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Monotonic and Continuous Real Function

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Summary. A continuation of [13] and [11]. We prove a few theorems about real functions monotonic and continuous on interval, on halfline and on the set of real numbers and continuity of the inverse function. At the beginning of the paper we show some facts about topological properties of the set of real numbers, halflines and intervals which rather belong to [14].

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The articles [15], [18], [2], [16], [5], [1], [3], [7], [19], [6], [12], [4], [17], [9], [14], [10], [13], and [8] provide the notation and terminology for this paper.

For simplicity, we use the following convention: X denotes a set, x_0, r, r_1, g, p denote real numbers, n denotes a natural number, a denotes a sequence of real numbers, and f denotes a partial function from $\mathbb{R}$ to $\mathbb{R}$.

Next we state several propositions:

(1) $\Omega_{\mathbb{R}}$ is closed.

(2) $\emptyset_{\mathbb{R}}$ is open.

(3) $\emptyset_{\mathbb{R}}$ is closed.

(4) $\Omega_{\mathbb{R}}$ is open.

(5) $[r, +\infty[$ is closed.

(6) $]-\infty, r]$ is closed.

(7) $]r, +\infty[$ is open.

(8) $]-\infty, r[$ is open.

Let us consider r. Note that $]r, +\infty[$ is open and $\mathrm{HL}(r)$ is open.

Let p, g be real numbers. Observe that $]p, g[$ is open.

Next we state a number of propositions:

(9) $0 < r$ and $g \in]x_0 - r, x_0 + r[$ iff there exists r_1 such that $g = x_0 + r_1$ and $|r_1| < r$.

(10) $0 < r$ and $g \in]x_0 - r, x_0 + r[$ iff $g - x_0 \in]-r, r[$.

(11) $]-\infty, p] = \{p\} \cup]-\infty, p[$.

(12) $[p, +\infty[= \{p\} \cup]p, +\infty[$.

(13) For every real number x_0 such that for every n holds $a(n) = x_0 - \frac{p}{n+1}$ holds a is convergent and $\lim a = x_0$.

(14) For every real number x_0 such that for every n holds $a(n) = x_0 + \frac{p}{n+1}$ holds a is convergent and $\lim a = x_0$.

(15) Suppose f is continuous in x_0 and $f(x_0) \neq r$ and there exists a neighbourhood N of x_0 such that $N \subseteq \mathrm{dom} f$. Then there exists a neighbourhood N of x_0 such that $N \subseteq \mathrm{dom} f$ and for every g such that $g \in N$ holds $f(g) \neq r$.

(16) If f is increasing on X and decreasing on X, then $f \restriction X$ is one-to-one.

(17) For every one-to-one partial function f from $\mathbb{R}$ to $\mathbb{R}$ such that f is increasing on X holds $(f \restriction X)^{-1}$ is increasing on $f^{\circ}X$.

(18) For every one-to-one partial function f from $\mathbb{R}$ to $\mathbb{R}$ such that f is decreasing on X holds $(f \restriction X)^{-1}$ is decreasing on $f^{\circ}X$.

(19) If $X \subseteq \mathrm{dom} f$ and f is monotone on X and there exists p such that $f^{\circ}X =]-\infty, p[$, then f is continuous on X.

(20) If $X \subseteq \mathrm{dom} f$ and f is monotone on X and there exists p such that $f^{\circ}X =]p, +\infty[$, then f is continuous on X.

(21) If $X \subseteq \mathrm{dom} f$ and f is monotone on X and there exists p such that $f^{\circ}X =]-\infty, p]$, then f is continuous on X.

(22) If $X \subseteq \mathrm{dom} f$ and f is monotone on X and there exists p such that $f^{\circ}X = [p, +\infty[$, then f is continuous on X.

(23) If $X \subseteq \mathrm{dom} f$ and f is monotone on X and there exist p, g such that $f^{\circ}X =]p, g[$, then f is continuous on X.

(24) If $X \subseteq \mathrm{dom} f$ and f is monotone on X and $f^{\circ}X = \mathbb{R}$, then f is continuous on X.

(25) Let f be an one-to-one partial function from $\mathbb{R}$ to $\mathbb{R}$. Suppose f is increasing on $]p, g[$ and decreasing on $]p, g[$ and $]p, g[\subseteq \mathrm{dom} f$. Then $(f \restriction]p, g[)^{-1}$ is continuous on $f^{\circ}]p, g[$.

(26) Let f be an one-to-one partial function from $\mathbb{R}$ to $\mathbb{R}$. Suppose f is increasing on $]-\infty, p[$ and decreasing on $]-\infty, p[$ and $]-\infty, p[\subseteq \mathrm{dom} f$. Then $(f \restriction]-\infty, p[)^{-1}$ is continuous on $f^{\circ}]-\infty, p[$.

(27) Let f be an one-to-one partial function from $\mathbb{R}$ to $\mathbb{R}$. Suppose f is increasing on $]p, +\infty[$ and decreasing on $]p, +\infty[$ and $]p, +\infty[\subseteq \mathrm{dom} f$. Then $(f \restriction]p, +\infty[)^{-1}$ is continuous on $f^{\circ}]p, +\infty[$.

(28) Let f be an one-to-one partial function from $\mathbb{R}$ to $\mathbb{R}$. Suppose f is increasing on $]-\infty, p]$ and decreasing on $]-\infty, p]$ and $]-\infty, p] \subseteq \mathrm{dom} f$. Then $(f \restriction]-\infty, p])^{-1}$ is continuous on $f^{\circ}]-\infty, p]$.

(29) Let f be an one-to-one partial function from $\mathbb{R}$ to $\mathbb{R}$. Suppose f is increasing on $[p, +\infty[$ and decreasing on $[p, +\infty[$ and $[p, +\infty[\subseteq \mathrm{dom} f$. Then $(f \restriction [p, +\infty[)^{-1}$ is continuous on $f^{\circ}[p, +\infty[$.

(30) Let f be an one-to-one partial function from $\mathbb{R}$ to $\mathbb{R}$. Suppose f is increasing on $\Omega_{\mathbb{R}}$, decreasing on $\Omega_{\mathbb{R}}$, and total. Then f^{-1} is continuous on $\mathrm{rng} f$.

(31) If f is continuous on $]p, g[$, increasing on $]p, g[$, and decreasing on $]p, g[$, then $\mathrm{rng}(f \restriction]p, g[)$ is open.

(32) If f is continuous on $]-\infty, p[$, increasing on $]-\infty, p[$, and decreasing on $]-\infty, p[$, then $\mathrm{rng}(f \restriction]-\infty, p[)$ is open.

(33) If f is continuous on $]p, +\infty[$, increasing on $]p, +\infty[$, and decreasing on $]p, +\infty[$, then $\mathrm{rng}(f \restriction]p, +\infty[)$ is open.

(34) If f is continuous on $\Omega_{\mathbb{R}}$, increasing on $\Omega_{\mathbb{R}}$, and decreasing on $\Omega_{\mathbb{R}}$, then $\mathrm{rng} f$ is open.

REFERENCES

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/nat_1.html.

[2] Grzegorz Bancerek. The ordinal numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/ordinal1.html.

[3] Czesław Byliński. Functions and their basic properties. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/funct_1.html.

[4] Czesław Byliński. Partial functions. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/partfun1.html.

[5] Krzysztof Hryniewiecki. Basic properties of real numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/real_1.html.

[6] Jarosław Kotowicz. Convergent sequences and the limit of sequences. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/seq_2.html.

[7] Jarosław Kotowicz. Real sequences and basic operations on them. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/seq_1.html.

[8] Jarosław Kotowicz. The limit of a real function at infinity. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/limfunc1.html.

[9] Jarosław Kotowicz. Partial functions from a domain to a domain. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/partfun2.html.

[10] Jarosław Kotowicz. Properties of real functions. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/rfunct_2.html.

[11] Jarosław Kotowicz and Konrad Raczkowski. Real function uniform continuity. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/fcont_2.html.

[12] Jan Popiołek. Some properties of functions modul and signum. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/absvalue.html.

[13] Konrad Raczkowski and Paweł Sadowski. Real function continuity. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/fcont_1.html.

[14] Konrad Raczkowski and Paweł Sadowski. Topological properties of subsets in real numbers. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/rcomp_1.html.

[15] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

[16] Andrzej Trybulec. Subsets of real numbers. *Journal of Formalized Mathematics*, Addenda, 2003. http://mizar.org/JFM/Addenda/numbers.html.

[17] Andrzej Trybulec and Czesław Byliński. Some properties of real numbers operations: min, max, square, and square root. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/square_1.html.

[18] Zinaida Trybulec. Properties of subsets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/subset_1.html.

[19] Edmund Woronowicz. Relations defined on sets. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/relset_1.html.

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