JOURNAL OF FORMALIZED MATHEMATICS
Volume 15, Released 2003, Published 2003
Inst. of Computer Science, Univ. of Białystok

High Speed Adder Algorithm with Radix-2^k Sub Signed-Digit Number

Masaaki Niimura
Shinshu University
Nagano

Yasushi Fuwa
Shinshu University
Nagano

Summary. In this article, a new adder algorithm using Radix-2^k sub signed-digit numbers is defined and properties for the hardware-realization is discussed.

Until now, we proposed Radix-2^k sub signed-digit numbers in consideration of the hardware realization. In this article, we proposed High Speed Adder Algorithm using this Radix-2^k sub signed-digit numbers. This method has two ways to speed up at hardware-realization. One is 'bit compare' at carry calculation, it is proposed in another article. Other is carry calculation between two numbers. We proposed that n digits Radix-2^k signed-digit numbers is expressed in $n+1$ digits Radix-2^k sub signed-digit numbers, and addition result of two $n+1$ digits Radix-2^k sub signed-digit numbers is expressed in $n+1$ digits. In this way, carry operation between two Radix-2^k sub signed-digit numbers can be processed at $n+1$ digit adder circuit and additional circuit to operate carry is not needed.

In the first section of this article, we prepared some useful theorems for operation of Radix-2^k numbers. In the second section, we proved some properties about carry on Radix-2^k sub signed-digit numbers. In the last section, we defined the new addition operation using Radix-2^k sub signed-digit numbers, and we clarified its correctness.

MML Identifier: `RADIX_4`.

WWW: `http://mizar.org/JFM/Vol15/radix_4.html`

The articles [9], [11], [10], [1], [4], [3], [8], [2], [6], [5], and [7] provide the notation and terminology for this paper.

1. Preliminaries

In this paper i, n, m, k, x, y denote natural numbers.

The following proposition is true

(1) For every natural number k such that $2 \leq k$ holds $2 < \mathrm{Radix}\,k$.

2. Carry Operation at $n+1$ Digits Radix-2^k Sub Signed-Digit Number

We now state several propositions:

(2) For all integers x, y and for every natural number k such that $3 \leq k$ holds $\mathrm{SDSubAddCarry}(\mathrm{SDSubAddCarry}(x,k) + \mathrm{SDSubAddCarry}(y,k), k) = 0$.

(3) If $2 \leq k$, then $\mathrm{DigA\_SDSub}(\mathrm{SD2SDSub}\,\mathrm{DecSD}(m,n,k), n+1) = \mathrm{SDSubAddCarry}(\mathrm{DigA}(\mathrm{DecSD}(m,n,k),n),k)$.

(4) If $2 \le k$ and m is represented by 1, k, then $\mathrm{DigA\_SDSub}(\mathrm{SD2SDSubDecSD}(m,1,k),1+1) = \mathrm{SDSubAddCarry}(m,k)$.

(5) Let k, x, n be natural numbers. Suppose $n \ge 1$ and $k \ge 3$ and x is represented by $n+1$, k. Then $\mathrm{DigA\_SDSub}(\mathrm{SD2SDSubDecSD}(x \bmod (\mathrm{Radix}\,k)^n,n,k),n+1) = \mathrm{SDSubAddCarry}(\mathrm{DigA}(\mathrm{DecSD}(x,n,k),n),k)$.

(6) If $2 \le k$ and m is represented by 1, k, then $\mathrm{DigA\_SDSub}(\mathrm{SD2SDSubDecSD}(m,1,k),1) = m - \mathrm{SDSubAddCarry}(m,k) \cdot \mathrm{Radix}\,k$.

(7) Let k, x, n be natural numbers. Suppose $n \ge 1$ and $k \ge 2$ and x is represented by $n+1, k$. Then $(\mathrm{Radix}\,k)^n \cdot \mathrm{DigA\_SDSub}(\mathrm{SD2SDSubDecSD}(x,n+1,k),n+1) = ((\mathrm{Radix}\,k)^n \cdot \mathrm{DigA}(\mathrm{DecSD}(x,n+1,k),n+1) - (\mathrm{Radix}\,k)^{n+1} \cdot \mathrm{SDSubAddCarry}(\mathrm{DigA}(\mathrm{DecSD}(x,n+1,k),n+1),k)) + (\mathrm{Radix}\,k)^n \cdot \mathrm{SDSubAddCarry}(\mathrm{DigA}(\mathrm{DecSD}(x,n+1,k),n),k)$.

3. Definition for Adder Operation on Radix-2^k Sub Signed-Digit Number

Let i, n, k be natural numbers, let x be a n-tuple of $k-\mathrm{SD\_Sub}$, and let y be a n-tuple of $k-\mathrm{SD\_Sub}$. Let us assume that $i \in \mathrm{Seg}\,n$ and $k \ge 2$. The functor $\mathrm{SDSubAddDigit}(x,y,i,k)$ yields an element of $k-\mathrm{SD\_Sub}$ and is defined by:

(Def. 1) $\mathrm{SDSubAddDigit}(x,y,i,k) = \mathrm{SDSubAddData}(\mathrm{DigA\_SDSub}(x,i) + \mathrm{DigA\_SDSub}(y,i),k) + \mathrm{SDSubAddCarry}(\mathrm{DigA\_SDSub}(x,i -' 1) + \mathrm{DigA\_SDSub}(y,i -' 1),k)$.

Let n, k be natural numbers and let x, y be n-tuples of $k-\mathrm{SD\_Sub}$. The functor $x' +' y$ yields a n-tuple of $k-\mathrm{SD\_Sub}$ and is defined by:

(Def. 2) For every i such that $i \in \mathrm{Seg}\,n$ holds $\mathrm{DigA\_SDSub}(x' +' y,i) = \mathrm{SDSubAddDigit}(x,y,i,k)$.

One can prove the following propositions:

(8) For every i such that $i \in \mathrm{Seg}\,n$ holds if $2 \le k$, then $\mathrm{SDSubAddDigit}(\mathrm{SD2SDSubDecSD}(x,n+1,k),\mathrm{SD2SDSubDecSD}(y,n+1,k),i,k) = \mathrm{SDSubAddDigit}(\mathrm{SD2SDSubDecSD}(x \bmod (\mathrm{Radix}\,k)^n,n,k),\mathrm{SD2SDSubDecSD}($ $(\mathrm{Radix}\,k)^n,n,k),i,k)$.

(9) Let given n. Suppose $n \ge 1$. Let given k, x, y. Suppose $k \ge 3$ and x is represented by n, k and y is represented by n, k. Then $x + y = \mathrm{SDSub2IntOut}\,\mathrm{SD2SDSubDecSD}(x,n,k)' +' \mathrm{SD2SDSubDecSD}(y,n,k)$.

References

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/nat_1.html`.

[2] Grzegorz Bancerek. Joining of decorated trees. *Journal of Formalized Mathematics*, 5, 1993. `http://mizar.org/JFM/Vol5/trees_4.html`.

[3] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/finseq_1.html`.

[4] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/funct_1.html`.

[5] Yoshinori Fujisawa and Yasushi Fuwa. Definitions of radix-2^k signed-digit number and its adder algorithm. *Journal of Formalized Mathematics*, 11, 1999. `http://mizar.org/JFM/Vol11/radix_1.html`.

[6] Andrzej Kondracki. The Chinese Remainder Theorem. *Journal of Formalized Mathematics*, 9, 1997. `http://mizar.org/JFM/Vol9/wsierp_1.html`.

[7] Masaaki Niimura and Yasushi Fuwa. Improvement of radix-2^k signed-digit number for high speed circuit. *Journal of Formalized Mathematics*, 15, 2003. `http://mizar.org/JFM/Vol15/radix_3.html`.

[8] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. *Journal of Formalized Mathematics*, 5, 1993. `http://mizar.org/JFM/Vol5/binarith.html`.

[9] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. `http://mizar.org/JFM/Axiomatics/tarski.html`.

[10] Michał J. Trybulec. Integers. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/int_1.html`.

[11] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/subset_1.html`.

Received January 3, 2003

Published January 2, 2004