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Gauges and Cages. Part II[1]

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The articles [23], [21], [2], [12], [24], [1], [4], [3], [19], [16], [22], [9], [20], [13], [10], [14], [15], [11], [18], [7], [17], [6], [5], and [8] provide the notation and terminology for this paper.

1. Preliminaries

For simplicity, we follow the rules: a, b, i, k, m, n denote natural numbers, r, s denote real numbers, D denotes a non empty subset of $\mathcal{E}^2_{\mathrm{T}}$, and C denotes a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$.

We now state the proposition

(1) For all sets A, B such that for every set x such that $x \in A$ there exists a set K such that $K \subseteq B$ and $x \subseteq \bigcup K$ holds $\bigcup A \subseteq \bigcup B$.

Let m be an even integer. Note that $m+2$ is even.

Let m be an odd integer. One can verify that $m+2$ is odd.

Let m be a non empty natural number. One can check that 2^m is even.

Let n be an even natural number and let m be a non empty natural number. One can verify that n^m is even.

One can prove the following propositions:

(2) If $r \neq 0$, then $\frac{1}{r} \cdot r^{i+1} = r^i$.

(3) If $\frac{r}{s}$ is an integer, then $-\lfloor \frac{r}{s} \rfloor = \lfloor \frac{-r}{s} \rfloor + 1$.

(4) If $\frac{r}{s}$ is an integer, then $-\lfloor \frac{r}{s} \rfloor = \lfloor \frac{-r}{s} \rfloor$.

(5) If $n > 0$ and $k \bmod n \neq 0$, then $-(k \div n) = (-k \div n) + 1$.

(6) If $n > 0$ and $k \bmod n = 0$, then $-(k \div n) = -k \div n$.

2. Gauges and Cages

Next we state a number of propositions:

(7) If $2 \leq m$ and $m < \mathrm{len}\,\mathrm{Gauge}(D,i)$ and $1 \leq a$ and $a \leq \mathrm{len}\,\mathrm{Gauge}(D,i)$ and $1 \leq b$ and $b \leq \mathrm{len}\,\mathrm{Gauge}(D,i+1)$, then $(\mathrm{Gauge}(D,i) \circ (m,a))_{\mathbf{1}} = (\mathrm{Gauge}(D,i+1) \circ (2 \cdot m -' 2, b))_{\mathbf{1}}$.

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(8) If $2 \leq n$ and $n < \operatorname{len}\operatorname{Gauge}(D,i)$ and $1 \leq a$ and $a \leq \operatorname{len}\operatorname{Gauge}(D,i)$ and $1 \leq b$ and $b \leq \operatorname{len}\operatorname{Gauge}(D,i+1)$, then $(\operatorname{Gauge}(D,i) \circ (a,n))_{\mathbf{2}} = (\operatorname{Gauge}(D,i+1) \circ (b, 2 \cdot n -' 2))_{\mathbf{2}}$.

(9) Let D be a compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $2 \leq m$ and $m+1 < \operatorname{len}\operatorname{Gauge}(D,i)$ and $2 \leq n$ and $n+1 < \operatorname{len}\operatorname{Gauge}(D,i)$. Then $\operatorname{cell}(\operatorname{Gauge}(D,i),m,n) = \operatorname{cell}(\operatorname{Gauge}(D,i+1), 2 \cdot m -' 2, 2 \cdot n -' 2) \cup \operatorname{cell}(\operatorname{Gauge}(D,i+1), 2 \cdot m -' 1, 2 \cdot n -' 2) \cup \operatorname{cell}(\operatorname{Gauge}(D,i+1), 2 \cdot m -' 2, 2 \cdot n -' 1) \cup \operatorname{cell}(\operatorname{Gauge}(D,i+1), 2 \cdot m -' 1, 2 \cdot n -' 1)$.

(10) Let D be a compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and k be a natural number. Suppose $2 \leq m$ and $m+1 < \operatorname{len}\operatorname{Gauge}(D,i)$ and $2 \leq n$ and $n+1 < \operatorname{len}\operatorname{Gauge}(D,i)$. Then $\operatorname{cell}(\operatorname{Gauge}(D,i),m,n) = \bigcup\{\operatorname{cell}(\operatorname{Gauge}(D,i+k),a,b); a$ ranges over natural numbers, b ranges over natural numbers: $(2^k \cdot m - 2^{k+1}) + 2 \leq a \ \wedge \ a \leq (2^k \cdot m - 2^k) + 1 \ \wedge \ (2^k \cdot n - 2^{k+1}) + 2 \leq b \ \wedge \ b \leq (2^k \cdot n - 2^k) + 1\}$.

(11) There exists a natural number i such that $1 \leq i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and $\mathrm{N}_{\max}(C) \in \operatorname{right\_cell}(\operatorname{Cage}(C,n), i, \operatorname{Gauge}(C,n))$.

(12) There exists a natural number i such that $1 \leq i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and $\mathrm{N}_{\max}(C) \in \operatorname{rightcell}(\operatorname{Cage}(C,n), i)$.

(13) There exists a natural number i such that $1 \leq i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and $\mathrm{E}_{\min}(C) \in \operatorname{right\_cell}(\operatorname{Cage}(C,n), i, \operatorname{Gauge}(C,n))$.

(14) There exists a natural number i such that $1 \leq i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and $\mathrm{E}_{\min}(C) \in \operatorname{rightcell}(\operatorname{Cage}(C,n), i)$.

(15) There exists a natural number i such that $1 \leq i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and $\mathrm{E}_{\max}(C) \in \operatorname{right\_cell}(\operatorname{Cage}(C,n), i, \operatorname{Gauge}(C,n))$.

(16) There exists a natural number i such that $1 \leq i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and $\mathrm{E}_{\max}(C) \in \operatorname{rightcell}(\operatorname{Cage}(C,n), i)$.

(17) There exists a natural number i such that $1 \leq i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and $\mathrm{S}_{\min}(C) \in \operatorname{right\_cell}(\operatorname{Cage}(C,n), i, \operatorname{Gauge}(C,n))$.

(18) There exists a natural number i such that $1 \leq i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and $\mathrm{S}_{\min}(C) \in \operatorname{rightcell}(\operatorname{Cage}(C,n), i)$.

(19) There exists a natural number i such that $1 \leq i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and $\mathrm{S}_{\max}(C) \in \operatorname{right\_cell}(\operatorname{Cage}(C,n), i, \operatorname{Gauge}(C,n))$.

(20) There exists a natural number i such that $1 \leq i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and $\mathrm{S}_{\max}(C) \in \operatorname{rightcell}(\operatorname{Cage}(C,n), i)$.

(21) There exists a natural number i such that $1 \leq i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and $\mathrm{W}_{\min}(C) \in \operatorname{right\_cell}(\operatorname{Cage}(C,n), i, \operatorname{Gauge}(C,n))$.

(22) There exists a natural number i such that $1 \leq i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and $\mathrm{W}_{\min}(C) \in \operatorname{rightcell}(\operatorname{Cage}(C,n), i)$.

(23) There exists a natural number i such that $1 \leq i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and $\mathrm{W}_{\max}(C) \in \operatorname{right\_cell}(\operatorname{Cage}(C,n), i, \operatorname{Gauge}(C,n))$.

(24) There exists a natural number i such that $1 \leq i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and $\mathrm{W}_{\max}(C) \in \operatorname{rightcell}(\operatorname{Cage}(C,n), i)$.

(25) There exists a natural number i such that $1 \leq i$ and $i \leq \operatorname{len}\operatorname{Gauge}(C,n)$ and $\mathrm{N}_{\min}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n))) = \operatorname{Gauge}(C,n) \circ (i, \operatorname{width}\operatorname{Gauge}(C,n))$.

(26) There exists a natural number i such that $1 \leq i$ and $i \leq \operatorname{len}\operatorname{Gauge}(C,n)$ and $\mathrm{N}_{\max}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n))) = \operatorname{Gauge}(C,n) \circ (i, \operatorname{width}\operatorname{Gauge}(C,n))$.

(27) There exists a natural number i such that $1 \leq i$ and $i \leq \operatorname{len}\operatorname{Gauge}(C,n)$ and $\operatorname{Gauge}(C,n) \circ (i, \operatorname{width}\operatorname{Gauge}(C,n)) \in \operatorname{rng}\operatorname{Cage}(C,n)$.

(28) There exists a natural number j such that $1 \leq j$ and $j \leq \operatorname{width}\operatorname{Gauge}(C,n)$ and $\mathrm{E}_{\mathrm{min}}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n))) = \operatorname{Gauge}(C,n) \circ (\operatorname{len}\operatorname{Gauge}(C,n), j)$.

(29) There exists a natural number j such that $1 \leq j$ and $j \leq \operatorname{width}\operatorname{Gauge}(C,n)$ and $\mathrm{E}_{\mathrm{max}}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n))) = \operatorname{Gauge}(C,n) \circ (\operatorname{len}\operatorname{Gauge}(C,n), j)$.

(30) There exists a natural number j such that $1 \leq j$ and $j \leq \operatorname{width}\operatorname{Gauge}(C,n)$ and $\operatorname{Gauge}(C,n) \circ (\operatorname{len}\operatorname{Gauge}(C,n), j) \in \operatorname{rng}\operatorname{Cage}(C,n)$.

(31) There exists a natural number i such that $1 \leq i$ and $i \leq \operatorname{len}\operatorname{Gauge}(C,n)$ and $\mathrm{S}_{\mathrm{min}}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n))) = \operatorname{Gauge}(C,n) \circ (i, 1)$.

(32) There exists a natural number i such that $1 \leq i$ and $i \leq \operatorname{len}\operatorname{Gauge}(C,n)$ and $\mathrm{S}_{\mathrm{max}}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n))) = \operatorname{Gauge}(C,n) \circ (i, 1)$.

(33) There exists a natural number i such that $1 \leq i$ and $i \leq \operatorname{len}\operatorname{Gauge}(C,n)$ and $\operatorname{Gauge}(C,n) \circ (i, 1) \in \operatorname{rng}\operatorname{Cage}(C,n)$.

(34) There exists a natural number j such that $1 \leq j$ and $j \leq \operatorname{width}\operatorname{Gauge}(C,n)$ and $\mathrm{W}_{\mathrm{min}}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n))) = \operatorname{Gauge}(C,n) \circ (1, j)$.

(35) There exists a natural number j such that $1 \leq j$ and $j \leq \operatorname{width}\operatorname{Gauge}(C,n)$ and $\mathrm{W}_{\mathrm{max}}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C,n))) = \operatorname{Gauge}(C,n) \circ (1, j)$.

(36) There exists a natural number j such that $1 \leq j$ and $j \leq \operatorname{width}\operatorname{Gauge}(C,n)$ and $\operatorname{Gauge}(C,n) \circ (1, j) \in \operatorname{rng}\operatorname{Cage}(C,n)$.

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