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Basic Properties of Real Numbers

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Summary. Basic facts of arithmetics of real numbers are presented: definitions and properties of the complement element, the inverse element, subtraction and division; some basic properties of the set REAL (e.g. density), and the scheme of separation for sets of reals.

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WWW: `http://mizar.org/JFM/Vol1/real_1.html`

The articles [2], [4], [1], and [3] provide the notation and terminology for this paper.

In this paper x, y, z, t denote real numbers.

Let us note that every element of $\mathbb{R}$ is real.

A real number is an element of $\mathbb{R}$.

Let x be a real number. Then $-x$ is a real number. Then x^{-1} is a real number.

Let x, y be real numbers. Then $x+y$ is a real number. Then $x \cdot y$ is a real number. Then $x-y$ is a real number. Then $\frac{x}{y}$ is a real number.

The following propositions are true:

(25)[1] $x-0=x$.

(26) $-0=0$.

(49)[2] If $x \leq y$, then $x-z \leq y-z$.

(50) $x \leq y$ iff $-y \leq -x$.

(52)[3] If $x \leq y$ and $z \leq 0$, then $y \cdot z \leq x \cdot z$.

(53) If $x+z \leq y+z$, then $x \leq y$.

(54) If $x-z \leq y-z$, then $x \leq y$.

(55) If $x \leq y$ and $z \leq t$, then $x+z \leq y+t$.

Let y, x be real numbers. Let us observe that $x<y$ if and only if:

(Def. 5)[4] $x \leq y$ and $x \neq y$.

We now state a number of propositions:

(66)[5] $x<0$ iff $0<-x$.

[1] The propositions (1)–(24) have been removed.
[2] The propositions (27)–(48) have been removed.
[3] The proposition (51) has been removed.
[4] The definitions (Def. 1)–(Def. 4) have been removed.
[5] The propositions (56)–(65) have been removed.

(67) If $x < y$ and $z \leq t$, then $x + z < y + t$.

(69)[6] If $0 < x$, then $y < y + x$.

(70) If $0 < z$ and $x < y$, then $x \cdot z < y \cdot z$.

(71) If $z < 0$ and $x < y$, then $y \cdot z < x \cdot z$.

(72) If $0 < z$, then $0 < z^{-1}$.

(73) If $0 < z$, then $x < y$ iff $\frac{x}{z} < \frac{y}{z}$.

(74) If $z < 0$, then $x < y$ iff $\frac{y}{z} < \frac{x}{z}$.

(75) If $x < y$, then there exists z such that $x < z$ and $z < y$.

(76) For every x there exists y such that $x < y$.

(77) For every x there exists y such that $y < x$.

The scheme *SepReal* concerns a unary predicate $\mathcal{P}$, and states that:

There exists a subset X of $\mathbb{R}$ such that for every real number x holds $x \in X$ iff $\mathcal{P}[x]$

for all values of the parameters.

Next we state four propositions:

(84)[7] $x + y \leq z$ iff $x \leq z - y$.

(86)[8] $x \leq y + z$ iff $x - y \leq z$.

(92)[9] If $x \leq y$ and $z \leq t$, then $x - t \leq y - z$ and if $x < y$ and $z \leq t$ or $x \leq y$ and $z < t$, then $x - t < y - z$.

(93) $0 \leq x \cdot x$.

REFERENCES

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[3] Andrzej Trybulec. Subsets of real numbers. *Journal of Formalized Mathematics*, Addenda, 2003. `http://mizar.org/JFM/Addenda/numbers.html`.

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[6] The proposition (68) has been removed.
[7] The propositions (78)–(83) have been removed.
[8] The proposition (85) has been removed.
[9] The propositions (87)–(91) have been removed.