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Some Basic Properties of Many Sorted Sets

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The articles [8], [2], [10], [1], [4], [5], [3], [9], [7], and [6] provide the notation and terminology for this paper.

1. Preliminaries

For simplicity, we follow the rules: i, I denote sets, f denotes a function, x, x_1, x_2, y, A, B, X, Y, Z denote many sorted sets indexed by I, J denotes a non empty set, and N_1 denotes a many sorted set indexed by J.

Next we state three propositions:

(1) For every set X and for every many sorted set M indexed by I such that $i \in I$ holds $\mathrm{dom}(M{+\cdot}(i \dot{\longmapsto} X)) = I$.

(2) If $f = \emptyset$, then f is a many sorted set indexed by $\emptyset$.

(3) If I is non empty, then there exists no X which is empty yielding and non-empty.

2. Singleton and unordered pairs

Let us consider I, A. The functor $\{A\}$ yielding a many sorted set indexed by I is defined by:

(Def. 1) For every i such that $i \in I$ holds $\{A\}(i) = \{A(i)\}$.

Let us consider I, A. One can check that $\{A\}$ is non-empty and locally-finite.

Let us consider I, A, B. The functor $\{A,B\}$ yielding a many sorted set indexed by I is defined by:

(Def. 2) For every i such that $i \in I$ holds $\{A,B\}(i) = \{A(i),B(i)\}$.

Let us notice that the functor $\{A,B\}$ is commutative.

Let us consider I, A, B. Note that $\{A,B\}$ is non-empty and locally-finite.

The following propositions are true:

(4) $X = \{y\}$ iff for every x holds $x \in X$ iff $x = y$.

(5) If for every x holds $x \in X$ iff $x = x_1$ or $x = x_2$, then $X = \{x_1,x_2\}$.

(6) If $X = \{x_1,x_2\}$, then for every x such that $x = x_1$ or $x = x_2$ holds $x \in X$.

(7) $\{N_1\} \neq \mathbf{0}_I$.

(8) If $x \in \{A\}$, then $x = A$.

(9) $x \in \{x\}$.

(10) If $x = A$ or $x = B$, then $x \in \{A, B\}$.

(11) $\{A\} \cup \{B\} = \{A, B\}$.

(12) $\{x, x\} = \{x\}$.

(14)[1] If $\{A\} \subseteq \{B\}$, then $A = B$.

(15) If $\{x\} = \{y\}$, then $x = y$.

(16) If $\{x\} = \{A, B\}$, then $x = A$ and $x = B$.

(17) If $\{x\} = \{A, B\}$, then $A = B$.

(18) $\{x\} \subseteq \{x, y\}$ and $\{y\} \subseteq \{x, y\}$.

(19) If $\{x\} \cup \{y\} = \{x\}$ or $\{x\} \cup \{y\} = \{y\}$, then $x = y$.

(20) $\{x\} \cup \{x, y\} = \{x, y\}$.

(22)[2] If I is non empty and $\{x\} \cap \{y\} = \mathbf{0}_I$, then $x \neq y$.

(23) If $\{x\} \cap \{y\} = \{x\}$ or $\{x\} \cap \{y\} = \{y\}$, then $x = y$.

(24) $\{x\} \cap \{x, y\} = \{x\}$ and $\{y\} \cap \{x, y\} = \{y\}$.

(25) If I is non empty and $\{x\} \setminus \{y\} = \{x\}$, then $x \neq y$.

(26) If $\{x\} \setminus \{y\} = \mathbf{0}_I$, then $x = y$.

(27) $\{x\} \setminus \{x, y\} = \mathbf{0}_I$ and $\{y\} \setminus \{x, y\} = \mathbf{0}_I$.

(28) If $\{x\} \subseteq \{y\}$, then $\{x\} = \{y\}$.

(29) If $\{x, y\} \subseteq \{A\}$, then $x = A$ and $y = A$.

(30) If $\{x, y\} \subseteq \{A\}$, then $\{x, y\} = \{A\}$.

(31) $2^{\{x\}} = \{\mathbf{0}_I, \{x\}\}$.

(32) $\{A\} \subseteq 2^A$.

(33) $\bigcup\{x\} = x$.

(34) $\bigcup\{\{x\}, \{y\}\} = \{x, y\}$.

(35) $\bigcup\{A, B\} = A \cup B$.

(36) $\{x\} \subseteq X$ iff $x \in X$.

(37) $\{x_1, x_2\} \subseteq X$ iff $x_1 \in X$ and $x_2 \in X$.

(38) If $A = \mathbf{0}_I$ or $A = \{x_1\}$ or $A = \{x_2\}$ or $A = \{x_1, x_2\}$, then $A \subseteq \{x_1, x_2\}$.

[1] The proposition (13) has been removed.
[2] The proposition (21) has been removed.

3. Sum of unordered pairs (or a singleton) and a set

Next we state several propositions:

(39) If $x \in A$ or $x = B$, then $x \in A \cup \{B\}$.

(40) $A \cup \{x\} \subseteq B$ iff $x \in B$ and $A \subseteq B$.

(41) If $\{x\} \cup X = X$, then $x \in X$.

(42) If $x \in X$, then $\{x\} \cup X = X$.

(43) $\{x, y\} \cup A = A$ iff $x \in A$ and $y \in A$.

(44) If I is non empty, then $\{x\} \cup X \neq \mathbf{0}_I$.

(45) If I is non empty, then $\{x, y\} \cup X \neq \mathbf{0}_I$.

4. Intersection of unordered pairs (or a singleton) and a set

We now state several propositions:

(46) If $X \cap \{x\} = \{x\}$, then $x \in X$.

(47) If $x \in X$, then $X \cap \{x\} = \{x\}$.

(48) $x \in X$ and $y \in X$ iff $\{x, y\} \cap X = \{x, y\}$.

(49) If I is non empty and $\{x\} \cap X = \mathbf{0}_I$, then $x \notin X$.

(50) If I is non empty and $\{x, y\} \cap X = \mathbf{0}_I$, then $x \notin X$ and $y \notin X$.

5. Difference of unordered pairs (or a singleton) and a set

The following propositions are true:

(51) If $y \in X \setminus \{x\}$, then $y \in X$.

(52) If I is non empty and $y \in X \setminus \{x\}$, then $y \neq x$.

(53) If I is non empty and $X \setminus \{x\} = X$, then $x \notin X$.

(54) If I is non empty and $\{x\} \setminus X = \{x\}$, then $x \notin X$.

(55) $\{x\} \setminus X = \mathbf{0}_I$ iff $x \in X$.

(56) If I is non empty and $\{x, y\} \setminus X = \{x\}$, then $x \notin X$.

(58)[3] If I is non empty and $\{x, y\} \setminus X = \{x, y\}$, then $x \notin X$ and $y \notin X$.

(59) $\{x, y\} \setminus X = \mathbf{0}_I$ iff $x \in X$ and $y \in X$.

(60) If $X = \mathbf{0}_I$ or $X = \{x\}$ or $X = \{y\}$ or $X = \{x, y\}$, then $X \setminus \{x, y\} = \mathbf{0}_I$.

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[3] The proposition (57) has been removed.

6. CARTESIAN PRODUCT

One can prove the following propositions:

(61) If $X = \mathbf{0}_I$ or $Y = \mathbf{0}_I$, then $[\![X,Y]\!] = \mathbf{0}_I$.

(62) If X is non-empty and Y is non-empty and $[\![X,Y]\!] = [\![Y,X]\!]$, then $X = Y$.

(63) If $[\![X,X]\!] = [\![Y,Y]\!]$, then $X = Y$.

(64) If Z is non-empty and if $[\![X,Z]\!] \subseteq [\![Y,Z]\!]$ or $[\![Z,X]\!] \subseteq [\![Z,Y]\!]$, then $X \subseteq Y$.

(65) If $X \subseteq Y$, then $[\![X,Z]\!] \subseteq [\![Y,Z]\!]$ and $[\![Z,X]\!] \subseteq [\![Z,Y]\!]$.

(66) If $x_1 \subseteq A$ and $x_2 \subseteq B$, then $[\![x_1,x_2]\!] \subseteq [\![A,B]\!]$.

(67) $[\![X \cup Y,Z]\!] = [\![X,Z]\!] \cup [\![Y,Z]\!]$ and $[\![Z,X \cup Y]\!] = [\![Z,X]\!] \cup [\![Z,Y]\!]$.

(68) $[\![x_1 \cup x_2,A \cup B]\!] = [\![x_1,A]\!] \cup [\![x_1,B]\!] \cup [\![x_2,A]\!] \cup [\![x_2,B]\!]$.

(69) $[\![X \cap Y,Z]\!] = [\![X,Z]\!] \cap [\![Y,Z]\!]$ and $[\![Z,X \cap Y]\!] = [\![Z,X]\!] \cap [\![Z,Y]\!]$.

(70) $[\![x_1 \cap x_2,A \cap B]\!] = [\![x_1,A]\!] \cap [\![x_2,B]\!]$.

(71) If $A \subseteq X$ and $B \subseteq Y$, then $[\![A,Y]\!] \cap [\![X,B]\!] = [\![A,B]\!]$.

(72) $[\![X \setminus Y,Z]\!] = [\![X,Z]\!] \setminus [\![Y,Z]\!]$ and $[\![Z,X \setminus Y]\!] = [\![Z,X]\!] \setminus [\![Z,Y]\!]$.

(73) $[\![x_1,x_2]\!] \setminus [\![A,B]\!] = [\![x_1 \setminus A,x_2]\!] \cup [\![x_1,x_2 \setminus B]\!]$.

(74) If $x_1 \cap x_2 = \mathbf{0}_I$ or $A \cap B = \mathbf{0}_I$, then $[\![x_1,A]\!] \cap [\![x_2,B]\!] = \mathbf{0}_I$.

(75) If X is non-empty, then $[\![\{x\},X]\!]$ is non-empty and $[\![X,\{x\}]\!]$ is non-empty.

(76) $[\![\{x,y\},X]\!] = [\![\{x\},X]\!] \cup [\![\{y\},X]\!]$ and $[\![X,\{x,y\}]\!] = [\![X,\{x\}]\!] \cup [\![X,\{y\}]\!]$.

(77) If x_1 is non-empty and A is non-empty and $[\![x_1,A]\!] = [\![x_2,B]\!]$, then $x_1 = x_2$ and $A = B$.

(78) If $X \subseteq [\![X,Y]\!]$ or $X \subseteq [\![Y,X]\!]$, then $X = \mathbf{0}_I$.

(79) If $A \in [\![x,y]\!]$ and $A \in [\![X,Y]\!]$, then $A \in [\![x \cap X,y \cap Y]\!]$.

(80) If $[\![x,X]\!] \subseteq [\![y,Y]\!]$ and $[\![x,X]\!]$ is non-empty, then $x \subseteq y$ and $X \subseteq Y$.

(81) If $A \subseteq X$, then $[\![A,A]\!] \subseteq [\![X,X]\!]$.

(82) If $X \cap Y = \mathbf{0}_I$, then $[\![X,Y]\!] \cap [\![Y,X]\!] = \mathbf{0}_I$.

(83) If A is non-empty and if $[\![A,B]\!] \subseteq [\![X,Y]\!]$ or $[\![B,A]\!] \subseteq [\![Y,X]\!]$, then $B \subseteq Y$.

(84) If $x \subseteq [\![A,B]\!]$ and $y \subseteq [\![X,Y]\!]$, then $x \cup y \subseteq [\![A \cup X,B \cup Y]\!]$.

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