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On the Upper and Lower Approximations of the Curve[1]

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The articles [28], [32], [2], [15], [1], [5], [6], [4], [31], [16], [29], [17], [27], [13], [3], [25], [26], [10], [11], [8], [30], [14], [20], [18], [12], [23], [22], [24], [7], [9], [19], and [21] provide the notation and terminology for this paper.

In this paper n denotes a natural number.

Let C be a simple closed curve. The functor $\mathrm{UpperAppr}(C)$ yields a sequence of subsets of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ and is defined as follows:

(Def. 1) For every natural number i holds $(\mathrm{UpperAppr}(C))(i) = \mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,i)))$.

The functor $\mathrm{LowerAppr}(C)$ yielding a sequence of subsets of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ is defined as follows:

(Def. 2) For every natural number i holds $(\mathrm{LowerAppr}(C))(i) = \mathrm{LowerArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,i)))$.

Let C be a simple closed curve. The functor $\mathrm{NorthArc}(C)$ yielding a subset of $\mathcal{E}^2_{\mathrm{T}}$ is defined by:

(Def. 3) $\mathrm{NorthArc}(C) = \mathrm{Li}\,\mathrm{UpperAppr}(C)$.

The functor $\mathrm{SouthArc}(C)$ yields a subset of $\mathcal{E}^2_{\mathrm{T}}$ and is defined by:

(Def. 4) $\mathrm{SouthArc}(C) = \mathrm{Li}\,\mathrm{LowerAppr}(C)$.

We now state a number of propositions:

(1) For all natural numbers n, m such that $n \leq m$ and $n \neq 0$ holds $\frac{n+1}{n} \geq \frac{m+1}{m}$.

(2) Let E be a compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and m, j be natural numbers. Suppose $1 \leq m$ and $m \leq n$ and $1 \leq j$ and $j \leq \mathrm{width}\,\mathrm{Gauge}(E,n)$. Then $\mathcal{L}(\mathrm{Gauge}(E,n) \circ (\mathrm{Center}\,\mathrm{Gauge}(E,n), \mathrm{width}\,\mathrm{Gauge}(E,n)), \mathrm{Gauge}(E,n) \circ (\mathrm{Center}\,\mathrm{Gauge}(E,n), j)) \subseteq \mathcal{L}(\mathrm{Gauge}(E,m) \circ (\mathrm{Center}\,\mathrm{Gauge}(E,m), \mathrm{width}\,\mathrm{Gauge}(E,m)), \mathrm{Gauge}(E,n) \circ (\mathrm{Center}\,\mathrm{Gauge}(E,n), j))$.

(3) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and i, j be natural numbers. Suppose $1 \leq i$ and $i \leq \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (i,j) \in \widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i, \mathrm{width}\,\mathrm{Gauge}(C,n)), \mathrm{Gauge}(C,n) \circ (i,j))$ meets $\widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$.

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(4) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and n be a natural number. Suppose $n > 0$. Let i, j be natural numbers. Suppose $1 \leq i$ and $i \leq \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (i,j) \in \widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i, \mathrm{width}\,\mathrm{Gauge}(C,n)), \mathrm{Gauge}(C,n) \circ (i,j))$ meets $\mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$.

(5) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and j be a natural number. Suppose $\mathrm{Gauge}(C,n+1) \circ (\mathrm{Center}\,\mathrm{Gauge}(C,n+1), j) \in \mathrm{LowerArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n+1)))$ and $1 \leq j$ and $j \leq \mathrm{width}\,\mathrm{Gauge}(C,n+1)$. Then $\mathcal{L}(\mathrm{Gauge}(C,1) \circ (\mathrm{Center}\,\mathrm{Gauge}(C,1), \mathrm{width}\,\mathrm{Gauge}(C,1)), \mathrm{Gauge}(C,n+1) \circ (\mathrm{Center}\,\mathrm{Gauge}(C,n+1), j))$ meets $\mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n+1)))$.

(6) Let C be a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$, f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$, and k be a natural number. Suppose $1 \leq k$ and $k+1 \leq \mathrm{len}\,f$ and f is a sequence which elements belong to $\mathrm{Gauge}(C,n)$. Then $\rho(f_k, f_{k+1}) = \frac{\text{N-bound}(C) - \text{S-bound}(C)}{2^n}$ or $\rho(f_k, f_{k+1}) = \frac{\text{E-bound}(C) - \text{W-bound}(C)}{2^n}$.

(7) Let M be a symmetric triangle metric structure, r be a real number, and p, q, x be elements of M. If $p \in \mathrm{Ball}(x,r)$ and $q \in \mathrm{Ball}(x,r)$, then $\rho(p,q) < 2 \cdot r$.

(8) Let A be a subset of $\mathcal{E}^n_{\mathrm{T}}$, p be a point of $\mathcal{E}^n_{\mathrm{T}}$, and p' be a point of $\mathcal{E}^n$. Suppose $p = p'$. Let s be a real number. Suppose $s > 0$. Then $p \in \overline{A}$ if and only if for every real number r such that $0 < r$ and $r < s$ holds $\mathrm{Ball}(p',r)$ meets A.

(9) For every compact connected non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\text{N-bound}(C) < \text{N-bound}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$.

(10) For every compact connected non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\text{E-bound}(C) < \text{E-bound}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$.

(11) For every compact connected non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\text{S-bound}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) < \text{S-bound}(C)$.

(12) For every compact connected non vertical non horizontal subset C of $\mathcal{E}^2_{\mathrm{T}}$ holds $\text{W-bound}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) < \text{W-bound}(C)$.

(13) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq k$ and $k \leq j$ and $j \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,k), \mathrm{Gauge}(C,n) \circ (i,j)) \cap \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i,k)\}$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,k), \mathrm{Gauge}(C,n) \circ (i,j)) \cap \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i,j)\}$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,k), \mathrm{Gauge}(C,n) \circ (i,j))$ meets $\mathrm{UpperArc}(C)$.

(14) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq k$ and $k \leq j$ and $j \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,k), \mathrm{Gauge}(C,n) \circ (i,j)) \cap \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i,k)\}$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,k), \mathrm{Gauge}(C,n) \circ (i,j)) \cap \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i,j)\}$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,k), \mathrm{Gauge}(C,n) \circ (i,j))$ meets $\mathrm{LowerArc}(C)$.

(15) Let C be a simple closed curve and i, j, k be natural numbers. Suppose that $1 < i$ and $i < \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $n > 0$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j), \mathrm{Gauge}(C,n) \circ (i,k)) \cap \mathrm{LowerArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) = \{\mathrm{Gauge}(C,n) \circ (i,k)\}$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j), \mathrm{Gauge}(C,n) \circ (i,k)) \cap \mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) = \{\mathrm{Gauge}(C,n) \circ (i,j)\}$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j), \mathrm{Gauge}(C,n) \circ (i,k))$ meets $\mathrm{UpperArc}(C)$.

(16) Let C be a simple closed curve and i, j, k be natural numbers. Suppose that $1 < i$ and $i < \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $n > 0$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j), \mathrm{Gauge}(C,n) \circ (i,k)) \cap \mathrm{LowerArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) = \{\mathrm{Gauge}(C,n) \circ (i,k)\}$ and $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j), \mathrm{Gauge}(C,n) \circ (i,k)) \cap \mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n))) = \{\mathrm{Gauge}(C,n) \circ (i,j)\}$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j), \mathrm{Gauge}(C,n) \circ (i,k))$ meets $\mathrm{LowerArc}(C)$.

(17) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < \operatorname{len}\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (i,k) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$ and $\mathrm{Gauge}(C,n) \circ (i,j) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j), \mathrm{Gauge}(C,n) \circ (i,k))$ meets $\mathrm{UpperArc}(C)$.

(18) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < \operatorname{len}\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\mathrm{Gauge}(C,n)$ and $\mathrm{Gauge}(C,n) \circ (i,k) \in \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n))$ and $\mathrm{Gauge}(C,n) \circ (i,j) \in \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n))$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j), \mathrm{Gauge}(C,n) \circ (i,k))$ meets $\mathrm{LowerArc}(C)$.

(19) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < \operatorname{len}\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\mathrm{Gauge}(C,n)$ and $n > 0$ and $\mathrm{Gauge}(C,n) \circ (i,k) \in \mathrm{LowerArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$ and $\mathrm{Gauge}(C,n) \circ (i,j) \in \mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j), \mathrm{Gauge}(C,n) \circ (i,k))$ meets $\mathrm{UpperArc}(C)$.

(20) Let C be a simple closed curve and i, j, k be natural numbers. Suppose $1 < i$ and $i < \operatorname{len}\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\mathrm{Gauge}(C,n)$ and $n > 0$ and $\mathrm{Gauge}(C,n) \circ (i,k) \in \mathrm{LowerArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$ and $\mathrm{Gauge}(C,n) \circ (i,j) \in \mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n)))$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i,j), \mathrm{Gauge}(C,n) \circ (i,k))$ meets $\mathrm{LowerArc}(C)$.

(21) Let C be a simple closed curve and i_1, i_2, j, k be natural numbers. Suppose that $1 < i_1$ and $i_1 \leq i_2$ and $i_2 < \operatorname{len}\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\mathrm{Gauge}(C,n)$ and $(\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,j), \mathrm{Gauge}(C,n) \circ (i_1,k)) \cup \mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,k), \mathrm{Gauge}(C,n) \circ (i_2,k))) \cap \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i_1,j)\}$ and $(\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,j), \mathrm{Gauge}(C,n) \circ (i_1,k)) \cup \mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,k), \mathrm{Gauge}(C,n) \circ (i_2,k))) \cap \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i_2,k)\}$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,j), \mathrm{Gauge}(C,n) \circ (i_1,k)) \cup \mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,k), \mathrm{Gauge}(C,n) \circ (i_2,k))$ meets $\mathrm{UpperArc}(C)$.

(22) Let C be a simple closed curve and i_1, i_2, j, k be natural numbers. Suppose that $1 < i_1$ and $i_1 \leq i_2$ and $i_2 < \operatorname{len}\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\mathrm{Gauge}(C,n)$ and $(\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,j), \mathrm{Gauge}(C,n) \circ (i_1,k)) \cup \mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,k), \mathrm{Gauge}(C,n) \circ (i_2,k))) \cap \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i_1,j)\}$ and $(\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,j), \mathrm{Gauge}(C,n) \circ (i_1,k)) \cup \mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,k), \mathrm{Gauge}(C,n) \circ (i_2,k))) \cap \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i_2,k)\}$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,j), \mathrm{Gauge}(C,n) \circ (i_1,k)) \cup \mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,k), \mathrm{Gauge}(C,n) \circ (i_2,k))$ meets $\mathrm{LowerArc}(C)$.

(23) Let C be a simple closed curve and i_1, i_2, j, k be natural numbers. Suppose that $1 < i_2$ and $i_2 \leq i_1$ and $i_1 < \operatorname{len}\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\mathrm{Gauge}(C,n)$ and $(\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,j), \mathrm{Gauge}(C,n) \circ (i_1,k)) \cup \mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,k), \mathrm{Gauge}(C,n) \circ (i_2,k))) \cap \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i_1,j)\}$ and $(\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,j), \mathrm{Gauge}(C,n) \circ (i_1,k)) \cup \mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,k), \mathrm{Gauge}(C,n) \circ (i_2,k))) \cap \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i_2,k)\}$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,j), \mathrm{Gauge}(C,n) \circ (i_1,k)) \cup \mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,k), \mathrm{Gauge}(C,n) \circ (i_2,k))$ meets $\mathrm{UpperArc}(C)$.

(24) Let C be a simple closed curve and i_1, i_2, j, k be natural numbers. Suppose that $1 < i_2$ and $i_2 \leq i_1$ and $i_1 < \operatorname{len}\mathrm{Gauge}(C,n)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\mathrm{Gauge}(C,n)$ and $(\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,j), \mathrm{Gauge}(C,n) \circ (i_1,k)) \cup \mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,k), \mathrm{Gauge}(C,n) \circ (i_2,k))) \cap \widetilde{\mathcal{L}}(\mathrm{UpperSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i_1,j)\}$ and $(\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,j), \mathrm{Gauge}(C,n) \circ (i_1,k)) \cup \mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,k), \mathrm{Gauge}(C,n) \circ (i_2,k))) \cap \widetilde{\mathcal{L}}(\mathrm{LowerSeq}(C,n)) = \{\mathrm{Gauge}(C,n) \circ (i_2,k)\}$. Then $\mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,j), \mathrm{Gauge}(C,n) \circ (i_1,k)) \cup \mathcal{L}(\mathrm{Gauge}(C,n) \circ (i_1,k), \mathrm{Gauge}(C,n) \circ (i_2,k))$ meets $\mathrm{LowerArc}(C)$.

(25) Let C be a simple closed curve and i_1, i_2, j, k be natural numbers. Suppose that $1 < i_1$ and $i_1 < \operatorname{len}\mathrm{Gauge}(C,n+1)$ and $1 < i_2$ and $i_2 < \operatorname{len}\mathrm{Gauge}(C,n+1)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\mathrm{Gauge}(C,n+1)$ and $\mathrm{Gauge}(C,n+1) \circ (i_1,k) \in \mathrm{LowerArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n+1)))$ and $\mathrm{Gauge}(C,n+1) \circ (i_2,j) \in \mathrm{UpperArc}(\widetilde{\mathcal{L}}(\mathrm{Cage}(C,n+1)))$. Then $\mathcal{L}(\mathrm{Gauge}(C,n+1) \circ (i_2,j), \mathrm{Gauge}(C,n+1) \circ (i_2,k)) \cup \mathcal{L}(\mathrm{Gauge}(C,n+1) \circ (i_2,k), \mathrm{Gauge}(C,n+1) \circ (i_1,k))$ meets $\mathrm{LowerArc}(C)$.

(26) Let C be a simple closed curve and i_1, i_2, j, k be natural numbers. Suppose that $1 < i_1$ and $i_1 < \operatorname{len}\operatorname{Gauge}(C, n+1)$ and $1 < i_2$ and $i_2 < \operatorname{len}\operatorname{Gauge}(C, n+1)$ and $1 \leq j$ and $j \leq k$ and $k \leq \operatorname{width}\operatorname{Gauge}(C, n+1)$ and $\operatorname{Gauge}(C, n+1) \circ (i_1, k) \in \operatorname{LowerArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C, n+1)))$ and $\operatorname{Gauge}(C, n+1) \circ (i_2, j) \in \operatorname{UpperArc}(\widetilde{\mathcal{L}}(\operatorname{Cage}(C, n+1)))$. Then $\mathcal{L}(\operatorname{Gauge}(C, n+1) \circ (i_2, j), \operatorname{Gauge}(C, n+1) \circ (i_2, k)) \cup \mathcal{L}(\operatorname{Gauge}(C, n+1) \circ (i_2, k), \operatorname{Gauge}(C, n+1) \circ (i_1, k))$ meets $\operatorname{UpperArc}(C)$.

(27) For every simple closed curve C and for every point p of $\mathcal{E}^2_{\mathrm{T}}$ such that $\text{W-bound}(C) < p_{\mathbf{1}}$ and $p_{\mathbf{1}} < \text{E-bound}(C)$ holds $p \notin \operatorname{NorthArc}(C)$ or $p \notin \operatorname{SouthArc}(C)$.

(28) For every simple closed curve C and for every point p of $\mathcal{E}^2_{\mathrm{T}}$ such that $p_{\mathbf{1}} = \frac{\text{W-bound}(C)+\text{E-bound}(C)}{2}$ holds $p \notin \operatorname{NorthArc}(C)$ or $p \notin \operatorname{SouthArc}(C)$.

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