $\{u,v\}$ is linearly independent and $u\neq v$ and $a\neq 0$, then $\{u,a\cdot v\}$ is linearly independent.

(23) If $\{u,v\}$ is linearly independent and $u\neq v$, then $\{u,-v\}$ is linearly independent.

(24) If $a\neq b$, then $\{a\cdot v,b\cdot v\}$ is linearly dependent.

(25) If $a\neq 1$, then $\{v,a\cdot v\}$ is linearly dependent.

(26) If $\{u,w,v\}$ is linearly independent and $u\neq v$ and $u\neq w$ and $v\neq w$, then $\{u,w,v-u\}$ is linearly independent.

(27) If $\{u,w,v\}$ is linearly independent and $u\neq v$ and $u\neq w$ and $v\neq w$, then $\{u,w-u,v-u\}$ is linearly independent.

(28) If $\{u,w,v\}$ is linearly independent and $u\neq v$ and $u\neq w$ and $v\neq w$, then $\{u,w,v+u\}$ is linearly independent.

(29) If $\{u,w,v\}$ is linearly independent and $u\neq v$ and $u\neq w$ and $v\neq w$, then $\{u,w+u,v+u\}$ is linearly independent.

(30) If $\{u,w,v\}$ is linearly independent and $u\neq v$ and $u\neq w$ and $v\neq w$ and $a\neq 0$, then $\{u,w,a\cdot v\}$ is linearly independent.

[1] The proposition (3) has been removed.
[2] The proposition (5) has been removed.

(31) If $\{u, w, v\}$ is linearly independent and $u \neq v$ and $u \neq w$ and $v \neq w$ and $a \neq 0$ and $b \neq 0$, then $\{u, a \cdot w, b \cdot v\}$ is linearly independent.

(32) If $\{u, w, v\}$ is linearly independent and $u \neq v$ and $u \neq w$ and $v \neq w$, then $\{u, w, -v\}$ is linearly independent.

(33) If $\{u, w, v\}$ is linearly independent and $u \neq v$ and $u \neq w$ and $v \neq w$, then $\{u, -w, -v\}$ is linearly independent.

(34) If $a \neq b$, then $\{a \cdot v, b \cdot v, w\}$ is linearly dependent.

(35) If $a \neq 1$, then $\{v, a \cdot v, w\}$ is linearly dependent.

(36) If $v \in \mathrm{Lin}(\{w\})$ and $v \neq 0_V$, then $\mathrm{Lin}(\{v\}) = \mathrm{Lin}(\{w\})$.

(37) Suppose $v_1 \neq v_2$ and $\{v_1, v_2\}$ is linearly independent and $v_1 \in \mathrm{Lin}(\{w_1, w_2\})$ and $v_2 \in \mathrm{Lin}(\{w_1, w_2\})$. Then $\mathrm{Lin}(\{w_1, w_2\}) = \mathrm{Lin}(\{v_1, v_2\})$ and $\{w_1, w_2\}$ is linearly independent and $w_1 \neq w_2$.

(38) If $w \neq 0_V$ and $\{v, w\}$ is linearly dependent, then there exists a such that $v = a \cdot w$.

(39) If $v \neq w$ and $\{v, w\}$ is linearly independent and $\{u, v, w\}$ is linearly dependent, then there exist a, b such that $u = a \cdot v + b \cdot w$.

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