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Correctness of Johnson Counter Circuits

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Summary. This article introduces the verification of the correctness for the operations and the specification of the Johnson counter. We formalize the concepts of 2-bit, 3-bit and 4-bit Johnson counter circuits with a reset input, and define the specification of the state transitions without the minor loop.

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WWW: `http://mizar.org/JFM/Vol11/gate_3.html`

The article [1] provides the notation and terminology for this paper.

The following propositions are true:

(1) Let s_0, s_1, s_2, s_3, n_0, n_3, n_4, n_5, q_1, q_2, n_1, n_2 be sets such that NE s_0 iff NE AND2(NOT1 q_2, NOT1 q_1) and NE s_1 iff NE AND2(NOT1 q_2, q_1) and NE s_2 iff NE AND2(q_2, NOT1 q_1) and NE s_3 iff NE AND2(q_2, q_1) and NE n_0 iff NE AND2(NOT1 n_2, NOT1 n_1) and NE n_3 iff NE AND2(NOT1 n_2, n_1) and NE n_4 iff NE AND2(n_2, NOT1 n_1) and NE n_5 iff NE AND2(n_2, n_1) and NE n_1 iff NE NOT1 q_2 and NE n_2 iff NE q_1. Then

- (i) NE n_3 iff NE s_0,
- (ii) NE n_5 iff NE s_1,
- (iii) NE n_4 iff NE s_3, and
- (iv) NE n_0 iff NE s_2.

(2) Let s_0, s_1, s_2, s_3, n_0, n_3, n_4, n_5, q_1, q_2, n_1, n_2, R be sets such that NE s_0 iff NE AND2(NOT1 q_2, NOT1 q_1) and NE s_1 iff NE AND2(NOT1 q_2, q_1) and NE s_2 iff NE AND2(q_2, NOT1 q_1) and NE s_3 iff NE AND2(q_2, q_1) and NE n_0 iff NE AND2(NOT1 n_2, NOT1 n_1) and NE n_3 iff NE AND2(NOT1 n_2, n_1) and NE n_4 iff NE AND2(n_2, NOT1 n_1) and NE n_5 iff NE AND2(n_2, n_1) and NE n_1 iff NE AND2(NOT1 q_2, R) and NE n_2 iff NE AND2(q_1, R). Then

- (i) NE n_3 iff NE AND2(s_0, R),
- (ii) NE n_5 iff NE AND2(s_1, R),
- (iii) NE n_4 iff NE AND2(s_3, R), and
- (iv) NE n_0 iff NE OR2(AND2(s_2, R), NOT1 R).

(3) Let s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, n_0, n_3, n_4, n_5, n_7, n_8, n_9, n_{10}, q_1, q_2, q_3, n_1, n_2, n_6 be sets such that NE s_0 iff NE AND3(NOT1 q_3, NOT1 q_2, NOT1 q_1) and NE s_1 iff NE AND3(NOT1 q_3, NOT1 q_2, q_1) and NE s_2 iff NE AND3(NOT1 q_3, q_2, NOT1 q_1) and NE s_3 iff NE AND3(NOT1 q_3, q_2, q_1) and NE s_4 iff NE AND3(q_3, NOT1 q_2, NOT1 q_1) and NE s_5 iff NE AND3(q_3, NOT1 q_2, q_1) and NE s_6 iff NE AND3(q_3, q_2, NOT1 q_1) and NE s_7 iff NE AND3(q_3, q_2, q_1) and NE n_0 iff NE AND3(NOT1 n_6, NOT1 n_2, NOT1 n_1) and NE n_3 iff NE AND3(NOT1 n_6, NOT1 n_2, n_1) and NE n_4 iff NE AND3(NOT1 n_6, n_2, NOT1 n_1) and NE n_5 iff NE AND3(NOT1 n_6, n_2, n_1) and NE n_7 iff NE AND3(n_6, NOT1 n_2, NOT1 n_1) and NE n_8 iff NE AND3(n_6, NOT1 n_2, n_1) and NE n_9 iff NE AND3(n_6, n_2, NOT1 n_1) and NE n_{10} iff NE AND3(n_6, n_2, n_1) and NE n_1 iff NE NOT1 q_3 and NE n_2 iff NE q_1 and NE n_6 iff NE q_2. Then

(i) NE n_3 iff NE s_0,

(ii) NE n_5 iff NE s_1,

(iii) NE n_{10} iff NE s_3,

(iv) NE n_9 iff NE s_7,

(v) NE n_7 iff NE s_6,

(vi) NE n_0 iff NE s_4,

(vii) NE n_4 iff NE s_5, and

(viii) NE n_8 iff NE s_2.

(4) Let s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, n_0, n_3, n_4, n_5, n_7, n_8, n_9, n_{10}, q_1, q_2, q_3, n_1, n_2, n_6, R be sets such that NE s_0 iff NE AND3(NOT1 q_3, NOT1 q_2, NOT1 q_1) and NE s_1 iff NE AND3(NOT1 q_3, NOT1 q_2, q_1) and NE s_2 iff NE AND3(NOT1 q_3, q_2, NOT1 q_1) and NE s_3 iff NE AND3(NOT1 q_3, q_2, q_1) and NE s_4 iff NE AND3(q_3, NOT1 q_2, NOT1 q_1) and NE s_5 iff NE AND3(q_3, NOT1 q_2, q_1) and NE s_6 iff NE AND3(q_3, q_2, NOT1 q_1) and NE s_7 iff NE AND3(q_3, q_2, q_1) and NE n_0 iff NE AND3(NOT1 n_6, NOT1 n_2, NOT1 n_1) and NE n_3 iff NE AND3(NOT1 n_6, NOT1 n_2, n_1) and NE n_4 iff NE AND3(NOT1 n_6, n_2, NOT1 n_1) and NE n_5 iff NE AND3(NOT1 n_6, n_2, n_1) and NE n_7 iff NE AND3(n_6, NOT1 n_2, NOT1 n_1) and NE n_8 iff NE AND3(n_6, NOT1 n_2, n_1) and NE n_9 iff NE AND3(n_6, n_2, NOT1 n_1) and NE n_{10} iff NE AND3(n_6, n_2, n_1) and NE n_1 iff NE AND2(NOT1 q_3, R) and NE n_2 iff NE AND2(q_1, R) and NE n_6 iff NE AND2(q_2, R). Then

(i) NE n_3 iff NE AND2(s_0, R),

(ii) NE n_5 iff NE AND2(s_1, R),

(iii) NE n_{10} iff NE AND2(s_3, R),

(iv) NE n_9 iff NE AND2(s_7, R),

(v) NE n_7 iff NE AND2(s_6, R),

(vi) NE n_0 iff NE OR2(AND2(s_4, R), NOT1 R),

(vii) NE n_4 iff NE AND2(s_5, R), and

(viii) NE n_8 iff NE AND2(s_2, R).

(5) Let s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9, s_{10}, s_{11}, s_{12}, s_{13}, s_{14}, s_{15}, n_0, n_3, n_4, n_5, n_7, n_8, n_9, n_{10}, n_{12}, n_{13}, n_{14}, n_{15}, n_{16}, n_{17}, n_{18}, n_{19}, q_1, q_2, q_3, q_4, n_1, n_2, n_6, n_{11} be sets such that NE s_0 iff NE AND4(NOT1 q_4, NOT1 q_3, NOT1 q_2, NOT1 q_1) and NE s_1 iff NE AND4(NOT1 q_4, NOT1 q_3, NOT1 q_2, q_1) and NE s_2 iff NE AND4(NOT1 q_4, NOT1 q_3, q_2, NOT1 q_1) and NE s_3 iff NE AND4(NOT1 q_4, NOT1 q_3, q_2, q_1) and NE s_4 iff NE AND4(NOT1 q_4, q_3, NOT1 q_2, NOT1 q_1) and NE s_5 iff NE AND4(NOT1 q_4, q_3, NOT1 q_2, q_1) and NE s_6 iff NE AND4(NOT1 q_4, q_3, q_2, NOT1 q_1) and NE s_7 iff NE AND4(NOT1 q_4, q_3, q_2, q_1) and NE s_8 iff NE AND4(q_4, NOT1 q_3, NOT1 q_2, NOT1 q_1) and NE s_9 iff NE AND4(q_4, NOT1 q_3, NOT1 q_2, q_1) and NE s_{10} iff NE AND4(q_4, NOT1 q_3, q_2, NOT1 q_1) and NE s_{11} iff NE AND4(q_4, NOT1 q_3, q_2, q_1) and NE s_{12} iff NE AND4(q_4, q_3, NOT1 q_2, NOT1 q_1) and NE s_{13} iff NE AND4(q_4, q_3, NOT1 q_2, q_1) and NE s_{14} iff NE AND4(q_4, q_3, q_2, NOT1 q_1) and NE s_{15} iff NE AND4(q_4, q_3, q_2, q_1) and NE n_0 iff NE AND4(NOT1 n_{11}, NOT1 n_6, NOT1 n_2, NOT1 n_1) and NE n_3 iff NE AND4(NOT1 n_{11}, NOT1 n_6, NOT1 n_2, n_1) and NE n_4 iff NE

AND4(NOT1 n_{11},NOT1 n_6,n_2,NOT1 n_1) and NE n_5 iff NE AND4(NOT1 n_{11},NOT1 n_6,n_2,n_1) and NE n_7 iff NE AND4(NOT1 n_{11},n_6,NOT1 n_2,NOT1 n_1) and NE n_8 iff NE AND4(NOT1 n_{11},n_6,NOT1 n_2,n_1) and NE n_9 iff NE AND4(NOT1 n_{11},n_6,n_2,NOT1 n_1) and NE n_{10} iff NE AND4(NOT1 n_{11},n_6,n_2,n_1) and NE n_{12} iff NE AND4(n_{11},NOT1 n_6,NOT1 n_2,NOT1 n_1) and NE n_{13} iff NE AND4(n_{11},NOT1 n_6,NOT1 n_2,n_1) and NE n_{14} iff NE AND4(n_{11},NOT1 n_6,n_2,NOT1 n_1) and NE n_{15} iff NE AND4(n_{11},NOT1 n_6,n_2,n_1) and NE n_{16} iff NE AND4(n_{11},n_6,NOT1 n_2,NOT1 n_1) and NE n_{17} iff NE AND4(n_{11},n_6,NOT1 n_2,n_1) and NE n_{18} iff NE AND4(n_{11},n_6,n_2,NOT1 n_1) and NE n_{19} iff NE AND4(n_{11},n_6,n_2,n_1) and NE n_1 iff NE NOT1 q_4 and NE n_2 iff NE q_1 and NE n_6 iff NE q_2 and NE n_{11} iff NE q_3. Then

(i) NE n_3 iff NE s_0,

(ii) NE n_5 iff NE s_1,

(iii) NE n_{10} iff NE s_3,

(iv) NE n_{19} iff NE s_7,

(v) NE n_{18} iff NE s_{15},

(vi) NE n_{16} iff NE s_{14},

(vii) NE n_{12} iff NE s_{12},

(viii) NE n_0 iff NE s_8,

(ix) NE n_8 iff NE s_2,

(x) NE n_{15} iff NE s_5,

(xi) NE n_9 iff NE s_{11},

(xii) NE n_{17} iff NE s_6,

(xiii) NE n_{14} iff NE s_{13},

(xiv) NE n_7 iff NE s_{10},

(xv) NE n_{13} iff NE s_4, and

(xvi) NE n_4 iff NE s_9.

(6) Let s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9, s_{10}, s_{11}, s_{12}, s_{13}, s_{14}, s_{15}, n_0, n_3, n_4, n_5, n_7, n_8, n_9, n_{10}, n_{12}, n_{13}, n_{14}, n_{15}, n_{16}, n_{17}, n_{18}, n_{19}, q_1, q_2, q_3, q_4, n_1, n_2, n_6, n_{11}, R be sets such that NE s_0 iff NE AND4(NOT1 q_4,NOT1 q_3,NOT1 q_2,NOT1 q_1) and NE s_1 iff NE AND4(NOT1 q_4,NOT1 q_3,NOT1 q_2,q_1) and NE s_2 iff NE AND4(NOT1 q_4,NOT1 q_3,q_2,NOT1 q_1) and NE s_3 iff NE AND4(NOT1 q_4,NOT1 q_3,q_2,q_1) and NE s_4 iff NE AND4(NOT1 q_4,q_3,NOT1 q_2,NOT1 q_1) and NE s_5 iff NE AND4(NOT1 q_4,q_3,NOT1 q_2,q_1) and NE s_6 iff NE AND4(NOT1 q_4,q_3,q_2,NOT1 q_1) and NE s_7 iff NE AND4(NOT1 q_4,q_3,q_2,q_1) and NE s_8 iff NE AND4(q_4,NOT1 q_3,NOT1 q_2,NOT1 q_1) and NE s_9 iff NE AND4(q_4,NOT1 q_3,NOT1 q_2,q_1) and NE s_{10} iff NE AND4(q_4,NOT1 q_3,q_2,NOT1 q_1) and NE s_{11} iff NE AND4(q_4,NOT1 q_3,q_2,q_1) and NE s_{12} iff NE AND4(q_4,q_3,NOT1 q_2,NOT1 q_1) and NE s_{13} iff NE AND4(q_4,q_3,NOT1 q_2,q_1) and NE s_{14} iff NE AND4(q_4,q_3,q_2,NOT1 q_1) and NE s_{15} iff NE AND4(q_4,q_3,q_2,q_1) and NE n_0 iff NE AND4(NOT1 n_{11},NOT1 n_6,NOT1 n_2,NOT1 n_1) and NE n_3 iff NE AND4(NOT1 n_{11},NOT1 n_6,NOT1 n_2,n_1) and NE n_4 iff NE AND4(NOT1 n_{11},NOT1 n_6,n_2,NOT1 n_1) and NE n_5 iff NE AND4(NOT1 n_{11},NOT1 n_6,n_2,n_1) and NE n_7 iff NE AND4(NOT1 n_{11},n_6,NOT1 n_2,NOT1 n_1) and NE n_8 iff NE AND4(NOT1 n_{11},n_6,NOT1 n_2,n_1) and NE n_9 iff NE AND4(NOT1 n_{11},n_6,n_2,NOT1 n_1) and NE n_{10} iff NE AND4(NOT1 n_{11},n_6,n_2,n_1) and NE n_{12} iff NE AND4(n_{11},NOT1 n_6,NOT1 n_2,NOT1 n_1) and NE n_{13} iff NE AND4(n_{11},NOT1 n_6,NOT1 n_2,n_1) and NE n_{14} iff NE AND4(n_{11},NOT1 n_6,n_2,NOT1 n_1) and NE n_{15} iff NE AND4(n_{11},NOT1 n_6,n_2,n_1) and NE n_{16} iff NE AND4(n_{11},n_6,NOT1 n_2,NOT1 n_1) and NE n_{17} iff NE AND4(n_{11},n_6,NOT1 n_2,n_1) and NE n_{18} iff NE AND4(n_{11},n_6,n_2,NOT1 n_1) and NE n_{19} iff NE AND4(n_{11},n_6,n_2,n_1) and NE n_1 iff NE AND2(NOT1 q_4,R) and NE n_2 iff NE AND2(q_1,R) and NE n_6 iff NE AND2(q_2,R) and NE n_{11} iff NE AND2(q_3,R). Then

(i) NE n_3 iff NE AND2(s_0,R),

(ii) NE n_5 iff NE AND2(s_1,R),

(iii) NE n_{10} iff NE AND2(s_3,R),

(iv) NE n_{19} iff NE AND2(s_7, R),

(v) NE n_{18} iff NE AND2(s_{15}, R),

(vi) NE n_{16} iff NE AND2(s_{14}, R),

(vii) NE n_{12} iff NE AND2(s_{12}, R),

(viii) NE n_0 iff NE OR2(AND2(s_8, R), NOT1 R),

(ix) NE n_8 iff NE AND2(s_2, R),

(x) NE n_{15} iff NE AND2(s_5, R),

(xi) NE n_9 iff NE AND2(s_{11}, R),

(xii) NE n_{17} iff NE AND2(s_6, R),

(xiii) NE n_{14} iff NE AND2(s_{13}, R),

(xiv) NE n_7 iff NE AND2(s_{10}, R),

(xv) NE n_{13} iff NE AND2(s_4, R), and

(xvi) NE n_4 iff NE AND2(s_9, R).

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