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Some Properties of Cells on Go Board

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The articles [20], [7], [22], [2], [18], [23], [5], [6], [1], [4], [8], [21], [9], [17], [3], [13], [12], [19], [10], [11], [14], [15], and [16] provide the notation and terminology for this paper.

We adopt the following convention: i, i_1, i_2, j, j_1, j_2, k, n denote natural numbers, D denotes a non empty set, and f denotes a finite sequence of elements of D.

Let E be a non empty set, let S be a non empty set of finite sequences of the carrier of $\mathcal{E}^2_{\mathrm{T}}$, let F be a function from E into S, and let e be an element of E. Then $F(e)$ is a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$.

Let F be a function. The functor Values F yields a set and is defined by:

(Def. 1) $\mathrm{Values}\,F = \bigcup(\mathrm{rng}_{\kappa}\,F(\kappa))$.

One can prove the following proposition

(1) For every finite sequence M of elements of D^* holds $M(i)$ is a finite sequence of elements of D.

Let D be a set. Observe that every finite sequence of elements of D^* is finite sequence yielding. One can check that every function which is finite sequence yielding is also function yielding.

One can prove the following proposition

(3)[1] For every finite sequence M of elements of D^* holds $\mathrm{Values}\,M = \bigcup\{\mathrm{rng}\,f; f$ ranges over elements of D^*: $f \in \mathrm{rng}\,M\}$.

Let D be a non empty set and let M be a finite sequence of elements of D^*. One can check that Values M is finite.

One can prove the following propositions:

(4) For every matrix M over D such that $i \in \mathrm{dom}\,M$ and $M(i) = f$ holds $\mathrm{len}\,f = \mathrm{width}\,M$.

(5) For every matrix M over D such that $i \in \mathrm{dom}\,M$ and $M(i) = f$ and $j \in \mathrm{dom}\,f$ holds $\langle i, j\rangle \in$ the indices of M.

(6) For every matrix M over D such that $\langle i, j\rangle \in$ the indices of M and $M(i) = f$ holds $\mathrm{len}\,f = \mathrm{width}\,M$ and $j \in \mathrm{dom}\,f$.

(7) For every matrix M over D holds $\mathrm{Values}\,M = \{M \circ (i, j) : \langle i, j\rangle \in$ the indices of $M\}$.

(8) For every non empty set D and for every matrix M over D holds $\mathrm{card}\,\mathrm{Values}\,M \leq \mathrm{len}\,M \cdot \mathrm{width}\,M$.

[1] The proposition (2) has been removed.

In the sequel f is a finite sequence of elements of $\mathcal{E}^2_{\rm T}$ and G is a Go-board.
The following propositions are true:

(9) For every matrix G over $\mathcal{E}^2_{\rm T}$ such that f is a sequence which elements belong to G holds $\operatorname{rng} f \subseteq \operatorname{Values} G$.

(10) For all Go-boards G_1, G_2 such that $\operatorname{Values} G_1 \subseteq \operatorname{Values} G_2$ and $\langle i_1, j_1 \rangle \in$ the indices of G_1 and $1 \leq j_2$ and $j_2 \leq \operatorname{width} G_2$ and $G_1 \circ (i_1, j_1) = G_2 \circ (1, j_2)$ holds $i_1 = 1$.

(11) For all Go-boards G_1, G_2 such that $\operatorname{Values} G_1 \subseteq \operatorname{Values} G_2$ and $\langle i_1, j_1 \rangle \in$ the indices of G_1 and $1 \leq j_2$ and $j_2 \leq \operatorname{width} G_2$ and $G_1 \circ (i_1, j_1) = G_2 \circ (\operatorname{len} G_2, j_2)$ holds $i_1 = \operatorname{len} G_1$.

(12) For all Go-boards G_1, G_2 such that $\operatorname{Values} G_1 \subseteq \operatorname{Values} G_2$ and $\langle i_1, j_1 \rangle \in$ the indices of G_1 and $1 \leq i_2$ and $i_2 \leq \operatorname{len} G_2$ and $G_1 \circ (i_1, j_1) = G_2 \circ (i_2, 1)$ holds $j_1 = 1$.

(13) For all Go-boards G_1, G_2 such that $\operatorname{Values} G_1 \subseteq \operatorname{Values} G_2$ and $\langle i_1, j_1 \rangle \in$ the indices of G_1 and $1 \leq i_2$ and $i_2 \leq \operatorname{len} G_2$ and $G_1 \circ (i_1, j_1) = G_2 \circ (i_2, \operatorname{width} G_2)$ holds $j_1 = \operatorname{width} G_1$.

(14) Let G_1, G_2 be Go-boards. Suppose $\operatorname{Values} G_1 \subseteq \operatorname{Values} G_2$ and $1 \leq i_1$ and $i_1 < \operatorname{len} G_1$ and $1 \leq j_1$ and $j_1 \leq \operatorname{width} G_1$ and $1 \leq i_2$ and $i_2 < \operatorname{len} G_2$ and $1 \leq j_2$ and $j_2 \leq \operatorname{width} G_2$ and $G_1 \circ (i_1, j_1) = G_2 \circ (i_2, j_2)$. Then $(G_2 \circ (i_2 + 1, j_2))_{\mathbf{1}} \leq (G_1 \circ (i_1 + 1, j_1))_{\mathbf{1}}$.

(15) Let G_1, G_2 be Go-boards. Suppose $G_1 \circ (i_1 -' 1, j_1) \in \operatorname{Values} G_2$ and $1 < i_1$ and $i_1 \leq \operatorname{len} G_1$ and $1 \leq j_1$ and $j_1 \leq \operatorname{width} G_1$ and $1 < i_2$ and $i_2 \leq \operatorname{len} G_2$ and $1 \leq j_2$ and $j_2 \leq \operatorname{width} G_2$ and $G_1 \circ (i_1, j_1) = G_2 \circ (i_2, j_2)$. Then $(G_1 \circ (i_1 -' 1, j_1))_{\mathbf{1}} \leq (G_2 \circ (i_2 -' 1, j_2))_{\mathbf{1}}$.

(16) Let G_1, G_2 be Go-boards. Suppose $G_1 \circ (i_1, j_1 + 1) \in \operatorname{Values} G_2$ and $1 \leq i_1$ and $i_1 \leq \operatorname{len} G_1$ and $1 \leq j_1$ and $j_1 < \operatorname{width} G_1$ and $1 \leq i_2$ and $i_2 \leq \operatorname{len} G_2$ and $1 \leq j_2$ and $j_2 < \operatorname{width} G_2$ and $G_1 \circ (i_1, j_1) = G_2 \circ (i_2, j_2)$. Then $(G_2 \circ (i_2, j_2 + 1))_{\mathbf{2}} \leq (G_1 \circ (i_1, j_1 + 1))_{\mathbf{2}}$.

(17) Let G_1, G_2 be Go-boards. Suppose $\operatorname{Values} G_1 \subseteq \operatorname{Values} G_2$ and $1 \leq i_1$ and $i_1 \leq \operatorname{len} G_1$ and $1 < j_1$ and $j_1 \leq \operatorname{width} G_1$ and $1 \leq i_2$ and $i_2 \leq \operatorname{len} G_2$ and $1 < j_2$ and $j_2 \leq \operatorname{width} G_2$ and $G_1 \circ (i_1, j_1) = G_2 \circ (i_2, j_2)$. Then $(G_1 \circ (i_1, j_1 -' 1))_{\mathbf{2}} \leq (G_2 \circ (i_2, j_2 -' 1))_{\mathbf{2}}$.

(18) Let G_1, G_2 be Go-boards. Suppose $\operatorname{Values} G_1 \subseteq \operatorname{Values} G_2$ and $\langle i_1, j_1 \rangle \in$ the indices of G_1 and $\langle i_2, j_2 \rangle \in$ the indices of G_2 and $G_1 \circ (i_1, j_1) = G_2 \circ (i_2, j_2)$. Then $\operatorname{cell}(G_2, i_2, j_2) \subseteq \operatorname{cell}(G_1, i_1, j_1)$.

(19) Let G_1, G_2 be Go-boards. Suppose $\operatorname{Values} G_1 \subseteq \operatorname{Values} G_2$ and $\langle i_1, j_1 \rangle \in$ the indices of G_1 and $\langle i_2, j_2 \rangle \in$ the indices of G_2 and $G_1 \circ (i_1, j_1) = G_2 \circ (i_2, j_2)$. Then $\operatorname{cell}(G_2, i_2 -' 1, j_2) \subseteq \operatorname{cell}(G_1, i_1 -' 1, j_1)$.

(20) Let G_1, G_2 be Go-boards. Suppose $\operatorname{Values} G_1 \subseteq \operatorname{Values} G_2$ and $\langle i_1, j_1 \rangle \in$ the indices of G_1 and $\langle i_2, j_2 \rangle \in$ the indices of G_2 and $G_1 \circ (i_1, j_1) = G_2 \circ (i_2, j_2)$. Then $\operatorname{cell}(G_2, i_2, j_2 -' 1) \subseteq \operatorname{cell}(G_1, i_1, j_1 -' 1)$.

(21) Let f be a standard special circular sequence. Suppose f is a sequence which elements belong to G. Then Values the Go-board of $f \subseteq \operatorname{Values} G$.

Let us consider f, G, k. Let us assume that $1 \leq k$ and $k + 1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G. The functor right_cell(f, k, G) yields a subset of $\mathcal{E}^2_{\rm T}$ and is defined by the condition (Def. 2).

(Def. 2) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $\langle i_1, j_1 \rangle \in$ the indices of G and $\langle i_2, j_2 \rangle \in$ the indices of G and $f_k = G \circ (i_1, j_1)$ and $f_{k+1} = G \circ (i_2, j_2)$. Then

(i) $i_1 = i_2$ and $j_1 + 1 = j_2$ and right_cell$(f, k, G) = \operatorname{cell}(G, i_1, j_1)$, or

(ii) $i_1 + 1 = i_2$ and $j_1 = j_2$ and right_cell$(f, k, G) = \operatorname{cell}(G, i_1, j_1 -' 1)$, or

(iii) $i_1 = i_2 + 1$ and $j_1 = j_2$ and right_cell$(f, k, G) = \operatorname{cell}(G, i_2, j_2)$, or

(iv) $i_1 = i_2$ and $j_1 = j_2 + 1$ and right_cell$(f, k, G) = \operatorname{cell}(G, i_1 -' 1, j_2)$.

The functor left_cell(f,k,G) yielding a subset of $\mathcal{E}^2_{\mathrm{T}}$ is defined by the condition (Def. 3).

(Def. 3) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $\langle i_1, j_1\rangle \in$ the indices of G and $\langle i_2, j_2\rangle \in$ the indices of G and $f_k = G \circ (i_1, j_1)$ and $f_{k+1} = G \circ (i_2, j_2)$. Then

- (i) $i_1 = i_2$ and $j_1 + 1 = j_2$ and left_cell$(f,k,G) =$ cell$(G, i_1 -' 1, j_1)$, or
- (ii) $i_1 + 1 = i_2$ and $j_1 = j_2$ and left_cell$(f,k,G) =$ cell(G, i_1, j_1), or
- (iii) $i_1 = i_2 + 1$ and $j_1 = j_2$ and left_cell$(f,k,G) =$ cell$(G, i_2, j_2 -' 1)$, or
- (iv) $i_1 = i_2$ and $j_1 = j_2 + 1$ and left_cell$(f,k,G) =$ cell(G, i_1, j_2).

One can prove the following propositions:

(22) Suppose that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G and $\langle i, j\rangle \in$ the indices of G and $\langle i, j+1\rangle \in$ the indices of G and $f_k = G \circ (i, j)$ and $f_{k+1} = G \circ (i, j+1)$. Then left_cell$(f,k,G) =$ cell$(G, i -' 1, j)$.

(23) Suppose that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G and $\langle i, j\rangle \in$ the indices of G and $\langle i, j+1\rangle \in$ the indices of G and $f_k = G \circ (i, j)$ and $f_{k+1} = G \circ (i, j+1)$. Then right_cell$(f,k,G) =$ cell(G, i, j).

(24) Suppose that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G and $\langle i, j\rangle \in$ the indices of G and $\langle i+1, j\rangle \in$ the indices of G and $f_k = G \circ (i, j)$ and $f_{k+1} = G \circ (i+1, j)$. Then left_cell$(f,k,G) =$ cell(G, i, j).

(25) Suppose that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G and $\langle i, j\rangle \in$ the indices of G and $\langle i+1, j\rangle \in$ the indices of G and $f_k = G \circ (i, j)$ and $f_{k+1} = G \circ (i+1, j)$. Then right_cell$(f,k,G) =$ cell$(G, i, j -' 1)$.

(26) Suppose that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G and $\langle i, j\rangle \in$ the indices of G and $\langle i+1, j\rangle \in$ the indices of G and $f_k = G \circ (i+1, j)$ and $f_{k+1} = G \circ (i, j)$. Then left_cell$(f,k,G) =$ cell$(G, i, j -' 1)$.

(27) Suppose that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G and $\langle i, j\rangle \in$ the indices of G and $\langle i+1, j\rangle \in$ the indices of G and $f_k = G \circ (i+1, j)$ and $f_{k+1} = G \circ (i, j)$. Then right_cell$(f,k,G) =$ cell(G, i, j).

(28) Suppose that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G and $\langle i, j+1\rangle \in$ the indices of G and $\langle i, j\rangle \in$ the indices of G and $f_k = G \circ (i, j+1)$ and $f_{k+1} = G \circ (i, j)$. Then left_cell$(f,k,G) =$ cell(G, i, j).

(29) Suppose that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G and $\langle i, j+1\rangle \in$ the indices of G and $\langle i, j\rangle \in$ the indices of G and $f_k = G \circ (i, j+1)$ and $f_{k+1} = G \circ (i, j)$. Then right_cell$(f,k,G) =$ cell$(G, i -' 1, j)$.

(30) If $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G, then left_cell$(f,k,G) \cap$ right_cell$(f,k,G) = \mathcal{L}(f,k)$.

(31) If $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G, then right_cell(f,k,G) is closed.

(32) Suppose $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G and $k+1 \leq n$. Then left_cell$(f,k,G) =$ left_cell$(f \restriction n, k, G)$ and right_cell$(f,k,G) =$ right_cell$(f \restriction n, k, G)$.

(33) Suppose $1 \leq k$ and $k+1 \leq \operatorname{len}(f_{\lfloor n})$ and $n \leq \operatorname{len} f$ and f is a sequence which elements belong to G. Then left_cell$(f, k+n, G) =$ left_cell$(f_{\lfloor n}, k, G)$ and right_cell$(f, k+n, G) =$ right_cell$(f_{\lfloor n}, k, G)$.

(34) Let G be a Go-board and f be a standard special circular sequence. Suppose $1 \leq n$ and $n+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G. Then left_cell$(f,n,G) \subseteq$ leftcell(f,n) and right_cell$(f,n,G) \subseteq$ rightcell(f,n).

Let us consider f, G, k. Let us assume that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G. The functor front_right_cell(f,k,G) yields a subset of $\mathcal{E}^2_{\mathrm{T}}$ and is defined by the condition (Def. 4).

(Def. 4) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $\langle i_1, j_1\rangle \in$ the indices of G and $\langle i_2, j_2\rangle \in$ the indices of G and $f_k = G \circ (i_1, j_1)$ and $f_{k+1} = G \circ (i_2, j_2)$. Then

- (i) $i_1 = i_2$ and $j_1 + 1 = j_2$ and front_right_cell$(f,k,G) = \mathrm{cell}(G, i_2, j_2)$, or
- (ii) $i_1 + 1 = i_2$ and $j_1 = j_2$ and front_right_cell$(f,k,G) = \mathrm{cell}(G, i_2, j_2 -' 1)$, or
- (iii) $i_1 = i_2 + 1$ and $j_1 = j_2$ and front_right_cell$(f,k,G) = \mathrm{cell}(G, i_2 -' 1, j_2)$, or
- (iv) $i_1 = i_2$ and $j_1 = j_2 + 1$ and front_right_cell$(f,k,G) = \mathrm{cell}(G, i_2 -' 1, j_2 -' 1)$.

The functor front_left_cell(f,k,G) yielding a subset of $\mathcal{E}^2_{\mathrm{T}}$ is defined by the condition (Def. 5).

(Def. 5) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $\langle i_1, j_1\rangle \in$ the indices of G and $\langle i_2, j_2\rangle \in$ the indices of G and $f_k = G \circ (i_1, j_1)$ and $f_{k+1} = G \circ (i_2, j_2)$. Then

- (i) $i_1 = i_2$ and $j_1 + 1 = j_2$ and front_left_cell$(f,k,G) = \mathrm{cell}(G, i_2 -' 1, j_2)$, or
- (ii) $i_1 + 1 = i_2$ and $j_1 = j_2$ and front_left_cell$(f,k,G) = \mathrm{cell}(G, i_2, j_2)$, or
- (iii) $i_1 = i_2 + 1$ and $j_1 = j_2$ and front_left_cell$(f,k,G) = \mathrm{cell}(G, i_2 -' 1, j_2 -' 1)$, or
- (iv) $i_1 = i_2$ and $j_1 = j_2 + 1$ and front_left_cell$(f,k,G) = \mathrm{cell}(G, i_2, j_2 -' 1)$.

The following propositions are true:

(35) Suppose that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G and $\langle i, j\rangle \in$ the indices of G and $\langle i, j+1\rangle \in$ the indices of G and $f_k = G \circ (i, j)$ and $f_{k+1} = G \circ (i, j+1)$. Then front_left_cell$(f,k,G) = \mathrm{cell}(G, i -' 1, j+1)$.

(36) Suppose that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G and $\langle i, j\rangle \in$ the indices of G and $\langle i, j+1\rangle \in$ the indices of G and $f_k = G \circ (i, j)$ and $f_{k+1} = G \circ (i, j+1)$. Then front_right_cell$(f,k,G) = \mathrm{cell}(G, i, j+1)$.

(37) Suppose that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G and $\langle i, j\rangle \in$ the indices of G and $\langle i+1, j\rangle \in$ the indices of G and $f_k = G \circ (i, j)$ and $f_{k+1} = G \circ (i+1, j)$. Then front_left_cell$(f,k,G) = \mathrm{cell}(G, i+1, j)$.

(38) Suppose that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G and $\langle i, j\rangle \in$ the indices of G and $\langle i+1, j\rangle \in$ the indices of G and $f_k = G \circ (i, j)$ and $f_{k+1} = G \circ (i+1, j)$. Then front_right_cell$(f,k,G) = \mathrm{cell}(G, i+1, j -' 1)$.

(39) Suppose that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G and $\langle i, j\rangle \in$ the indices of G and $\langle i+1, j\rangle \in$ the indices of G and $f_k = G \circ (i+1, j)$ and $f_{k+1} = G \circ (i, j)$. Then front_left_cell$(f,k,G) = \mathrm{cell}(G, i -' 1, j -' 1)$.

(40) Suppose that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G and $\langle i, j\rangle \in$ the indices of G and $\langle i+1, j\rangle \in$ the indices of G and $f_k = G \circ (i+1, j)$ and $f_{k+1} = G \circ (i, j)$. Then front_right_cell$(f,k,G) = \mathrm{cell}(G, i -' 1, j)$.

(41) Suppose that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G and $\langle i, j+1\rangle \in$ the indices of G and $\langle i, j\rangle \in$ the indices of G and $f_k = G \circ (i, j+1)$ and $f_{k+1} = G \circ (i, j)$. Then front_left_cell$(f,k,G) = \mathrm{cell}(G, i, j -' 1)$.

(42) Suppose that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G and $\langle i, j+1\rangle \in$ the indices of G and $\langle i, j\rangle \in$ the indices of G and $f_k = G \circ (i, j+1)$ and $f_{k+1} = G \circ (i, j)$. Then front_right_cell$(f,k,G) = \mathrm{cell}(G, i -' 1, j -' 1)$.

(43) Suppose $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and f is a sequence which elements belong to G and $k+1 \leq n$. Then front_left_cell$(f,k,G) =$ front_left_cell$(f {\restriction} n, k, G)$ and front_right_cell$(f,k,G) =$ front_right_cell$(f {\restriction} n, k, G)$.

Let D be a set, let f be a finite sequence of elements of D, let G be a matrix over D, and let us consider k. We say that f turns right k, G if and only if the condition (Def. 6) is satisfied.

(Def. 6) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $\langle i_1, j_1\rangle \in$ the indices of G and $\langle i_2, j_2\rangle \in$ the indices of G and $f_k = G \circ (i_1, j_1)$ and $f_{k+1} = G \circ (i_2, j_2)$. Then

- (i) $i_1 = i_2$ and $j_1 + 1 = j_2$ and $\langle i_2 + 1, j_2\rangle \in$ the indices of G and $f_{k+2} = G \circ (i_2 + 1, j_2)$, or
- (ii) $i_1 + 1 = i_2$ and $j_1 = j_2$ and $\langle i_2, j_2 -' 1\rangle \in$ the indices of G and $f_{k+2} = G \circ (i_2, j_2 -' 1)$, or
- (iii) $i_1 = i_2 + 1$ and $j_1 = j_2$ and $\langle i_2, j_2 + 1\rangle \in$ the indices of G and $f_{k+2} = G \circ (i_2, j_2 + 1)$, or
- (iv) $i_1 = i_2$ and $j_1 = j_2 + 1$ and $\langle i_2 -' 1, j_2\rangle \in$ the indices of G and $f_{k+2} = G \circ (i_2 -' 1, j_2)$.

We say that f turns left k, G if and only if the condition (Def. 7) is satisfied.

(Def. 7) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $\langle i_1, j_1\rangle \in$ the indices of G and $\langle i_2, j_2\rangle \in$ the indices of G and $f_k = G \circ (i_1, j_1)$ and $f_{k+1} = G \circ (i_2, j_2)$. Then

- (i) $i_1 = i_2$ and $j_1 + 1 = j_2$ and $\langle i_2 -' 1, j_2\rangle \in$ the indices of G and $f_{k+2} = G \circ (i_2 -' 1, j_2)$, or
- (ii) $i_1 + 1 = i_2$ and $j_1 = j_2$ and $\langle i_2, j_2 + 1\rangle \in$ the indices of G and $f_{k+2} = G \circ (i_2, j_2 + 1)$, or
- (iii) $i_1 = i_2 + 1$ and $j_1 = j_2$ and $\langle i_2, j_2 -' 1\rangle \in$ the indices of G and $f_{k+2} = G \circ (i_2, j_2 -' 1)$, or
- (iv) $i_1 = i_2$ and $j_1 = j_2 + 1$ and $\langle i_2 + 1, j_2\rangle \in$ the indices of G and $f_{k+2} = G \circ (i_2 + 1, j_2)$.

We say that f goes straight k, G if and only if the condition (Def. 8) is satisfied.

(Def. 8) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $\langle i_1, j_1\rangle \in$ the indices of G and $\langle i_2, j_2\rangle \in$ the indices of G and $f_k = G \circ (i_1, j_1)$ and $f_{k+1} = G \circ (i_2, j_2)$. Then

- (i) $i_1 = i_2$ and $j_1 + 1 = j_2$ and $\langle i_2, j_2 + 1\rangle \in$ the indices of G and $f_{k+2} = G \circ (i_2, j_2 + 1)$, or
- (ii) $i_1 + 1 = i_2$ and $j_1 = j_2$ and $\langle i_2 + 1, j_2\rangle \in$ the indices of G and $f_{k+2} = G \circ (i_2 + 1, j_2)$, or
- (iii) $i_1 = i_2 + 1$ and $j_1 = j_2$ and $\langle i_2 -' 1, j_2\rangle \in$ the indices of G and $f_{k+2} = G \circ (i_2 -' 1, j_2)$, or
- (iv) $i_1 = i_2$ and $j_1 = j_2 + 1$ and $\langle i_2, j_2 -' 1\rangle \in$ the indices of G and $f_{k+2} = G \circ (i_2, j_2 -' 1)$.

We adopt the following convention: D denotes a set, f, f_1, f_2 denote finite sequences of elements of D, and G denotes a matrix over D.

The following propositions are true:

(44) If $1 \le k$ and $k + 2 \le \operatorname{len} f$ and $k + 2 \le n$ and $f \restriction n$ turns right k, G, then f turns right k, G.

(45) If $1 \le k$ and $k + 2 \le \operatorname{len} f$ and $k + 2 \le n$ and $f \restriction n$ turns left k, G, then f turns left k, G.

(46) If $1 \le k$ and $k + 2 \le \operatorname{len} f$ and $k + 2 \le n$ and $f \restriction n$ goes straight k, G, then f goes straight k, G.

(47) Suppose that $1 < k$ and $k + 1 \le \operatorname{len} f_1$ and $k + 1 \le \operatorname{len} f_2$ and f_1 is a sequence which elements belong to G and $f_1 \restriction k = f_2 \restriction k$ and f_1 turns right $k -' 1$, G and f_2 turns right $k -' 1$, G. Then $f_1 \restriction (k + 1) = f_2 \restriction (k + 1)$.

(48) Suppose that $1 < k$ and $k + 1 \le \operatorname{len} f_1$ and $k + 1 \le \operatorname{len} f_2$ and f_1 is a sequence which elements belong to G and $f_1 \restriction k = f_2 \restriction k$ and f_1 turns left $k -' 1$, G and f_2 turns left $k -' 1$, G. Then $f_1 \restriction (k + 1) = f_2 \restriction (k + 1)$.

(49) Suppose that $1 < k$ and $k + 1 \le \operatorname{len} f_1$ and $k + 1 \le \operatorname{len} f_2$ and f_1 is a sequence which elements belong to G and $f_1 \restriction k = f_2 \restriction k$ and f_1 goes straight $k -' 1$, G and f_2 goes straight $k -' 1$, G. Then $f_1 \restriction (k + 1) = f_2 \restriction (k + 1)$.

(50) For every non empty set D and for every matrix M over D such that $1 \le i$ and $i \le \operatorname{len} M$ and $1 \le j$ and $j \le \operatorname{width} M$ holds $M \circ (i, j) \in \operatorname{Values} M$.

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