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Left and Right Component of the Complement of a Special Closed Curve

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Summary. In the article the concept of the left and right component are introduced. These are the auxiliary notions needed in the proof of Jordan Curve Theorem.

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The articles [18], [20], [3], [4], [2], [15], [6], [5], [10], [1], [17], [21], [16], [8], [9], [11], [12], [13], [7], [19], and [14] provide the notation and terminology for this paper.

For simplicity, we use the following convention: f is a non constant standard special circular sequence, i, j, k are natural numbers, a, b are natural numbers, p, q are points of $\mathcal{E}^2_{\mathrm{T}}$, and G is a Go-board.

One can prove the following propositions:

(1) $a -' a = 0$.

(2) $a -' b \leq a$.

(3) Let G_1 be a non empty topological space and A_1, A_2, B be subsets of G_1. Suppose A_1 is a component of B and A_2 is a component of B. Then $A_1 = A_2$ or A_1 misses A_2.

(4) Let G_1 be a non empty topological space, A, B be non empty subsets of G_1, and A_3 be a subset of $G_1 \restriction B$. If $A = A_3$, then $G_1 \restriction A = G_1 \restriction B \restriction A_3$.

(5) Let G_1 be a non empty topological space, A be a non empty subset of G_1, and B be a non empty subset of G_1. Suppose $A \subseteq B$ and A is connected. Then there exists a subset C of G_1 such that C is a component of B and $A \subseteq C$.

(6) Let G_1 be a non empty topological space, A, C, D be subsets of G_1, and B be a subset of G_1. Suppose B is connected and C is a component of D and $A \subseteq C$ and A meets B and $B \subseteq D$. Then $B \subseteq C$.

(7) $\mathcal{L}(p,q)$ is convex.

(8) $\mathcal{L}(p,q)$ is connected.

Let us note that there exists a subset of $\mathcal{E}^2_{\mathrm{T}}$ which is convex and non empty.

One can prove the following propositions:

(9) For all convex subsets P, Q of $\mathcal{E}^2_{\mathrm{T}}$ holds $P \cap Q$ is convex.

(10) For every finite sequence f of elements of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{Rev}(\mathbf{X}\text{-coordinate}(f)) = \mathbf{X}\text{-coordinate}(\mathrm{Rev}(f))$.

(11) For every finite sequence f of elements of $\mathcal{E}^2_{\mathrm{T}}$ holds $\mathrm{Rev}(\mathbf{Y}\text{-coordinate}(f)) = \mathbf{Y}\text{-coordinate}(\mathrm{Rev}(f))$.

Let us mention that there exists a finite sequence which is non constant.

Let f be a non constant finite sequence. One can verify that $\mathrm{Rev}(f)$ is non constant.

Let f be a standard special circular sequence. Then $\mathrm{Rev}(f)$ is a standard special circular sequence.

One can prove the following propositions:

(12) If $i \geq 1$ and $j \geq 1$ and $i + j = \mathrm{len}\, f$, then $\mathrm{leftcell}(f, i) = \mathrm{rightcell}(\mathrm{Rev}(f), j)$.

(13) If $i \geq 1$ and $j \geq 1$ and $i + j = \mathrm{len}\, f$, then $\mathrm{leftcell}(\mathrm{Rev}(f), i) = \mathrm{rightcell}(f, j)$.

(14) Suppose $1 \leq k$ and $k + 1 \leq \mathrm{len}\, f$. Then there exist i, j such that $i \leq \mathrm{len}$ the Go-board of f and $j \leq \mathrm{width}$ the Go-board of f and cell(the Go-board of f, $i, j) = \mathrm{leftcell}(f, k)$.

(15) If $j \leq \mathrm{width}\, G$, then $\mathrm{Int}\,\mathrm{hstrip}(G, j)$ is convex.

(16) If $i \leq \mathrm{len}\, G$, then $\mathrm{Int}\,\mathrm{vstrip}(G, i)$ is convex.

(17) If $i \leq \mathrm{len}\, G$ and $j \leq \mathrm{width}\, G$, then $\mathrm{Int}\,\mathrm{cell}(G, i, j) \neq \emptyset$.

(18) If $1 \leq k$ and $k + 1 \leq \mathrm{len}\, f$, then $\mathrm{Int}\,\mathrm{leftcell}(f, k) \neq \emptyset$.

(19) If $1 \leq k$ and $k + 1 \leq \mathrm{len}\, f$, then $\mathrm{Int}\,\mathrm{rightcell}(f, k) \neq \emptyset$.

(20) If $i \leq \mathrm{len}\, G$ and $j \leq \mathrm{width}\, G$, then $\mathrm{Int}\,\mathrm{cell}(G, i, j)$ is convex.

(21) If $i \leq \mathrm{len}\, G$ and $j \leq \mathrm{width}\, G$, then $\mathrm{Int}\,\mathrm{cell}(G, i, j)$ is connected.

(22) If $1 \leq k$ and $k + 1 \leq \mathrm{len}\, f$, then $\mathrm{Int}\,\mathrm{leftcell}(f, k)$ is connected.

(23) If $1 \leq k$ and $k + 1 \leq \mathrm{len}\, f$, then $\mathrm{Int}\,\mathrm{rightcell}(f, k)$ is connected.

Let us consider f. The functor $\mathrm{LeftComp}(f)$ yields a subset of $\mathcal{E}^2_{\mathrm{T}}$ and is defined by:

(Def. 1) $\mathrm{LeftComp}(f)$ is a component of $(\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ and $\mathrm{Int}\,\mathrm{leftcell}(f, 1) \subseteq \mathrm{LeftComp}(f)$.

The functor $\mathrm{RightComp}(f)$ yielding a subset of $\mathcal{E}^2_{\mathrm{T}}$ is defined as follows:

(Def. 2) $\mathrm{RightComp}(f)$ is a component of $(\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ and $\mathrm{Int}\,\mathrm{rightcell}(f, 1) \subseteq \mathrm{RightComp}(f)$.

The following propositions are true:

(24) For every k such that $1 \leq k$ and $k + 1 \leq \mathrm{len}\, f$ holds $\mathrm{Int}\,\mathrm{leftcell}(f, k) \subseteq \mathrm{LeftComp}(f)$.

(25) The Go-board of $\mathrm{Rev}(f) =$ the Go-board of f.

(26) $\mathrm{RightComp}(f) = \mathrm{LeftComp}(\mathrm{Rev}(f))$.

(27) $\mathrm{RightComp}(\mathrm{Rev}(f)) = \mathrm{LeftComp}(f)$.

(28) For every k such that $1 \leq k$ and $k + 1 \leq \mathrm{len}\, f$ holds $\mathrm{Int}\,\mathrm{rightcell}(f, k) \subseteq \mathrm{RightComp}(f)$.

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