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Propositional Calculus[1]

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Summary. We develop the classical propositional calculus, following [3].

MML Identifier: LUKASI_1.

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The articles [4], [1], and [2] provide the notation and terminology for this paper.

We follow the rules: p, q, r, s are elements of CQC-WFF and X is a subset of CQC-WFF.

One can prove the following propositions:

(1) $(p \Rightarrow q) \Rightarrow ((q \Rightarrow r) \Rightarrow (p \Rightarrow r)) \in \mathrm{Taut}$.

(2) If $p \Rightarrow q \in \mathrm{Taut}$, then $(q \Rightarrow r) \Rightarrow (p \Rightarrow r) \in \mathrm{Taut}$.

(3) If $p \Rightarrow q \in \mathrm{Taut}$ and $q \Rightarrow r \in \mathrm{Taut}$, then $p \Rightarrow r \in \mathrm{Taut}$.

(4) $p \Rightarrow p \in \mathrm{Taut}$.

(5) $q \Rightarrow (p \Rightarrow q) \in \mathrm{Taut}$.

(6) $((p \Rightarrow q) \Rightarrow r) \Rightarrow (q \Rightarrow r) \in \mathrm{Taut}$.

(7) $q \Rightarrow ((q \Rightarrow p) \Rightarrow p) \in \mathrm{Taut}$.

(8) $(s \Rightarrow (q \Rightarrow p)) \Rightarrow (q \Rightarrow (s \Rightarrow p)) \in \mathrm{Taut}$.

(9) $(q \Rightarrow r) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r)) \in \mathrm{Taut}$.

(10) $(q \Rightarrow (q \Rightarrow r)) \Rightarrow (q \Rightarrow r) \in \mathrm{Taut}$.

(11) $(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r)) \in \mathrm{Taut}$.

(12) $\neg \mathrm{VERUM} \Rightarrow p \in \mathrm{Taut}$.

(13) If $q \in \mathrm{Taut}$, then $p \Rightarrow q \in \mathrm{Taut}$.

(14) If $p \in \mathrm{Taut}$, then $(p \Rightarrow q) \Rightarrow q \in \mathrm{Taut}$.

(15) If $s \Rightarrow (q \Rightarrow p) \in \mathrm{Taut}$, then $q \Rightarrow (s \Rightarrow p) \in \mathrm{Taut}$.

(16) If $s \Rightarrow (q \Rightarrow p) \in \mathrm{Taut}$ and $q \in \mathrm{Taut}$, then $s \Rightarrow p \in \mathrm{Taut}$.

(17) If $s \Rightarrow (q \Rightarrow p) \in \mathrm{Taut}$ and $q \in \mathrm{Taut}$ and $s \in \mathrm{Taut}$, then $p \in \mathrm{Taut}$.

(18) If $q \Rightarrow (q \Rightarrow r) \in \mathrm{Taut}$, then $q \Rightarrow r \in \mathrm{Taut}$.

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(19) If $p \Rightarrow (q \Rightarrow r) \in \mathrm{Taut}$, then $(p \Rightarrow q) \Rightarrow (p \Rightarrow r) \in \mathrm{Taut}$.

(20) If $p \Rightarrow (q \Rightarrow r) \in \mathrm{Taut}$ and $p \Rightarrow q \in \mathrm{Taut}$, then $p \Rightarrow r \in \mathrm{Taut}$.

(21) If $p \Rightarrow (q \Rightarrow r) \in \mathrm{Taut}$ and $p \Rightarrow q \in \mathrm{Taut}$ and $p \in \mathrm{Taut}$, then $r \in \mathrm{Taut}$.

(22) If $p \Rightarrow (q \Rightarrow r) \in \mathrm{Taut}$ and $p \Rightarrow (r \Rightarrow s) \in \mathrm{Taut}$, then $p \Rightarrow (q \Rightarrow s) \in \mathrm{Taut}$.

(23) $p \Rightarrow \mathrm{VERUM} \in \mathrm{Taut}$.

(24) $(\neg p \Rightarrow \neg q) \Rightarrow (q \Rightarrow p) \in \mathrm{Taut}$.

(25) $\neg\neg p \Rightarrow p \in \mathrm{Taut}$.

(26) $(p \Rightarrow q) \Rightarrow (\neg q \Rightarrow \neg p) \in \mathrm{Taut}$.

(27) $p \Rightarrow \neg\neg p \in \mathrm{Taut}$.

(28) $(\neg\neg p \Rightarrow q) \Rightarrow (p \Rightarrow q) \in \mathrm{Taut}$ and $(p \Rightarrow q) \Rightarrow (\neg\neg p \Rightarrow q) \in \mathrm{Taut}$.

(29) $(p \Rightarrow \neg\neg q) \Rightarrow (p \Rightarrow q) \in \mathrm{Taut}$ and $(p \Rightarrow q) \Rightarrow (p \Rightarrow \neg\neg q) \in \mathrm{Taut}$.

(30) $(p \Rightarrow \neg q) \Rightarrow (q \Rightarrow \neg p) \in \mathrm{Taut}$.

(31) $(\neg p \Rightarrow q) \Rightarrow (\neg q \Rightarrow p) \in \mathrm{Taut}$.

(32) $(p \Rightarrow \neg p) \Rightarrow \neg p \in \mathrm{Taut}$.

(33) $\neg p \Rightarrow (p \Rightarrow q) \in \mathrm{Taut}$.

(34) $p \Rightarrow q \in \mathrm{Taut}$ iff $\neg q \Rightarrow \neg p \in \mathrm{Taut}$.

(35) If $\neg p \Rightarrow \neg q \in \mathrm{Taut}$, then $q \Rightarrow p \in \mathrm{Taut}$.

(36) $p \in \mathrm{Taut}$ iff $\neg\neg p \in \mathrm{Taut}$.

(37) $p \Rightarrow q \in \mathrm{Taut}$ iff $p \Rightarrow \neg\neg q \in \mathrm{Taut}$.

(38) $p \Rightarrow q \in \mathrm{Taut}$ iff $\neg\neg p \Rightarrow q \in \mathrm{Taut}$.

(39) If $p \Rightarrow \neg q \in \mathrm{Taut}$, then $q \Rightarrow \neg p \in \mathrm{Taut}$.

(40) If $\neg p \Rightarrow q \in \mathrm{Taut}$, then $\neg q \Rightarrow p \in \mathrm{Taut}$.

(41) $\vdash (p \Rightarrow q) \Rightarrow ((q \Rightarrow r) \Rightarrow (p \Rightarrow r))$.

(42) If $\vdash p \Rightarrow q$, then $\vdash (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$.

(43) If $\vdash p \Rightarrow q$ and $\vdash q \Rightarrow r$, then $\vdash p \Rightarrow r$.

(44) $\vdash p \Rightarrow p$.

(45) $\vdash p \Rightarrow (q \Rightarrow p)$.

(46) If $\vdash p$, then $\vdash q \Rightarrow p$.

(47) $\vdash (s \Rightarrow (q \Rightarrow p)) \Rightarrow (q \Rightarrow (s \Rightarrow p))$.

(48) If $\vdash p \Rightarrow (q \Rightarrow r)$, then $\vdash q \Rightarrow (p \Rightarrow r)$.

(49) If $\vdash p \Rightarrow (q \Rightarrow r)$ and $\vdash q$, then $\vdash p \Rightarrow r$.

(50) $\vdash p \Rightarrow \mathrm{VERUM}$ and $\vdash \neg\,\mathrm{VERUM} \Rightarrow p$.

(51) $\vdash p \Rightarrow ((p \Rightarrow q) \Rightarrow q)$.

(52) $\vdash (q \Rightarrow (q \Rightarrow r)) \Rightarrow (q \Rightarrow r)$.

(53) If $\vdash q \Rightarrow (q \Rightarrow r)$, then $\vdash q \Rightarrow r$.

(54) $\vdash (p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$.

(55) If $\vdash p \Rightarrow (q \Rightarrow r)$, then $\vdash (p \Rightarrow q) \Rightarrow (p \Rightarrow r)$.

(56) If $\vdash p \Rightarrow (q \Rightarrow r)$ and $\vdash p \Rightarrow q$, then $\vdash p \Rightarrow r$.

(57) $\vdash ((p \Rightarrow q) \Rightarrow r) \Rightarrow (q \Rightarrow r)$.

(58) If $\vdash (p \Rightarrow q) \Rightarrow r$, then $\vdash q \Rightarrow r$.

(59) $\vdash (p \Rightarrow q) \Rightarrow ((r \Rightarrow p) \Rightarrow (r \Rightarrow q))$.

(60) If $\vdash p \Rightarrow q$, then $\vdash (r \Rightarrow p) \Rightarrow (r \Rightarrow q)$.

(61) $\vdash (p \Rightarrow q) \Rightarrow (\neg q \Rightarrow \neg p)$.

(62) $\vdash (\neg p \Rightarrow \neg q) \Rightarrow (q \Rightarrow p)$.

(63) $\vdash \neg p \Rightarrow \neg q$ iff $\vdash q \Rightarrow p$.

(64) $\vdash p \Rightarrow \neg\neg p$.

(65) $\vdash \neg\neg p \Rightarrow p$.

(66) $\vdash \neg\neg p$ iff $\vdash p$.

(67) $\vdash (\neg\neg p \Rightarrow q) \Rightarrow (p \Rightarrow q)$.

(68) $\vdash \neg\neg p \Rightarrow q$ iff $\vdash p \Rightarrow q$.

(69) $\vdash (p \Rightarrow \neg\neg q) \Rightarrow (p \Rightarrow q)$.

(70) $\vdash p \Rightarrow \neg\neg q$ iff $\vdash p \Rightarrow q$.

(71) $\vdash (p \Rightarrow \neg q) \Rightarrow (q \Rightarrow \neg p)$.

(72) If $\vdash p \Rightarrow \neg q$, then $\vdash q \Rightarrow \neg p$.

(73) $\vdash (\neg p \Rightarrow q) \Rightarrow (\neg q \Rightarrow p)$.

(74) If $\vdash \neg p \Rightarrow q$, then $\vdash \neg q \Rightarrow p$.

(75) If $X \vdash p \Rightarrow q$, then $X \vdash (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$.

(76) If $X \vdash p \Rightarrow q$ and $X \vdash q \Rightarrow r$, then $X \vdash p \Rightarrow r$.

(77) $X \vdash p \Rightarrow p$.

(78) If $X \vdash p$, then $X \vdash q \Rightarrow p$.

(79) If $X \vdash p$, then $X \vdash (p \Rightarrow q) \Rightarrow q$.

(80) If $X \vdash p \Rightarrow (q \Rightarrow r)$, then $X \vdash q \Rightarrow (p \Rightarrow r)$.

(81) If $X \vdash p \Rightarrow (q \Rightarrow r)$ and $X \vdash q$, then $X \vdash p \Rightarrow r$.

(82) If $X \vdash p \Rightarrow (p \Rightarrow q)$, then $X \vdash p \Rightarrow q$.

(83) If $X \vdash (p \Rightarrow q) \Rightarrow r$, then $X \vdash q \Rightarrow r$.

(84) If $X \vdash p \Rightarrow (q \Rightarrow r)$, then $X \vdash (p \Rightarrow q) \Rightarrow (p \Rightarrow r)$.

(85) If $X \vdash p \Rightarrow (q \Rightarrow r)$ and $X \vdash p \Rightarrow q$, then $X \vdash p \Rightarrow r$.

(86) $X \vdash \neg p \Rightarrow \neg q$ iff $X \vdash q \Rightarrow p$.

(87) $X \vdash \neg\neg p$ iff $X \vdash p$.

(88) $X \vdash p \Rightarrow \neg\neg q$ iff $X \vdash p \Rightarrow q$.

(89) $X \vdash \neg\neg p \Rightarrow q$ iff $X \vdash p \Rightarrow q$.

(90) If $X \vdash p \Rightarrow \neg q$, then $X \vdash q \Rightarrow \neg p$.

(91) If $X \vdash \neg p \Rightarrow q$, then $X \vdash \neg q \Rightarrow p$.

(92) If $X \vdash p \Rightarrow \neg q$ and $X \vdash q$, then $X \vdash \neg p$.

(93) If $X \vdash \neg p \Rightarrow q$ and $X \vdash \neg q$, then $X \vdash p$.

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