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Extensions of Mappings on Generator Set

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Summary. The aim of the article is to prove the fact that if extensions of mappings on generator set are equal then these mappings are equal. The article contains the properties of epimorphisms and monomorphisms between Many Sorted Algebras.

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The articles [13], [16], [17], [18], [6], [8], [7], [1], [2], [3], [4], [14], [11], [15], [5], [10], [9], and [12] provide the notation and terminology for this paper.

1. Preliminaries

For simplicity, we follow the rules: S is a non void non empty many sorted signature, U_1, U_2, U_3 are non-empty algebras over S, I is a set, A is a many sorted set indexed by I, and B, C are non-empty many sorted sets indexed by I.

One can prove the following propositions:

(1) For every binary relation R and for all sets X, Y such that $X \subseteq Y$ holds $(R{\restriction}Y)^{\circ}X = R^{\circ}X$.

(3)[1] For every function yielding function f holds $\mathrm{dom}(\mathrm{dom}_{\kappa} f(\kappa)) = \mathrm{dom}\, f$.

(4) For every function yielding function f holds $\mathrm{dom}(\mathrm{rng}_{\kappa} f(\kappa)) = \mathrm{dom}\, f$.

2. Facts about Many Sorted Functions

Next we state several propositions:

(5) Let F be a many sorted function from A into B and X be a many sorted subset indexed by A. If $A \subseteq X$, then $F \restriction X = F$.

(6) Let A, B be many sorted sets indexed by I, M be a many sorted subset indexed by A, and F be a many sorted function from A into B. Then $F \circ M \subseteq F \circ A$.

(7) Let F be a many sorted function from A into B and M_1, M_2 be many sorted subsets indexed by A. If $M_1 \subseteq M_2$, then $(F \restriction M_2) \circ M_1 = F \circ M_1$.

(8) Let F be a many sorted function from A into B, G be a many sorted function from B into C, and X be a many sorted subset indexed by A. Then $(G \circ F) \restriction X = G \circ (F \restriction X)$.

[1] The proposition (2) has been removed.

(9) Let A, B be many sorted sets indexed by I. Suppose A is transformable to B. Let F be a many sorted function from A into B and C be a many sorted set indexed by I. Suppose B is a many sorted subset indexed by C. Then F is a many sorted function from A into C.

(10) Let F be a many sorted function from A into B and X be a many sorted subset indexed by A. If F is "1-1", then $F \restriction X$ is "1-1".

3. DOM'S AND RNG'S OF MANY SORTED FUNCTIONS

Let us consider I and let F be a many sorted function indexed by I. Then $\mathrm{dom}_\kappa F(\kappa)$ is a many sorted set indexed by I. Then $\mathrm{rng}_\kappa F(\kappa)$ is a many sorted set indexed by I.

We now state several propositions:

(11) For every many sorted function F from A into B and for every many sorted subset X indexed by A holds $\mathrm{dom}_\kappa(F \restriction X)(\kappa) \subseteq \mathrm{dom}_\kappa F(\kappa)$.

(12) For every many sorted function F from A into B and for every many sorted subset X indexed by A holds $\mathrm{rng}_\kappa(F \restriction X)(\kappa) \subseteq \mathrm{rng}_\kappa F(\kappa)$.

(13) Let A, B be many sorted sets indexed by I and F be a many sorted function from A into B. Then F is onto if and only if $\mathrm{rng}_\kappa F(\kappa) = B$.

(14) For every non-empty many sorted set X indexed by the carrier of S holds $\mathrm{rng}_\kappa(\mathrm{Reverse}(X))(\kappa) = X$.

(15) Let F be a many sorted function from A into B, G be a many sorted function from B into C, and X be a non-empty many sorted subset indexed by B. If $\mathrm{rng}_\kappa F(\kappa) \subseteq X$, then $(G \restriction X) \circ F = G \circ F$.

4. OTHER PROPERTIES OF "ONTO" AND "1-1"

The following propositions are true:

(16) Let F be a many sorted function from A into B. Then F is onto if and only if for every C and for all many sorted functions G, H from B into C such that $G \circ F = H \circ F$ holds $G = H$.

(17) Let F be a many sorted function from A into B. Suppose A is non-empty and B is non-empty. Then F is "1-1" if and only if for every many sorted set C indexed by I and for all many sorted functions G, H from C into A such that $F \circ G = F \circ H$ holds $G = H$.

5. EXTENSIONS OF MAPPINGS ON GENERATOR SET

Next we state three propositions:

(18) Let X be a non-empty many sorted set indexed by the carrier of S and h_1, h_2 be many sorted functions from $\mathrm{Free}(X)$ into U_1. Suppose h_1 is a homomorphism of $\mathrm{Free}(X)$ into U_1 and h_2 is a homomorphism of $\mathrm{Free}(X)$ into U_1 and $h_1 \restriction \mathrm{FreeGenerator}(X) = h_2 \restriction \mathrm{FreeGenerator}(X)$. Then $h_1 = h_2$.

(19) Let F be a many sorted function from U_1 into U_2. Suppose F is a homomorphism of U_1 into U_2. Suppose F is an epimorphism of U_1 onto U_2. Let U_3 be a non-empty algebra over S and h_1, h_2 be many sorted functions from U_2 into U_3. Suppose h_1 is a homomorphism of U_2 into U_3 and h_2 is a homomorphism of U_2 into U_3. If $h_1 \circ F = h_2 \circ F$, then $h_1 = h_2$.

(20) Let F be a many sorted function from U_2 into U_3. Suppose F is a homomorphism of U_2 into U_3. Then F is a monomorphism of U_2 into U_3 if and only if for every non-empty algebra U_1 over S and for all many sorted functions h_1, h_2 from U_1 into U_2 such that h_1 is a homomorphism of U_1 into U_2 and h_2 is a homomorphism of U_1 into U_2 holds if $F \circ h_1 = F \circ h_2$, then $h_1 = h_2$.

Let us consider S, U_1. Observe that there exists a generator set of U_1 which is non-empty.

One can prove the following three propositions:

(21) Let U_1 be an algebra over S and A, B be subsets of U_1. Suppose A is a many sorted subset indexed by B. Then $\mathrm{Gen}(A)$ is a subalgebra of $\mathrm{Gen}(B)$.

(22) Let U_1 be an algebra over S, U_2 be a subalgebra of U_1, B_1 be a subset of U_1, and B_2 be a subset of U_2. If $B_1 = B_2$, then $\mathrm{Gen}(B_1) = \mathrm{Gen}(B_2)$.

(23) Let U_1 be a strict non-empty algebra over S, U_2 be a non-empty algebra over S, G_1 be a generator set of U_1, and h_1, h_2 be many sorted functions from U_1 into U_2. Suppose h_1 is a homomorphism of U_1 into U_2 and h_2 is a homomorphism of U_1 into U_2 and $h_1 \restriction G_1 = h_2 \restriction G_1$. Then $h_1 = h_2$.

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