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Subgroup and Cosets of Subgroups

Wojciech A. Trybulec
Warsaw University

Summary. We introduce notion of subgroup, coset of a subgroup, sets of left and right cosets of a subgroup. We define multiplication of two subset of a group, subset of reverse elemens of a group, intersection of two subgroups. We define the notion of an index of a subgroup and prove Lagrange theorem which states that in a finite group the order of the group equals the order of a subgroup multiplied by the index of the subgroup. Some theorems that belong rather to [1] are proved.

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The articles [10], [5], [15], [8], [12], [6], [13], [7], [3], [14], [1], [9], [16], [4], [17], [2], and [11] provide the notation and terminology for this paper.

In this paper x is a set, G is a non empty 1-sorted structure, and A is a subset of G.

Next we state the proposition

(3)[1] If G is finite, then A is finite.

For simplicity, we adopt the following convention: Y, Z denote sets, k denotes a natural number, G denotes a group, g, h denote elements of G, and A denotes a subset of G.

Let us consider G, A. The functor A^{-1} yielding a subset of G is defined as follows:

(Def. 1) $A^{-1} = \{g^{-1} : g \in A\}$.

We now state several propositions:

(5)[2] $x \in A^{-1}$ iff there exists g such that $x = g^{-1}$ and $g \in A$.

(6) $\{g\}^{-1} = \{g^{-1}\}$.

(7) $\{g,h\}^{-1} = \{g^{-1},h^{-1}\}$.

(8) $(\emptyset_{\text{the carrier of } G})^{-1} = \emptyset$.

(9) $(\Omega_{\text{the carrier of } G})^{-1} =$ the carrier of G.

(10) $A \neq \emptyset$ iff $A^{-1} \neq \emptyset$.

We adopt the following rules: G is a non empty groupoid, A, B, C are subsets of G, and a, b, g, g_1, g_2, h are elements of G.

Let us consider G and let us consider A, B. The functor $A \cdot B$ yielding a subset of G is defined by:

[1] The propositions (1) and (2) have been removed.
[2] The proposition (4) has been removed.

(Def. 2) $A \cdot B = \{g \cdot h : g \in A \wedge h \in B\}$.

Next we state a number of propositions:

(12)[3] $x \in A \cdot B$ iff there exist g, h such that $x = g \cdot h$ and $g \in A$ and $h \in B$.

(13) $A \neq \emptyset$ and $B \neq \emptyset$ iff $A \cdot B \neq \emptyset$.

(14) If G is associative, then $(A \cdot B) \cdot C = A \cdot (B \cdot C)$.

(15) For every group G and for all subsets A, B of G holds $(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$.

(16) $A \cdot (B \cup C) = A \cdot B \cup A \cdot C$.

(17) $(A \cup B) \cdot C = A \cdot C \cup B \cdot C$.

(18) $A \cdot (B \cap C) \subseteq (A \cdot B) \cap (A \cdot C)$.

(19) $(A \cap B) \cdot C \subseteq (A \cdot C) \cap (B \cdot C)$.

(20) $\emptyset_{\text{the carrier of } G} \cdot A = \emptyset$ and $A \cdot \emptyset_{\text{the carrier of } G} = \emptyset$.

(21) Let G be a group and A be a subset of G. Suppose $A \neq \emptyset$. Then $\Omega_{\text{the carrier of } G} \cdot A =$ the carrier of G and $A \cdot \Omega_{\text{the carrier of } G} =$ the carrier of G.

(22) $\{g\} \cdot \{h\} = \{g \cdot h\}$.

(23) $\{g\} \cdot \{g_1, g_2\} = \{g \cdot g_1, g \cdot g_2\}$.

(24) $\{g_1, g_2\} \cdot \{g\} = \{g_1 \cdot g, g_2 \cdot g\}$.

(25) $\{g, h\} \cdot \{g_1, g_2\} = \{g \cdot g_1, g \cdot g_2, h \cdot g_1, h \cdot g_2\}$.

(26) Let G be a group and A be a subset of G. Suppose that
 (i) for all elements g_1, g_2 of G such that $g_1 \in A$ and $g_2 \in A$ holds $g_1 \cdot g_2 \in A$, and
 (ii) for every element g of G such that $g \in A$ holds $g^{-1} \in A$.

 Then $A \cdot A = A$.

(27) For every group G and for every subset A of G such that for every element g of G such that $g \in A$ holds $g^{-1} \in A$ holds $A^{-1} = A$.

(28) If for all a, b such that $a \in A$ and $b \in B$ holds $a \cdot b = b \cdot a$, then $A \cdot B = B \cdot A$.

(29) If G is a commutative group, then $A \cdot B = B \cdot A$.

(30) For every commutative group G and for all subsets A, B of G holds $(A \cdot B)^{-1} = A^{-1} \cdot B^{-1}$.

Let us consider G, g, A. The functor $g \cdot A$ yields a subset of G and is defined as follows:

(Def. 3) $g \cdot A = \{g\} \cdot A$.

The functor $A \cdot g$ yielding a subset of G is defined by:

(Def. 4) $A \cdot g = A \cdot \{g\}$.

We now state several propositions:

(33)[4] $x \in g \cdot A$ iff there exists h such that $x = g \cdot h$ and $h \in A$.

(34) $x \in A \cdot g$ iff there exists h such that $x = h \cdot g$ and $h \in A$.

(35) If G is associative, then $(g \cdot A) \cdot B = g \cdot (A \cdot B)$.

[3] The proposition (11) has been removed.

[4] The propositions (31) and (32) have been removed.

(36) If G is associative, then $(A \cdot g) \cdot B = A \cdot (g \cdot B)$.

(37) If G is associative, then $(A \cdot B) \cdot g = A \cdot (B \cdot g)$.

(38) If G is associative, then $(g \cdot h) \cdot A = g \cdot (h \cdot A)$.

(39) If G is associative, then $(g \cdot A) \cdot h = g \cdot (A \cdot h)$.

(40) If G is associative, then $(A \cdot g) \cdot h = A \cdot (g \cdot h)$.

(41) $\emptyset_{\text{the carrier of } G} \cdot a = \emptyset$ and $a \cdot \emptyset_{\text{the carrier of } G} = \emptyset$.

We adopt the following convention: G is a group-like non empty groupoid, g, g_1, g_2 are elements of G, and A is a subset of G.

One can prove the following propositions:

(42) Let G be a group and a be an element of G. Then $\Omega_{\text{the carrier of } G} \cdot a =$ the carrier of G and $a \cdot \Omega_{\text{the carrier of } G} =$ the carrier of G.

(43) $1_G \cdot A = A$ and $A \cdot 1_G = A$.

(44) If G is a commutative group, then $g \cdot A = A \cdot g$.

Let G be a group-like non empty groupoid. A group-like non empty groupoid is said to be a subgroup of G if it satisfies the conditions (Def. 5).

(Def. 5)(i) The carrier of it $\subseteq$ the carrier of G, and

(ii) the multiplication of it $=$ (the multiplication of G)$\restriction[\!:$the carrier of it, the carrier of it$:\!]$.

In the sequel H denotes a subgroup of G and h, h_1, h_2 denote elements of H.

One can prove the following propositions:

(48)[5] If G is finite, then H is finite.

(49) If $x \in H$, then $x \in G$.

(50) $h \in G$.

(51) h is an element of G.

(52) If $h_1 = g_1$ and $h_2 = g_2$, then $h_1 \cdot h_2 = g_1 \cdot g_2$.

Let G be a group. Note that every subgroup of G is associative.

For simplicity, we adopt the following rules: G, G_1, G_2, G_3 are groups, a, b, g, g_1, g_2 are elements of G, A, B are subsets of G, I, H, H_1, H_2, H_3 are subgroups of G, and h is an element of H.

We now state several propositions:

(53) $1_H = 1_G$.

(54) $1_{(H_1)} = 1_{(H_2)}$.

(55) $1_G \in H$.

(56) $1_{(H_1)} \in H_2$.

(57) If $h = g$, then $h^{-1} = g^{-1}$.

(58) $\cdot_H^{-1} = \cdot_G^{-1} \restriction$the carrier of H.

(59) If $g_1 \in H$ and $g_2 \in H$, then $g_1 \cdot g_2 \in H$.

(60) If $g \in H$, then $g^{-1} \in H$.

[5] The propositions (45)–(47) have been removed.

Let us consider G. Note that there exists a subgroup of G which is strict.
One can prove the following two propositions:

(61) Suppose $A \neq \emptyset$ and for all g_1, g_2 such that $g_1 \in A$ and $g_2 \in A$ holds $g_1 \cdot g_2 \in A$ and for every g such that $g \in A$ holds $g^{-1} \in A$. Then there exists a strict subgroup H of G such that the carrier of $H = A$.

(62) If G is a commutative group, then H is commutative.

Let G be a commutative group. Observe that every subgroup of G is commutative.
Next we state several propositions:

(63) G is a subgroup of G.

(64) If G_1 is a subgroup of G_2 and G_2 is a subgroup of G_1, then the groupoid of $G_1 =$ the groupoid of G_2.

(65) If G_1 is a subgroup of G_2 and G_2 is a subgroup of G_3, then G_1 is a subgroup of G_3.

(66) If the carrier of $H_1 \subseteq$ the carrier of H_2, then H_1 is a subgroup of H_2.

(67) If for every g such that $g \in H_1$ holds $g \in H_2$, then H_1 is a subgroup of H_2.

(68) If the carrier of $H_1 =$ the carrier of H_2, then the groupoid of $H_1 =$ the groupoid of H_2.

(69) If for every g holds $g \in H_1$ iff $g \in H_2$, then the groupoid of $H_1 =$ the groupoid of H_2.

Let us consider G and let H_1, H_2 be strict subgroups of G. Let us observe that $H_1 = H_2$ if and only if:

(Def. 6) For every g holds $g \in H_1$ iff $g \in H_2$.

We now state two propositions:

(70) Let G be a strict group and H be a strict subgroup of G. If the carrier of $H =$ the carrier of G, then $H = G$.

(71) If for every element g of G holds $g \in H$, then the groupoid of $H =$ the groupoid of G.

Let us consider G. The functor $\{\mathbf{1}\}_G$ yielding a strict subgroup of G is defined by:

(Def. 7) The carrier of $\{\mathbf{1}\}_G = \{1_G\}$.

Let us consider G. The functor Ω_G yielding a strict subgroup of G is defined by:

(Def. 8) $\Omega_G =$ the groupoid of G.

The following propositions are true:

(75)[6] $\{\mathbf{1}\}_H = \{\mathbf{1}\}_G$.

(76) $\{\mathbf{1}\}_{(H_1)} = \{\mathbf{1}\}_{(H_2)}$.

(77) $\{\mathbf{1}\}_G$ is a subgroup of H.

(78) For every strict group G holds every subgroup of G is a subgroup of Ω_G.

(79) Every strict group G is a subgroup of Ω_G.

(80) $\{\mathbf{1}\}_G$ is finite.

[6] The propositions (72)–(74) have been removed.

Let X be a non empty set. One can verify that there exists a subset of X which is finite and non empty.

One can prove the following propositions:

(81) $\mathrm{ord}(\{\mathbf{1}\}_G) = 1$.

(82) For every strict subgroup H of G such that H is finite and $\mathrm{ord}(H) = 1$ holds $H = \{\mathbf{1}\}_G$.

(83) $\mathrm{Ord}(H) \leq \mathrm{Ord}(G)$.

(84) If G is finite, then $\mathrm{ord}(H) \leq \mathrm{ord}(G)$.

(85) For every strict group G and for every strict subgroup H of G such that G is finite and $\mathrm{ord}(G) = \mathrm{ord}(H)$ holds $H = G$.

Let us consider G, H. The functor $\overline{H}$ yields a subset of G and is defined by:

(Def. 9) $\overline{H} =$ the carrier of H.

One can prove the following propositions:

(87)[7] $\overline{H} \neq \emptyset$.

(88) $x \in \overline{H}$ iff $x \in H$.

(89) If $g_1 \in \overline{H}$ and $g_2 \in \overline{H}$, then $g_1 \cdot g_2 \in \overline{H}$.

(90) If $g \in \overline{H}$, then $g^{-1} \in \overline{H}$.

(91) $\overline{H} \cdot \overline{H} = \overline{H}$.

(92) $\overline{H}^{-1} = \overline{H}$.

(93)(i) If $\overline{H_1} \cdot \overline{H_2} = \overline{H_2} \cdot \overline{H_1}$, then there exists a strict subgroup H of G such that the carrier of $H = \overline{H_1} \cdot \overline{H_2}$, and

(ii) if there exists H such that the carrier of $H = \overline{H_1} \cdot \overline{H_2}$, then $\overline{H_1} \cdot \overline{H_2} = \overline{H_2} \cdot \overline{H_1}$.

(94) If G is a commutative group, then there exists a strict subgroup H of G such that the carrier of $H = \overline{H_1} \cdot \overline{H_2}$.

Let us consider G, H_1, H_2. The functor $H_1 \cap H_2$ yields a strict subgroup of G and is defined by:

(Def. 10) The carrier of $H_1 \cap H_2 = \overline{H_1} \cap \overline{H_2}$.

Next we state several propositions:

(97)[8](i) For every subgroup H of G such that $H = H_1 \cap H_2$ holds the carrier of $H =$ (the carrier of H_1) $\cap$ (the carrier of H_2), and

(ii) for every strict subgroup H of G such that the carrier of $H =$ (the carrier of H_1) $\cap$ (the carrier of H_2) holds $H = H_1 \cap H_2$.

(98) $\overline{H_1 \cap H_2} = \overline{H_1} \cap \overline{H_2}$.

(99) $x \in H_1 \cap H_2$ iff $x \in H_1$ and $x \in H_2$.

(100) For every strict subgroup H of G holds $H \cap H = H$.

(101) $H_1 \cap H_2 = H_2 \cap H_1$.

Let us consider G, H_1, H_2. Let us notice that the functor $H_1 \cap H_2$ is commutative.

Next we state a number of propositions:

[7] The proposition (86) has been removed.

[8] The propositions (95) and (96) have been removed.

(102) $(H_1 \cap H_2) \cap H_3 = H_1 \cap (H_2 \cap H_3)$.

(103) $\{\mathbf{1}\}_G \cap H = \{\mathbf{1}\}_G$ and $H \cap \{\mathbf{1}\}_G = \{\mathbf{1}\}_G$.

(104) For every strict group G and for every strict subgroup H of G holds $H \cap \Omega_G = H$ and $\Omega_G \cap H = H$.

(105) For every strict group G holds $\Omega_G \cap \Omega_G = G$.

(106) $H_1 \cap H_2$ is a subgroup of H_1 and $H_1 \cap H_2$ is a subgroup of H_2.

(107) For every strict subgroup H_1 of G holds H_1 is a subgroup of H_2 iff $H_1 \cap H_2 = H_1$.

(108) If H_1 is a subgroup of H_2, then $H_1 \cap H_3$ is a subgroup of H_2.

(109) If H_1 is a subgroup of H_2 and a subgroup of H_3, then H_1 is a subgroup of $H_2 \cap H_3$.

(110) If H_1 is a subgroup of H_2, then $H_1 \cap H_3$ is a subgroup of $H_2 \cap H_3$.

(111) If H_1 is finite or H_2 is finite, then $H_1 \cap H_2$ is finite.

Let us consider G, H, A. The functor $A \cdot H$ yielding a subset of G is defined by:

(Def. 11) $A \cdot H = A \cdot \overline{H}$.

The functor $H \cdot A$ yields a subset of G and is defined by:

(Def. 12) $H \cdot A = \overline{H} \cdot A$.

One can prove the following propositions:

(114)[9] $x \in A \cdot H$ iff there exist g_1, g_2 such that $x = g_1 \cdot g_2$ and $g_1 \in A$ and $g_2 \in H$.

(115) $x \in H \cdot A$ iff there exist g_1, g_2 such that $x = g_1 \cdot g_2$ and $g_1 \in H$ and $g_2 \in A$.

(116) $(A \cdot B) \cdot H = A \cdot (B \cdot H)$.

(117) $(A \cdot H) \cdot B = A \cdot (H \cdot B)$.

(118) $(H \cdot A) \cdot B = H \cdot (A \cdot B)$.

(119) $(A \cdot H_1) \cdot H_2 = A \cdot (H_1 \cdot \overline{H_2})$.

(120) $(H_1 \cdot A) \cdot H_2 = H_1 \cdot (A \cdot H_2)$.

(121) $(H_1 \cdot \overline{H_2}) \cdot A = H_1 \cdot (H_2 \cdot A)$.

(122) If G is a commutative group, then $A \cdot H = H \cdot A$.

Let us consider G, H, a. The functor $a \cdot H$ yielding a subset of G is defined as follows:

(Def. 13) $a \cdot H = a \cdot \overline{H}$.

The functor $H \cdot a$ yields a subset of G and is defined by:

(Def. 14) $H \cdot a = \overline{H} \cdot a$.

We now state a number of propositions:

(125)[10] $x \in a \cdot H$ iff there exists g such that $x = a \cdot g$ and $g \in H$.

(126) $x \in H \cdot a$ iff there exists g such that $x = g \cdot a$ and $g \in H$.

(127) $(a \cdot b) \cdot H = a \cdot (b \cdot H)$.

(128) $(a \cdot H) \cdot b = a \cdot (H \cdot b)$.

(129) $(H \cdot a) \cdot b = H \cdot (a \cdot b)$.

(130) $a \in a \cdot H$ and $a \in H \cdot a$.

(132)[11] $1_G \cdot H = \overline{H}$ and $H \cdot 1_G = \overline{H}$.

[9] The propositions (112) and (113) have been removed.
[10] The propositions (123) and (124) have been removed.
[11] The proposition (131) has been removed.

(133) $\{\mathbf{1}\}_G \cdot a = \{a\}$ and $a \cdot \{\mathbf{1}\}_G = \{a\}$.

(134) $a \cdot \Omega_G =$ the carrier of G and $\Omega_G \cdot a =$ the carrier of G.

(135) If G is a commutative group, then $a \cdot H = H \cdot a$.

(136) $a \in H$ iff $a \cdot H = \overline{H}$.

(137) $a \cdot H = b \cdot H$ iff $b^{-1} \cdot a \in H$.

(138) $a \cdot H = b \cdot H$ iff $a \cdot H$ meets $b \cdot H$.

(139) $(a \cdot b) \cdot H \subseteq a \cdot H \cdot (b \cdot H)$.

(140) $\overline{H} \subseteq a \cdot H \cdot (a^{-1} \cdot H)$ and $\overline{H} \subseteq a^{-1} \cdot H \cdot (a \cdot H)$.

(141) $a^2 \cdot H \subseteq a \cdot H \cdot (a \cdot H)$.

(142) $a \in H$ iff $H \cdot a = \overline{H}$.

(143) $H \cdot a = H \cdot b$ iff $b \cdot a^{-1} \in H$.

(144) $H \cdot a = H \cdot b$ iff $H \cdot a$ meets $H \cdot b$.

(145) $(H \cdot a) \cdot b \subseteq H \cdot a \cdot (H \cdot b)$.

(146) $\overline{H} \subseteq H \cdot a \cdot (H \cdot a^{-1})$ and $\overline{H} \subseteq H \cdot a^{-1} \cdot (H \cdot a)$.

(147) $H \cdot a^2 \subseteq H \cdot a \cdot (H \cdot a)$.

(148) $a \cdot (H_1 \cap H_2) = (a \cdot H_1) \cap (a \cdot H_2)$.

(149) $(H_1 \cap H_2) \cdot a = (H_1 \cdot a) \cap (H_2 \cdot a)$.

(150) There exists a strict subgroup H_1 of G such that the carrier of $H_1 = a \cdot H_2 \cdot a^{-1}$.

(151) $a \cdot H \approx b \cdot H$.

(152) $a \cdot H \approx H \cdot b$.

(153) $H \cdot a \approx H \cdot b$.

(154) $\overline{H} \approx a \cdot H$ and $\overline{H} \approx H \cdot a$.

(155) $\mathrm{Ord}(H) = \overline{\overline{a \cdot H}}$ and $\mathrm{Ord}(H) = \overline{\overline{H \cdot a}}$.

(156) If H is finite, then there exist finite sets B, C such that $B = a \cdot H$ and $C = H \cdot a$ and $\mathrm{ord}(H) = \mathrm{card}\,B$ and $\mathrm{ord}(H) = \mathrm{card}\,C$.

The scheme *SubFamComp* deals with a set $\mathcal{A}$, a family $\mathcal{B}$ of subsets of $\mathcal{A}$, a family $\mathcal{C}$ of subsets of $\mathcal{A}$, and a unary predicate $\mathcal{P}$, and states that:

$\mathcal{B} = \mathcal{C}$

provided the following conditions are met:

- For every subset X of $\mathcal{A}$ holds $X \in \mathcal{B}$ iff $\mathcal{P}[X]$, and
- For every subset X of $\mathcal{A}$ holds $X \in \mathcal{C}$ iff $\mathcal{P}[X]$.

Let us consider G, H. The left cosets of H yielding a family of subsets of G is defined by:

(Def. 15) $A \in$ the left cosets of H iff there exists a such that $A = a \cdot H$.

The right cosets of H yields a family of subsets of G and is defined as follows:

(Def. 16) $A \in$ the right cosets of H iff there exists a such that $A = H \cdot a$.

The following propositions are true:

(164)[12] If G is finite, then the right cosets of H is finite and the left cosets of H is finite.

(165) $\overline{H} \in$ the left cosets of H and $\overline{H} \in$ the right cosets of H.

(166) The left cosets of $H \approx$ the right cosets of H.

(167) $\bigcup$(the left cosets of H) $=$ the carrier of G and $\bigcup$(the right cosets of H) $=$ the carrier of G.

(168) The left cosets of $\{\mathbf{1}\}_G = \{\{a\}\}$.

(169) The right cosets of $\{\mathbf{1}\}_G = \{\{a\}\}$.

(170) For every strict subgroup H of G such that the left cosets of $H = \{\{a\}\}$ holds $H = \{\mathbf{1}\}_G$.

(171) For every strict subgroup H of G such that the right cosets of $H = \{\{a\}\}$ holds $H = \{\mathbf{1}\}_G$.

(172) The left cosets of $\Omega_G = \{$the carrier of $G\}$ and the right cosets of $\Omega_G = \{$the carrier of $G\}$.

(173) Let G be a strict group and H be a strict subgroup of G. If the left cosets of $H = \{$the carrier of $G\}$, then $H = G$.

(174) Let G be a strict group and H be a strict subgroup of G. If the right cosets of $H = \{$the carrier of $G\}$, then $H = G$.

Let us consider G, H. The functor $|\bullet : H|$ yields a cardinal number and is defined as follows:

(Def. 17) $|\bullet : H| = \overline{\overline{\text{the left cosets of } H}}$.

We now state the proposition

(175) $|\bullet : H| = \overline{\overline{\text{the left cosets of } H}}$ and $|\bullet : H| = \overline{\overline{\text{the right cosets of } H}}$.

Let us consider G, H. Let us assume that the left cosets of H is finite. The functor $|\bullet : H|_{\mathbb{N}}$ yielding a natural number is defined as follows:

(Def. 18) There exists a finite set B such that $B =$ the left cosets of H and $|\bullet : H|_{\mathbb{N}} = \operatorname{card} B$.

The following proposition is true

(176) Suppose the left cosets of H is finite. Then

(i) there exists a finite set B such that $B =$ the left cosets of H and $|\bullet : H|_{\mathbb{N}} = \operatorname{card} B$, and

(ii) there exists a finite set C such that $C =$ the right cosets of H and $|\bullet : H|_{\mathbb{N}} = \operatorname{card} C$.

Let D be a non empty set and let d be an element of D. Then $\{d\}$ is an element of $\operatorname{Fin} D$.
The following two propositions are true:

(177) If G is finite, then $\operatorname{ord}(G) = \operatorname{ord}(H) \cdot |\bullet : H|_{\mathbb{N}}$.

(178) If G is finite, then $\operatorname{ord}(H) \mid \operatorname{ord}(G)$.

In the sequel J denotes a subgroup of H.
The following propositions are true:

(179) If G is finite and $I = J$, then $|\bullet : I|_{\mathbb{N}} = |\bullet : J|_{\mathbb{N}} \cdot |\bullet : H|_{\mathbb{N}}$.

(180) $|\bullet : \Omega_G|_{\mathbb{N}} = 1$.

(181) Let G be a strict group and H be a strict subgroup of G. If the left cosets of H is finite and $|\bullet : H|_{\mathbb{N}} = 1$, then $H = G$.

(182) $|\bullet : \{\mathbf{1}\}_G| = \operatorname{Ord}(G)$.

[12] The propositions (157)–(163) have been removed.

(183) If G is finite, then $|\bullet : \{\mathbf{1}\}_G|_{\mathbb{N}} = \mathrm{ord}(G)$.

(184) For every strict subgroup H of G such that G is finite and $|\bullet : H|_{\mathbb{N}} = \mathrm{ord}(G)$ holds $H = \{\mathbf{1}\}_G$.

(185) For every strict subgroup H of G such that the left cosets of H is finite and $|\bullet : H| = \mathrm{Ord}(G)$ holds G is finite and $H = \{\mathbf{1}\}_G$.

(186) Let X be a finite set. Suppose that for every Y such that $Y \in X$ there exists a finite set B such that $B = Y$ and $\mathrm{card}\, B = k$ and for every Z such that $Z \in X$ and $Y \neq Z$ holds $Y \approx Z$ and Y misses Z. Then there exists a finite set C such that $C = \bigcup X$ and $\mathrm{card}\, C = k \cdot \mathrm{card}\, X$.

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