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Groups, Rings, Left- and Right-Modules[1]

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Summary. The notion of group was defined as a group structure introduced in the article [1]. The article contains the basic properties of groups, rings, left- and right-modules of an associative ring.

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The articles [3], [1], and [2] provide the notation and terminology for this paper.

A group is an add-associative right zeroed right complementable non empty loop structure.

The following two propositions are true:

(13)[1] Let K be an add-associative right zeroed right complementable right distributive right unital non empty double loop structure and a be an element of K. Then $a \cdot -\mathbf{1}_K = -a$.

(14) Let K be an add-associative right zeroed right complementable left distributive left unital non empty double loop structure and a be an element of K. Then $(-\mathbf{1}_K) \cdot a = -a$.

For simplicity, we use the following convention: R is an Abelian add-associative right zeroed right complementable associative left unital right unital distributive non empty double loop structure, F is a non degenerated field-like ring, x is a scalar of F, V is an add-associative right zeroed right complementable vector space-like non empty vector space structure over F, and v is a vector of V.

One can prove the following two propositions:

(25)[2] $x \cdot v = 0_V$ iff $x = 0_F$ or $v = 0_V$.

(26) If $x \neq 0_F$, then $x^{-1} \cdot (x \cdot v) = v$.

In the sequel V denotes an add-associative right zeroed right complementable right module-like non empty right module structure over R, x denotes a scalar of R, and v, w denote vectors of V.

Next we state four propositions:

(37)[3] $v \cdot 0_R = 0_V$ and $v \cdot -\mathbf{1}_R = -v$ and $0_V \cdot x = 0_V$.

(38) $-v \cdot x = v \cdot -x$ and $w - v \cdot x = w + v \cdot -x$.

(39) $(-v) \cdot x = -v \cdot x$.

[1] Supported by RPBP.III-24.C6.

[1] The propositions (1)–(12) have been removed.
[2] The propositions (15)–(24) have been removed.
[3] The propositions (27)–(36) have been removed.

(40) $(v-w)\cdot x = v\cdot x - w\cdot x$.

In the sequel F is a non degenerated field-like ring, x is a scalar of F, V is an add-associative right zeroed right complementable right module-like non empty right module structure over F, and v is a vector of V.

Next we state two propositions:

(42)[4] $v\cdot x = 0_V$ iff $x = 0_F$ or $v = 0_V$.

(43) If $x \neq 0_F$, then $v\cdot x\cdot x^{-1} = v$.

References

[1] Eugeniusz Kusak, Wojciech Leończuk, and Michał Muzalewski. Abelian groups, fields and vector spaces. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/vectsp_1.html`.

[2] Michał Muzalewski. Construction of rings and left-, right-, and bi-modules over a ring. *Journal of Formalized Mathematics*, 2, 1990. `http://mizar.org/JFM/Vol2/vectsp_2.html`.

[3] Wojciech A. Trybulec. Vectors in real linear space. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/rlvect_1.html`.

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[4] The proposition (41) has been removed.