JOURNAL OF FORMALIZED MATHEMATICS
Volume 2, Released 1990, Published 2003
Inst. of Computer Science, Univ. of Białystok

Ordered Rings - Part II

Michał Muzalewski
Warsaw University
Białystok

Lesław W. Szczerba
Siedlce University

Summary. This series of papers is devoted to the notion of the ordered ring, and one of its most important cases: the notion of ordered field. It follows the results of [5]. The idea of the notion of order in the ring is based on that of positive cone i.e. the set of positive elements. Positive cone has to contain at least squares of all elements, and has to be closed under sum and product. Therefore the key notions of this theory are that of square, sum of squares, product of squares, etc. and finally elements generated from squares by means of sums and products. Part II contains classification of sums of such elements.

MML Identifier: O_RING_2.

WWW: http://mizar.org/JFM/Vol2/o_ring_2.html

The articles [2], [1], [6], [3], and [4] provide the notation and terminology for this paper.

In this paper R denotes a non empty double loop structure and x, y denote scalars of R.

Next we state a number of propositions:

(1) If x is a square and y is a square or x is a sum of squares and y is a square, then $x+y$ is a sum of squares.

(2) Suppose that

(i) x is a sum of products of squares and y is a square, or

(ii) x is a sum of products of squares and y is a product of squares.

Then $x+y$ is a sum of products of squares.

(3) Suppose that

(i) x is an amalgam of squares and y is a product of squares, or

(ii) x is an amalgam of squares and y is an amalgam of squares, or

(iii) x is a sum of amalgams of squares and y is a square, or

(iv) x is a sum of amalgams of squares and y is a product of squares, or

(v) x is a sum of amalgams of squares and y is an amalgam of squares.

Then $x+y$ is a sum of amalgams of squares.

(4) Suppose that

(i) x is a square and y is a sum of squares, or

(ii) x is a square and y is a product of squares, or

(iii) x is a square and y is a sum of products of squares, or

(iv) x is a square and y is an amalgam of squares, or

(v) x is a square and y is a sum of amalgams of squares, or

(vi) x is a square and y is generated from squares.

Then $x+y$ is generated from squares.

(5) Suppose that

(i) x is a sum of squares and y is a sum of squares, or

(ii) x is a sum of squares and y is a product of squares, or

(iii) x is a sum of squares and y is a sum of products of squares, or

(iv) x is a sum of squares and y is an amalgam of squares, or

(v) x is a sum of squares and y is a sum of amalgams of squares, or

(vi) x is a sum of squares and y is generated from squares.

Then $x+y$ is generated from squares.

(6) Suppose that x is a product of squares and y is a square or x is a product of squares and y is a sum of squares or x is a product of squares and y is a product of squares or x is a product of squares and y is a sum of products of squares or x is a product of squares and y is an amalgam of squares or x is a product of squares and y is a sum of amalgams of squares or x is a product of squares and y is generated from squares. Then $x+y$ is generated from squares.

(7) Suppose that

(i) x is a sum of products of squares and y is a sum of squares, or

(ii) x is a sum of products of squares and y is a sum of products of squares, or

(iii) x is a sum of products of squares and y is an amalgam of squares, or

(iv) x is a sum of products of squares and y is a sum of amalgams of squares, or

(v) x is a sum of products of squares and y is generated from squares.

Then $x+y$ is generated from squares.

(8) Suppose that

(i) x is an amalgam of squares and y is a square, or

(ii) x is an amalgam of squares and y is a sum of squares, or

(iii) x is an amalgam of squares and y is a sum of products of squares, or

(iv) x is an amalgam of squares and y is a sum of amalgams of squares, or

(v) x is an amalgam of squares and y is generated from squares.

Then $x+y$ is generated from squares.

(9) Suppose that

(i) x is a sum of amalgams of squares and y is a sum of squares, or

(ii) x is a sum of amalgams of squares and y is a sum of products of squares, or

(iii) x is a sum of amalgams of squares and y is a sum of amalgams of squares, or

(iv) x is a sum of amalgams of squares and y is generated from squares.

Then $x+y$ is generated from squares.

(10) Suppose that x is generated from squares and y is a square or x is generated from squares and y is a sum of squares or x is generated from squares and y is a product of squares or x is generated from squares and y is a sum of products of squares or x is generated from squares and y is an amalgam of squares or x is generated from squares and y is a sum of amalgams of squares or x is generated from squares and y is generated from squares. Then $x+y$ is generated from squares.

References

[1] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/finseq_1.html`.

[2] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/funct_1.html`.

[3] Eugeniusz Kusak, Wojciech Leończuk, and Michał Muzalewski. Abelian groups, fields and vector spaces. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/vectsp_1.html`.

[4] Michał Muzalewski and Lesław W. Szczerba. Ordered rings - part I. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/o_ring_1.html`.

[5] Wanda Szmielew. *From Affine to Euclidean Geometry*, volume 27. PWN – D.Reidel Publ. Co., Warszawa – Dordrecht, 1983.

[6] Wojciech A. Trybulec. Vectors in real linear space. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/rlvect_1.html`.

Received October 11, 1990

Published January 2, 2004