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Properties of the Internal Approximation of Jordan's Curve[1]

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The articles [19], [25], [14], [10], [1], [16], [2], [3], [24], [11], [18], [9], [26], [6], [17], [7], [8], [12], [13], [20], [15], [4], [5], [21], [23], and [22] provide the notation and terminology for this paper.

The following propositions are true:

(1) For every non constant standard special circular sequence f holds $\mathrm{BDD}\,\widetilde{\mathcal{L}}(f) = \mathrm{RightComp}(f)$ or $\mathrm{BDD}\,\widetilde{\mathcal{L}}(f) = \mathrm{LeftComp}(f)$.

(2) For every non constant standard special circular sequence f holds $\mathrm{UBD}\,\widetilde{\mathcal{L}}(f) = \mathrm{RightComp}(f)$ or $\mathrm{UBD}\,\widetilde{\mathcal{L}}(f) = \mathrm{LeftComp}(f)$.

(3) Let G be a Go-board, f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$, and k be a natural number. Suppose $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$ and f is a sequence which elements belong to G. Then $\mathrm{left\_cell}(f,k,G)$ is closed.

(4) Let G be a Go-board, p be a point of $\mathcal{E}^2_{\mathrm{T}}$, and i, j be natural numbers. Suppose $1 \leq i$ and $i+1 \leq \mathrm{len}\, G$ and $1 \leq j$ and $j+1 \leq \mathrm{width}\, G$. Then $p \in \mathrm{Int\,cell}(G,i,j)$ if and only if the following conditions are satisfied:

(i) $(G \circ (i,j))_{\mathbf{1}} < p_{\mathbf{1}}$,

(ii) $p_{\mathbf{1}} < (G \circ (i+1,j))_{\mathbf{1}}$,

(iii) $(G \circ (i,j))_{\mathbf{2}} < p_{\mathbf{2}}$, and

(iv) $p_{\mathbf{2}} < (G \circ (i,j+1))_{\mathbf{2}}$.

(5) For every non constant standard special circular sequence f holds $\mathrm{BDD}\,\widetilde{\mathcal{L}}(f)$ is connected.

Let f be a non constant standard special circular sequence. One can check that $\mathrm{BDD}\,\widetilde{\mathcal{L}}(f)$ is connected.

Let C be a simple closed curve and let n be a natural number. The functor $\mathrm{SpanStart}(C,n)$ yields a point of $\mathcal{E}^2_{\mathrm{T}}$ and is defined as follows:

(Def. 1) $\mathrm{SpanStart}(C,n) = \mathrm{Gauge}(C,n) \circ (\text{X-SpanStart}(C,n), \text{Y-SpanStart}(C,n))$.

We now state four propositions:

(6) Let C be a simple closed curve and n be a natural number. If n is sufficiently large for C, then $(\mathrm{Span}(C,n))_1 = \mathrm{SpanStart}(C,n)$.

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(7) For every simple closed curve C and for every natural number n such that n is sufficiently large for C holds $\mathrm{SpanStart}(C,n) \in \mathrm{BDD}\,C$.

(8) Let C be a simple closed curve and n, k be natural numbers. Suppose n is sufficiently large for C. Suppose $1 \leq k$ and $k+1 \leq \mathrm{len}\,\mathrm{Span}(C,n)$. Then $\mathrm{right\_cell}(\mathrm{Span}(C,n),k,\mathrm{Gauge}(C,n))$ misses C and $\mathrm{left\_cell}(\mathrm{Span}(C,n),k,\mathrm{Gauge}(C,n))$ meets C.

(9) Let C be a simple closed curve and n be a natural number. If n is sufficiently large for C, then C misses $\widetilde{\mathcal{L}}(\mathrm{Span}(C,n))$.

Let C be a simple closed curve and let n be a natural number. Observe that $\overline{\mathrm{RightComp}(\mathrm{Span}(C,n))}$ is compact.

One can prove the following propositions:

(10) Let C be a simple closed curve and n be a natural number. If n is sufficiently large for C, then C meets $\mathrm{LeftComp}(\mathrm{Span}(C,n))$.

(11) Let C be a simple closed curve and n be a natural number. If n is sufficiently large for C, then C misses $\mathrm{RightComp}(\mathrm{Span}(C,n))$.

(12) For every simple closed curve C and for every natural number n such that n is sufficiently large for C holds $C \subseteq \mathrm{LeftComp}(\mathrm{Span}(C,n))$.

(13) For every simple closed curve C and for every natural number n such that n is sufficiently large for C holds $C \subseteq \mathrm{UBD}\,\widetilde{\mathcal{L}}(\mathrm{Span}(C,n))$.

(14) For every simple closed curve C and for every natural number n such that n is sufficiently large for C holds $\mathrm{BDD}\,\widetilde{\mathcal{L}}(\mathrm{Span}(C,n)) \subseteq \mathrm{BDD}\,C$.

(15) For every simple closed curve C and for every natural number n such that n is sufficiently large for C holds $\mathrm{UBD}\,C \subseteq \mathrm{UBD}\,\widetilde{\mathcal{L}}(\mathrm{Span}(C,n))$.

(16) For every simple closed curve C and for every natural number n such that n is sufficiently large for C holds $\mathrm{RightComp}(\mathrm{Span}(C,n)) \subseteq \mathrm{BDD}\,C$.

(17) For every simple closed curve C and for every natural number n such that n is sufficiently large for C holds $\mathrm{UBD}\,C \subseteq \mathrm{LeftComp}(\mathrm{Span}(C,n))$.

(18) Let C be a simple closed curve and n be a natural number. If n is sufficiently large for C, then $\mathrm{UBD}\,C$ misses $\mathrm{BDD}\,\widetilde{\mathcal{L}}(\mathrm{Span}(C,n))$.

(19) Let C be a simple closed curve and n be a natural number. If n is sufficiently large for C, then $\mathrm{UBD}\,C$ misses $\mathrm{RightComp}(\mathrm{Span}(C,n))$.

(20) Let C be a simple closed curve, P be a subset of $\mathcal{E}^2_\mathrm{T}$, and n be a natural number. Suppose n is sufficiently large for C. If P is outside component of C, then P misses $\widetilde{\mathcal{L}}(\mathrm{Span}(C,n))$.

(21) Let C be a simple closed curve and n be a natural number. If n is sufficiently large for C, then $\mathrm{UBD}\,C$ misses $\widetilde{\mathcal{L}}(\mathrm{Span}(C,n))$.

(22) For every simple closed curve C and for every natural number n such that n is sufficiently large for C holds $\widetilde{\mathcal{L}}(\mathrm{Span}(C,n)) \subseteq \mathrm{BDD}\,C$.

(23) Let C be a simple closed curve and i, j, k, n be natural numbers. Suppose n is sufficiently large for C and $1 \leq k$ and $k \leq \mathrm{len}\,\mathrm{Span}(C,n)$ and $\langle i,\, j\rangle \in$ the indices of $\mathrm{Gauge}(C,n)$ and $(\mathrm{Span}(C,n))_k = \mathrm{Gauge}(C,n) \circ (i,j)$. Then $i > 1$.

(24) Let C be a simple closed curve and i, j, k, n be natural numbers. Suppose n is sufficiently large for C and $1 \leq k$ and $k \leq \mathrm{len}\,\mathrm{Span}(C,n)$ and $\langle i,\, j\rangle \in$ the indices of $\mathrm{Gauge}(C,n)$ and $(\mathrm{Span}(C,n))_k = \mathrm{Gauge}(C,n) \circ (i,j)$. Then $i < \mathrm{len}\,\mathrm{Gauge}(C,n)$.

(25) Let C be a simple closed curve and i, j, k, n be natural numbers. Suppose n is sufficiently large for C and $1 \leq k$ and $k \leq \operatorname{len}\operatorname{Span}(C,n)$ and $\langle i, j\rangle \in$ the indices of $\operatorname{Gauge}(C,n)$ and $(\operatorname{Span}(C,n))_k = \operatorname{Gauge}(C,n) \circ (i,j)$. Then $j > 1$.

(26) Let C be a simple closed curve and i, j, k, n be natural numbers. Suppose n is sufficiently large for C and $1 \leq k$ and $k \leq \operatorname{len}\operatorname{Span}(C,n)$ and $\langle i, j\rangle \in$ the indices of $\operatorname{Gauge}(C,n)$ and $(\operatorname{Span}(C,n))_k = \operatorname{Gauge}(C,n) \circ (i,j)$. Then $j < \operatorname{width}\operatorname{Gauge}(C,n)$.

(27) For every simple closed curve C and for every natural number n such that n is sufficiently large for C holds $\text{Y-SpanStart}(C,n) < \operatorname{width}\operatorname{Gauge}(C,n)$.

(28) Let C be a compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and n, m be natural numbers. If $m \geq n$ and $n \geq 1$, then $\text{X-SpanStart}(C,m) = 2^{m-'n} \cdot (\text{X-SpanStart}(C,n) - 2) + 2$.

(29) Let C be a compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and n, m be natural numbers. Suppose $n \leq m$ and n is sufficiently large for C. Then m is sufficiently large for C.

(30) Let G be a Go-board, f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$, and i, j be natural numbers. Suppose f is a sequence which elements belong to G and special and $i \leq \operatorname{len} G$ and $j \leq \operatorname{width} G$. Then $\operatorname{cell}(G,i,j) \setminus \widetilde{\mathcal{L}}(f)$ is connected.

(31) Let C be a simple closed curve and n, k be natural numbers. Suppose n is sufficiently large for C and $\text{Y-SpanStart}(C,n) \leq k$ and $k \leq 2^{n-'\text{ApproxIndex}\,C} \cdot (\text{Y-InitStart}\,C -' 2) + 2$. Then $\operatorname{cell}(\operatorname{Gauge}(C,n), \text{X-SpanStart}(C,n) -' 1, k) \setminus \widetilde{\mathcal{L}}(\operatorname{Span}(C,n)) \subseteq \operatorname{BDD}\widetilde{\mathcal{L}}(\operatorname{Span}(C,n))$.

(32) Let C be a subset of $\mathcal{E}^2_{\mathrm{T}}$ and n, m, i be natural numbers. If $m \leq n$ and $1 < i$ and $i+1 < \operatorname{len}\operatorname{Gauge}(C,m)$, then $2^{n-'m} \cdot (i-2) + 2 + 1 < \operatorname{len}\operatorname{Gauge}(C,n)$.

(33) Let C be a simple closed curve and n, m be natural numbers. If n is sufficiently large for C and $n \leq m$, then $\operatorname{RightComp}(\operatorname{Span}(C,n))$ meets $\operatorname{RightComp}(\operatorname{Span}(C,m))$.

(34) Let G be a Go-board and f be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. Suppose f is a sequence which elements belong to G and special. Let i, j be natural numbers. If $i \leq \operatorname{len} G$ and $j \leq \operatorname{width} G$, then $\operatorname{Int}\operatorname{cell}(G,i,j) \subseteq (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$.

(35) Let C be a simple closed curve and n, m be natural numbers. If n is sufficiently large for C and $n \leq m$, then $\widetilde{\mathcal{L}}(\operatorname{Span}(C,m)) \subseteq \overline{\operatorname{LeftComp}(\operatorname{Span}(C,n))}$.

(36) Let C be a simple closed curve and n, m be natural numbers. If n is sufficiently large for C and $n \leq m$, then $\operatorname{RightComp}(\operatorname{Span}(C,n)) \subseteq \operatorname{RightComp}(\operatorname{Span}(C,m))$.

(37) Let C be a simple closed curve and n, m be natural numbers. If n is sufficiently large for C and $n \leq m$, then $\operatorname{LeftComp}(\operatorname{Span}(C,m)) \subseteq \operatorname{LeftComp}(\operatorname{Span}(C,n))$.

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