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Decomposing a Go-Board into Cells

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The articles [15], [5], [17], [8], [2], [12], [14], [1], [4], [3], [18], [9], [13], [6], [7], [10], [11], and [16] provide the notation and terminology for this paper.

For simplicity, we adopt the following convention: q denotes a point of $\mathcal{E}^2_{\rm T}$, i, i_1, i_2, j, j_1, j_2, k denote natural numbers, r, s denote real numbers, and G denotes a matrix over $\mathcal{E}^2_{\rm T}$.

Next we state the proposition

(1) Let M be a tabular finite sequence and given i, j. If $\langle i, j\rangle \in$ the indices of M, then $1 \leq i$ and $i \leq \operatorname{len} M$ and $1 \leq j$ and $j \leq \operatorname{width} M$.

Let G be a matrix over $\mathcal{E}^2_{\rm T}$ and let us consider i. The functor $\operatorname{vstrip}(G,i)$ yields a subset of $\mathcal{E}^2_{\rm T}$ and is defined by:

$$\text{(Def. 1)}\quad \operatorname{vstrip}(G,i) = \begin{cases} \{[r,s] : (G\circ(i,1))_{\mathbf{1}} \leq r \wedge r \leq (G\circ(i+1,1))_{\mathbf{1}}\}, & \text{if } 1 \leq i \text{ and } i < \operatorname{len} G,\\ \{[r,s] : (G\circ(i,1))_{\mathbf{1}} \leq r\}, & \text{if } i \geq \operatorname{len} G,\\ \{[r,s] : r \leq (G\circ(i+1,1))_{\mathbf{1}}\}, & \text{otherwise.}\end{cases}$$

The functor $\operatorname{hstrip}(G,i)$ yielding a subset of $\mathcal{E}^2_{\rm T}$ is defined by:

$$\text{(Def. 2)}\quad \operatorname{hstrip}(G,i) = \begin{cases} \{[r,s] : (G\circ(1,i))_{\mathbf{2}} \leq s \wedge s \leq (G\circ(1,i+1))_{\mathbf{2}}\}, & \text{if } 1 \leq i \text{ and } i < \operatorname{width} G,\\ \{[r,s] : (G\circ(1,i))_{\mathbf{2}} \leq s\}, & \text{if } i \geq \operatorname{width} G,\\ \{[r,s] : s \leq (G\circ(1,i+1))_{\mathbf{2}}\}, & \text{otherwise.}\end{cases}$$

The following propositions are true:

(2) If G is column $\mathbf{Y}$-constant and $1 \leq j$ and $j \leq \operatorname{width} G$ and $1 \leq i$ and $i \leq \operatorname{len} G$, then $(G\circ(i,j))_{\mathbf{2}} = (G\circ(1,j))_{\mathbf{2}}$.

(3) If G is line $\mathbf{X}$-constant and $1 \leq j$ and $j \leq \operatorname{width} G$ and $1 \leq i$ and $i \leq \operatorname{len} G$, then $(G\circ(i,j))_{\mathbf{1}} = (G\circ(i,1))_{\mathbf{1}}$.

(4) If G is column $\mathbf{X}$-increasing and $1 \leq j$ and $j \leq \operatorname{width} G$ and $1 \leq i_1$ and $i_1 < i_2$ and $i_2 \leq \operatorname{len} G$, then $(G\circ(i_1,j))_{\mathbf{1}} < (G\circ(i_2,j))_{\mathbf{1}}$.

(5) If G is line $\mathbf{Y}$-increasing and $1 \leq j_1$ and $j_1 < j_2$ and $j_2 \leq \operatorname{width} G$ and $1 \leq i$ and $i \leq \operatorname{len} G$, then $(G\circ(i,j_1))_{\mathbf{2}} < (G\circ(i,j_2))_{\mathbf{2}}$.

(6) If G is column $\mathbf{Y}$-constant and $1 \leq j$ and $j < \operatorname{width} G$ and $1 \leq i$ and $i \leq \operatorname{len} G$, then $\operatorname{hstrip}(G,j) = \{[r,s] : (G\circ(i,j))_{\mathbf{2}} \leq s \wedge s \leq (G\circ(i,j+1))_{\mathbf{2}}\}$.

(7) If G is non empty yielding and column $\mathbf{Y}$-constant and $1 \leq i$ and $i \leq \operatorname{len} G$, then $\operatorname{hstrip}(G, \operatorname{width} G) = \{[r,s] : (G \circ (i, \operatorname{width} G))_{\mathbf{2}} \leq s\}$.

(8) If G is non empty yielding and column $\mathbf{Y}$-constant and $1 \leq i$ and $i \leq \operatorname{len} G$, then $\operatorname{hstrip}(G, 0) = \{[r,s] : s \leq (G \circ (i,1))_{\mathbf{2}}\}$.

(9) If G is line $\mathbf{X}$-constant and $1 \leq i$ and $i < \operatorname{len} G$ and $1 \leq j$ and $j \leq \operatorname{width} G$, then $\operatorname{vstrip}(G,i) = \{[r,s] : (G \circ (i,j))_{\mathbf{1}} \leq r \wedge r \leq (G \circ (i+1,j))_{\mathbf{1}}\}$.

(10) If G is non empty yielding and line $\mathbf{X}$-constant and $1 \leq j$ and $j \leq \operatorname{width} G$, then $\operatorname{vstrip}(G, \operatorname{len} G) = \{[r,s] : (G \circ (\operatorname{len} G, j))_{\mathbf{1}} \leq r\}$.

(11) If G is non empty yielding and line $\mathbf{X}$-constant and $1 \leq j$ and $j \leq \operatorname{width} G$, then $\operatorname{vstrip}(G,0) = \{[r,s] : r \leq (G \circ (1,j))_{\mathbf{1}}\}$.

Let G be a matrix over $\mathcal{E}^2_{\mathrm{T}}$ and let us consider i, j. The functor $\operatorname{cell}(G,i,j)$ yields a subset of $\mathcal{E}^2_{\mathrm{T}}$ and is defined as follows:

(Def. 3) $\operatorname{cell}(G,i,j) = \operatorname{vstrip}(G,i) \cap \operatorname{hstrip}(G,j)$.

Let I_1 be a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. We say that I_1 is s.c.c. if and only if:

(Def. 4) For all i, j such that $i+1 < j$ but $i > 1$ and $j < \operatorname{len} I_1$ or $j+1 < \operatorname{len} I_1$ holds $\mathcal{L}(I_1,i)$ misses $\mathcal{L}(I_1,j)$.

Let I_1 be a non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. We say that I_1 is standard if and only if:

(Def. 5) I_1 is a sequence which elements belong to the Go-board of I_1.

Let us observe that there exists a non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ which is non constant, special, unfolded, circular, s.c.c., and standard.

Next we state two propositions:

(12) Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ and n be a natural number. Suppose $n \in \operatorname{dom} f$. Then there exist i, j such that $\langle i, j \rangle \in$ the indices of the Go-board of f and $f_n =$ the Go-board of $f \circ (i,j)$.

(13) Let f be a standard non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$ and n be a natural number. Suppose $n \in \operatorname{dom} f$ and $n+1 \in \operatorname{dom} f$. Let m, k, i, j be natural numbers. Suppose that

(i) $\langle m, k \rangle \in$ the indices of the Go-board of f,

(ii) $\langle i, j \rangle \in$ the indices of the Go-board of f,

(iii) $f_n =$ the Go-board of $f \circ (m,k)$, and

(iv) $f_{n+1} =$ the Go-board of $f \circ (i,j)$.

Then $|m-i| + |k-j| = 1$.

A special circular sequence is a special unfolded circular s.c.c. non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$.

In the sequel f denotes a standard special circular sequence.

Let us consider f, k. Let us assume that $1 \leq k$ and $k+1 \leq \operatorname{len} f$. The functor $\operatorname{rightcell}(f,k)$ yields a subset of $\mathcal{E}^2_{\mathrm{T}}$ and is defined by the condition (Def. 6).

(Def. 6) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose that

(i) $\langle i_1, j_1 \rangle \in$ the indices of the Go-board of f,

(ii) $\langle i_2, j_2 \rangle \in$ the indices of the Go-board of f,

(iii) $f_k =$ the Go-board of $f \circ (i_1, j_1)$, and

(iv) $f_{k+1} =$ the Go-board of $f \circ (i_2, j_2)$.

Then

(v) $i_1 = i_2$ and $j_1 + 1 = j_2$ and $\mathrm{rightcell}(f,k) = \mathrm{cell}($the Go-board of f, $i_1, j_1)$, or

(vi) $i_1 + 1 = i_2$ and $j_1 = j_2$ and $\mathrm{rightcell}(f,k) = \mathrm{cell}($the Go-board of f, $i_1, j_1 -' 1)$, or

(vii) $i_1 = i_2 + 1$ and $j_1 = j_2$ and $\mathrm{rightcell}(f,k) = \mathrm{cell}($the Go-board of f, $i_2, j_2)$, or

(viii) $i_1 = i_2$ and $j_1 = j_2 + 1$ and $\mathrm{rightcell}(f,k) = \mathrm{cell}($the Go-board of f, $i_1 -' 1, j_2)$.

The functor $\mathrm{leftcell}(f,k)$ yields a subset of $\mathcal{E}^2_{\mathrm{T}}$ and is defined by the condition (Def. 7).

(Def. 7) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose that

(i) $\langle i_1, j_1 \rangle \in$ the indices of the Go-board of f,

(ii) $\langle i_2, j_2 \rangle \in$ the indices of the Go-board of f,

(iii) $f_k =$ the Go-board of $f \circ (i_1, j_1)$, and

(iv) $f_{k+1} =$ the Go-board of $f \circ (i_2, j_2)$.

Then

(v) $i_1 = i_2$ and $j_1 + 1 = j_2$ and $\mathrm{leftcell}(f,k) = \mathrm{cell}($the Go-board of f, $i_1 -' 1, j_1)$, or

(vi) $i_1 + 1 = i_2$ and $j_1 = j_2$ and $\mathrm{leftcell}(f,k) = \mathrm{cell}($the Go-board of f, $i_1, j_1)$, or

(vii) $i_1 = i_2 + 1$ and $j_1 = j_2$ and $\mathrm{leftcell}(f,k) = \mathrm{cell}($the Go-board of f, $i_2, j_2 -' 1)$, or

(viii) $i_1 = i_2$ and $j_1 = j_2 + 1$ and $\mathrm{leftcell}(f,k) = \mathrm{cell}($the Go-board of f, $i_1, j_2)$.

Next we state a number of propositions:

(14) Suppose G is non empty yielding, line **X**-constant, and column **X**-increasing and $i < \mathrm{len}\, G$ and $1 \leq j$ and $j < \mathrm{width}\, G$. Then $\mathcal{L}(G \circ (i+1, j), G \circ (i+1, j+1)) \subseteq \mathrm{vstrip}(G, i)$.

(15) Suppose that

(i) G is non empty yielding, line **X**-constant, and column **X**-increasing,

(ii) $1 \leq i$,

(iii) $i \leq \mathrm{len}\, G$,

(iv) $1 \leq j$, and

(v) $j < \mathrm{width}\, G$.

Then $\mathcal{L}(G \circ (i, j), G \circ (i, j+1)) \subseteq \mathrm{vstrip}(G, i)$.

(16) Suppose G is non empty yielding, column **Y**-constant, and line **Y**-increasing and $j < \mathrm{width}\, G$ and $1 \leq i$ and $i < \mathrm{len}\, G$. Then $\mathcal{L}(G \circ (i, j+1), G \circ (i+1, j+1)) \subseteq \mathrm{hstrip}(G, j)$.

(17) Suppose that

(i) G is non empty yielding, column **Y**-constant, and line **Y**-increasing,

(ii) $1 \leq j$,

(iii) $j \leq \mathrm{width}\, G$,

(iv) $1 \leq i$, and

(v) $i < \mathrm{len}\, G$.

Then $\mathcal{L}(G \circ (i, j), G \circ (i+1, j)) \subseteq \mathrm{hstrip}(G, j)$.

(18) Suppose G is column **Y**-constant and line **Y**-increasing and $1 \leq i$ and $i \leq \mathrm{len}\, G$ and $1 \leq j$ and $j + 1 \leq \mathrm{width}\, G$. Then $\mathcal{L}(G \circ (i, j), G \circ (i, j+1)) \subseteq \mathrm{hstrip}(G, j)$.

(19) For every Go-board G such that $i < \mathrm{len}\, G$ and $1 \leq j$ and $j < \mathrm{width}\, G$ holds $\mathcal{L}(G \circ (i+1, j), G \circ (i+1, j+1)) \subseteq \mathrm{cell}(G, i, j)$.

(20) For every Go-board G such that $1 \leq i$ and $i \leq \mathrm{len}\, G$ and $1 \leq j$ and $j < \mathrm{width}\, G$ holds $\mathcal{L}(G \circ (i, j), G \circ (i, j+1)) \subseteq \mathrm{cell}(G, i, j)$.

(21) Suppose G is line $\mathbf{X}$-constant and column $\mathbf{X}$-increasing and $1 \leq j$ and $j \leq \mathrm{width}\, G$ and $1 \leq i$ and $i+1 \leq \mathrm{len}\, G$. Then $\mathcal{L}(G \circ (i, j), G \circ (i+1, j)) \subseteq \mathrm{vstrip}(G, i)$.

(22) For every Go-board G such that $j < \mathrm{width}\, G$ and $1 \leq i$ and $i < \mathrm{len}\, G$ holds $\mathcal{L}(G \circ (i, j+1), G \circ (i+1, j+1)) \subseteq \mathrm{cell}(G, i, j)$.

(23) For every Go-board G such that $1 \leq i$ and $i < \mathrm{len}\, G$ and $1 \leq j$ and $j \leq \mathrm{width}\, G$ holds $\mathcal{L}(G \circ (i, j), G \circ (i+1, j)) \subseteq \mathrm{cell}(G, i, j)$.

(24) Suppose G is non empty yielding, line $\mathbf{X}$-constant, and column $\mathbf{X}$-increasing and $i+1 \leq \mathrm{len}\, G$. Then $\mathrm{vstrip}(G, i) \cap \mathrm{vstrip}(G, i+1) = \{q : q_{\mathbf{1}} = (G \circ (i+1, 1))_{\mathbf{1}}\}$.

(25) Suppose G is non empty yielding, column $\mathbf{Y}$-constant, and line $\mathbf{Y}$-increasing and $j+1 \leq \mathrm{width}\, G$. Then $\mathrm{hstrip}(G, j) \cap \mathrm{hstrip}(G, j+1) = \{q : q_{\mathbf{2}} = (G \circ (1, j+1))_{\mathbf{2}}\}$.

(26) For every Go-board G such that $i < \mathrm{len}\, G$ and $1 \leq j$ and $j < \mathrm{width}\, G$ holds $\mathrm{cell}(G, i, j) \cap \mathrm{cell}(G, i+1, j) = \mathcal{L}(G \circ (i+1, j), G \circ (i+1, j+1))$.

(27) For every Go-board G such that $j < \mathrm{width}\, G$ and $1 \leq i$ and $i < \mathrm{len}\, G$ holds $\mathrm{cell}(G, i, j) \cap \mathrm{cell}(G, i, j+1) = \mathcal{L}(G \circ (i, j+1), G \circ (i+1, j+1))$.

(28) Suppose that

(i) $1 \leq k$,

(ii) $k+1 \leq \mathrm{len}\, f$,

(iii) $\langle i+1, j \rangle \in$ the indices of the Go-board of f,

(iv) $\langle i+1, j+1 \rangle \in$ the indices of the Go-board of f,

(v) $f_k =$ the Go-board of $f \circ (i+1, j)$, and

(vi) $f_{k+1} =$ the Go-board of $f \circ (i+1, j+1)$.

Then $\mathrm{leftcell}(f, k) = \mathrm{cell}($the Go-board of f, $i, j)$ and $\mathrm{rightcell}(f, k) = \mathrm{cell}($the Go-board of f, $i+1, j)$.

(29) Suppose that

(i) $1 \leq k$,

(ii) $k+1 \leq \mathrm{len}\, f$,

(iii) $\langle i, j+1 \rangle \in$ the indices of the Go-board of f,

(iv) $\langle i+1, j+1 \rangle \in$ the indices of the Go-board of f,

(v) $f_k =$ the Go-board of $f \circ (i, j+1)$, and

(vi) $f_{k+1} =$ the Go-board of $f \circ (i+1, j+1)$.

Then $\mathrm{leftcell}(f, k) = \mathrm{cell}($the Go-board of f, $i, j+1)$ and $\mathrm{rightcell}(f, k) = \mathrm{cell}($the Go-board of f, $i, j)$.

(30) Suppose that

(i) $1 \leq k$,

(ii) $k+1 \leq \mathrm{len}\, f$,

(iii) $\langle i, j+1 \rangle \in$ the indices of the Go-board of f,

(iv) $\langle i+1, j+1 \rangle \in$ the indices of the Go-board of f,

(v) $f_k =$ the Go-board of $f \circ (i+1, j+1)$, and

(vi) $f_{k+1} =$ the Go-board of $f \circ (i, j+1)$.

Then $\mathrm{leftcell}(f, k) = \mathrm{cell}($the Go-board of f, $i, j)$ and $\mathrm{rightcell}(f, k) = \mathrm{cell}($the Go-board of f, $i, j+1)$.

(31) Suppose that

(i) $1 \leq k$,

(ii) $k+1 \leq \operatorname{len} f$,

(iii) $\langle i+1, j+1 \rangle \in$ the indices of the Go-board of f,

(iv) $\langle i+1, j \rangle \in$ the indices of the Go-board of f,

(v) $f_k =$ the Go-board of $f \circ (i+1, j+1)$, and

(vi) $f_{k+1} =$ the Go-board of $f \circ (i+1, j)$.

Then $\mathrm{leftcell}(f,k) = \mathrm{cell}($the Go-board of f, $i+1, j)$ and $\mathrm{rightcell}(f,k) = \mathrm{cell}($the Go-board of f, $i, j)$.

(32) If $1 \leq k$ and $k+1 \leq \operatorname{len} f$, then $\mathrm{leftcell}(f,k) \cap \mathrm{rightcell}(f,k) = \mathcal{L}(f,k)$.

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