JOURNAL OF FORMALIZED MATHEMATICS
Volume 1, Released 1989, Published 2003
Inst. of Computer Science, Univ. of Białystok

Enumerated Sets

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Summary. We prove basic facts about enumerated sets: definitional theorems and their immediate consequences, some theorems related to the decomposition of an enumerated set into union of two sets, facts about removing elements that occur more than once, and facts about permutations of enumerated sets (with the length ≤ 4). The article includes also schemes enabling instantiation of up to nine universal quantifiers.

MML Identifier: ENUMSET1.

WWW: http://mizar.org/JFM/Vol1/enumset1.html

The article [1] provides the notation and terminology for this paper.

In this paper x, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, X are sets.

In this article we present several logical schemes. The scheme *UI1* deals with a set $\mathcal{A}$ and a unary predicate $\mathcal{P}$, and states that:

$\mathcal{P}[\mathcal{A}]$

provided the parameters have the following property:

- For every x_1 holds $\mathcal{P}[x_1]$.

The scheme *UI2* deals with sets $\mathcal{A}$, $\mathcal{B}$ and a binary predicate $\mathcal{P}$, and states that:

$\mathcal{P}[\mathcal{A},\mathcal{B}]$

provided the following condition is satisfied:

- For all x_1, x_2 holds $\mathcal{P}[x_1,x_2]$.

The scheme *UI3* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and a ternary predicate $\mathcal{P}$, and states that:

$\mathcal{P}[\mathcal{A},\mathcal{B},\mathcal{C}]$

provided the parameters meet the following condition:

- For all x_1, x_2, x_3 holds $\mathcal{P}[x_1,x_2,x_3]$.

The scheme *UI4* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$ and a 4-ary predicate $\mathcal{P}$, and states that:

$\mathcal{P}[\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D}]$

provided the following requirement is met:

- For all x_1, x_2, x_3, x_4 holds $\mathcal{P}[x_1,x_2,x_3,x_4]$.

The scheme *UI5* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$ and a 5-ary predicate $\mathcal{P}$, and states that:

$\mathcal{P}[\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\mathcal{E}]$

provided the following requirement is met:

- For all x_1, x_2, x_3, x_4, x_5 holds $\mathcal{P}[x_1,x_2,x_3,x_4,x_5]$.

The scheme *UI6* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, $\mathcal{F}$ and a 6-ary predicate $\mathcal{P}$, and states that:

$\mathcal{P}[\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\mathcal{E},\mathcal{F}]$

provided the parameters meet the following condition:

- For all x_1, x_2, x_3, x_4, x_5, x_6 holds $\mathcal{P}[x_1,x_2,x_3,x_4,x_5,x_6]$.

The scheme *UI7* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, $\mathcal{F}$, $\mathcal{G}$ and a 7-ary predicate $\mathcal{P}$, and states that:

$\mathcal{P}[\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\mathcal{E},\mathcal{F},\mathcal{G}]$

provided the parameters satisfy the following condition:

- For all x_1, x_2, x_3, x_4, x_5, x_6, x_7 holds $\mathcal{P}[x_1,x_2,x_3,x_4,x_5,x_6,x_7]$.

The scheme *UI8* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$ and a 8-ary predicate $\mathcal{P}$, and states that:

$\mathcal{P}[\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\mathcal{E},\mathcal{F},\mathcal{G},\mathcal{H}]$

provided the parameters satisfy the following condition:

- For all x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 holds $\mathcal{P}[x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8]$.

The scheme *UI9* deals with sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$, I and a 9-ary predicate $\mathcal{P}$, and states that:

$\mathcal{P}[\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\mathcal{E},\mathcal{F},\mathcal{G},\mathcal{H},I]$

provided the following condition is satisfied:

- For all sets x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 holds $\mathcal{P}[x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9]$.

Let us consider x_1, x_2, x_3. The functor $\{x_1,x_2,x_3\}$ yields a set and is defined as follows:

(Def. 1) $x \in \{x_1,x_2,x_3\}$ iff $x = x_1$ or $x = x_2$ or $x = x_3$.

Next we state three propositions:

(13)[1] If $x \in \{x_1,x_2,x_3\}$, then $x = x_1$ or $x = x_2$ or $x = x_3$.

(14) If $x = x_1$ or $x = x_2$ or $x = x_3$, then $x \in \{x_1,x_2,x_3\}$.

(15) For all x_1, x_2, x_3, X such that for every x holds $x \in X$ iff $x = x_1$ or $x = x_2$ or $x = x_3$ holds $X = \{x_1,x_2,x_3\}$.

Let us consider x_1, x_2, x_3, x_4. The functor $\{x_1,x_2,x_3,x_4\}$ yielding a set is defined by:

(Def. 2) $x \in \{x_1,x_2,x_3,x_4\}$ iff $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$.

We now state three propositions:

(18)[2] If $x \in \{x_1,x_2,x_3,x_4\}$, then $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$.

(19) If $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$, then $x \in \{x_1,x_2,x_3,x_4\}$.

(20) For all x_1, x_2, x_3, x_4, X such that for every x holds $x \in X$ iff $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ holds $X = \{x_1,x_2,x_3,x_4\}$.

Let us consider x_1, x_2, x_3, x_4, x_5. The functor $\{x_1,x_2,x_3,x_4,x_5\}$ yields a set and is defined by:

(Def. 3) $x \in \{x_1,x_2,x_3,x_4,x_5\}$ iff $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$.

One can prove the following three propositions:

(23)[3] If $x \in \{x_1,x_2,x_3,x_4,x_5\}$, then $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$.

(24) If $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$, then $x \in \{x_1,x_2,x_3,x_4,x_5\}$.

(25) For every set X such that for every x holds $x \in X$ iff $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$ holds $X = \{x_1,x_2,x_3,x_4,x_5\}$.

Let us consider x_1, x_2, x_3, x_4, x_5, x_6. The functor $\{x_1,x_2,x_3,x_4,x_5,x_6\}$ yields a set and is defined as follows:

(Def. 4) $x \in \{x_1,x_2,x_3,x_4,x_5,x_6\}$ iff $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$ or $x = x_6$.

One can prove the following three propositions:

(28)[4] If $x \in \{x_1,x_2,x_3,x_4,x_5,x_6\}$, then $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$ or $x = x_6$.

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[1] The propositions (1)–(12) have been removed.
[2] The propositions (16) and (17) have been removed.
[3] The propositions (21) and (22) have been removed.
[4] The propositions (26) and (27) have been removed.

(29) If $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$ or $x = x_6$, then $x \in \{x_1, x_2, x_3, x_4, x_5, x_6\}$.

(30) Let X be a set. Suppose that for every x holds $x \in X$ iff $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$ or $x = x_6$. Then $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$.

Let us consider x_1, x_2, x_3, x_4, x_5, x_6, x_7. The functor $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ yields a set and is defined as follows:

(Def. 5) $x \in \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ iff $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$ or $x = x_6$ or $x = x_7$.

The following propositions are true:

(33)[5] If $x \in \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$, then $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$ or $x = x_6$ or $x = x_7$.

(34) If $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$ or $x = x_6$ or $x = x_7$, then $x \in \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$.

(35) Let X be a set. Suppose that for every x holds $x \in X$ iff $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$ or $x = x_6$ or $x = x_7$. Then $X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$.

Let us consider x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8. The functor $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ yields a set and is defined as follows:

(Def. 6) $x \in \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ iff $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$ or $x = x_6$ or $x = x_7$ or $x = x_8$.

The following propositions are true:

(38)[6] If $x \in \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$, then $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$ or $x = x_6$ or $x = x_7$ or $x = x_8$.

(39) If $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$ or $x = x_6$ or $x = x_7$ or $x = x_8$, then $x \in \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$.

(40) Let X be a set. Suppose that for every x holds $x \in X$ iff $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$ or $x = x_6$ or $x = x_7$ or $x = x_8$. Then $X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$.

(41) $\{x_1, x_2\} = \{x_1\} \cup \{x_2\}$.

(42) $\{x_1, x_2, x_3\} = \{x_1\} \cup \{x_2, x_3\}$.

(43) $\{x_1, x_2, x_3\} = \{x_1, x_2\} \cup \{x_3\}$.

(44) $\{x_1, x_2, x_3, x_4\} = \{x_1\} \cup \{x_2, x_3, x_4\}$.

(45) $\{x_1, x_2, x_3, x_4\} = \{x_1, x_2\} \cup \{x_3, x_4\}$.

(46) $\{x_1, x_2, x_3, x_4\} = \{x_1, x_2, x_3\} \cup \{x_4\}$.

(47) $\{x_1, x_2, x_3, x_4, x_5\} = \{x_1\} \cup \{x_2, x_3, x_4, x_5\}$.

(48) $\{x_1, x_2, x_3, x_4, x_5\} = \{x_1, x_2\} \cup \{x_3, x_4, x_5\}$.

(49) $\{x_1, x_2, x_3, x_4, x_5\} = \{x_1, x_2, x_3\} \cup \{x_4, x_5\}$.

(50) $\{x_1, x_2, x_3, x_4, x_5\} = \{x_1, x_2, x_3, x_4\} \cup \{x_5\}$.

(51) $\{x_1, x_2, x_3, x_4, x_5, x_6\} = \{x_1\} \cup \{x_2, x_3, x_4, x_5, x_6\}$.

(52) $\{x_1, x_2, x_3, x_4, x_5, x_6\} = \{x_1, x_2\} \cup \{x_3, x_4, x_5, x_6\}$.

[5] The propositions (31) and (32) have been removed.
[6] The propositions (36) and (37) have been removed.

(53) $\{x_1,x_2,x_3,x_4,x_5,x_6\} = \{x_1,x_2,x_3\} \cup \{x_4,x_5,x_6\}$.

(54) $\{x_1,x_2,x_3,x_4,x_5,x_6\} = \{x_1,x_2,x_3,x_4\} \cup \{x_5,x_6\}$.

(55) $\{x_1,x_2,x_3,x_4,x_5,x_6\} = \{x_1,x_2,x_3,x_4,x_5\} \cup \{x_6\}$.

(56) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7\} = \{x_1\} \cup \{x_2,x_3,x_4,x_5,x_6,x_7\}$.

(57) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7\} = \{x_1,x_2\} \cup \{x_3,x_4,x_5,x_6,x_7\}$.

(58) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7\} = \{x_1,x_2,x_3\} \cup \{x_4,x_5,x_6,x_7\}$.

(59) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7\} = \{x_1,x_2,x_3,x_4\} \cup \{x_5,x_6,x_7\}$.

(60) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7\} = \{x_1,x_2,x_3,x_4,x_5\} \cup \{x_6,x_7\}$.

(61) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7\} = \{x_1,x_2,x_3,x_4,x_5,x_6\} \cup \{x_7\}$.

(62) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\} = \{x_1\} \cup \{x_2,x_3,x_4,x_5,x_6,x_7,x_8\}$.

(63) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\} = \{x_1,x_2\} \cup \{x_3,x_4,x_5,x_6,x_7,x_8\}$.

(64) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\} = \{x_1,x_2,x_3\} \cup \{x_4,x_5,x_6,x_7,x_8\}$.

(65) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\} = \{x_1,x_2,x_3,x_4\} \cup \{x_5,x_6,x_7,x_8\}$.

(66) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\} = \{x_1,x_2,x_3,x_4,x_5\} \cup \{x_6,x_7,x_8\}$.

(67) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\} = \{x_1,x_2,x_3,x_4,x_5,x_6\} \cup \{x_7,x_8\}$.

(68) $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\} = \{x_1,x_2,x_3,x_4,x_5,x_6,x_7\} \cup \{x_8\}$.

(69) $\{x_1,x_1\} = \{x_1\}$.

(70) $\{x_1,x_1,x_2\} = \{x_1,x_2\}$.

(71) $\{x_1,x_1,x_2,x_3\} = \{x_1,x_2,x_3\}$.

(72) $\{x_1,x_1,x_2,x_3,x_4\} = \{x_1,x_2,x_3,x_4\}$.

(73) $\{x_1,x_1,x_2,x_3,x_4,x_5\} = \{x_1,x_2,x_3,x_4,x_5\}$.

(74) $\{x_1,x_1,x_2,x_3,x_4,x_5,x_6\} = \{x_1,x_2,x_3,x_4,x_5,x_6\}$.

(75) $\{x_1,x_1,x_2,x_3,x_4,x_5,x_6,x_7\} = \{x_1,x_2,x_3,x_4,x_5,x_6,x_7\}$.

(76) $\{x_1,x_1,x_1\} = \{x_1\}$.

(77) $\{x_1,x_1,x_1,x_2\} = \{x_1,x_2\}$.

(78) $\{x_1,x_1,x_1,x_2,x_3\} = \{x_1,x_2,x_3\}$.

(79) $\{x_1,x_1,x_1,x_2,x_3,x_4\} = \{x_1,x_2,x_3,x_4\}$.

(80) $\{x_1,x_1,x_1,x_2,x_3,x_4,x_5\} = \{x_1,x_2,x_3,x_4,x_5\}$.

(81) $\{x_1,x_1,x_1,x_2,x_3,x_4,x_5,x_6\} = \{x_1,x_2,x_3,x_4,x_5,x_6\}$.

(82) $\{x_1,x_1,x_1,x_1\} = \{x_1\}$.

(83) $\{x_1,x_1,x_1,x_1,x_2\} = \{x_1,x_2\}$.

(84) $\{x_1,x_1,x_1,x_1,x_2,x_3\} = \{x_1,x_2,x_3\}$.

(85) $\{x_1,x_1,x_1,x_1,x_2,x_3,x_4\} = \{x_1,x_2,x_3,x_4\}$.

(86) $\{x_1,x_1,x_1,x_1,x_2,x_3,x_4,x_5\} = \{x_1,x_2,x_3,x_4,x_5\}$.

(87) $\{x_1,x_1,x_1,x_1,x_1\} = \{x_1\}$.

(88) $\{x_1,x_1,x_1,x_1,x_1,x_2\} = \{x_1,x_2\}$.

(89) $\{x_1,x_1,x_1,x_1,x_1,x_2,x_3\} = \{x_1,x_2,x_3\}$.

(90) $\{x_1,x_1,x_1,x_1,x_1,x_2,x_3,x_4\} = \{x_1,x_2,x_3,x_4\}$.

(91) $\{x_1,x_1,x_1,x_1,x_1,x_1\} = \{x_1\}$.

(92) $\{x_1,x_1,x_1,x_1,x_1,x_1,x_2\} = \{x_1,x_2\}$.

(93) $\{x_1,x_1,x_1,x_1,x_1,x_1,x_2,x_3\} = \{x_1,x_2,x_3\}$.

(94) $\{x_1,x_1,x_1,x_1,x_1,x_1,x_1\} = \{x_1\}$.

(95) $\{x_1,x_1,x_1,x_1,x_1,x_1,x_1,x_2\} = \{x_1,x_2\}$.

(96) $\{x_1,x_1,x_1,x_1,x_1,x_1,x_1,x_1\} = \{x_1\}$.

(98)[7] $\{x_1,x_2,x_3\} = \{x_1,x_3,x_2\}$.

(99) $\{x_1,x_2,x_3\} = \{x_2,x_1,x_3\}$.

(100) $\{x_1,x_2,x_3\} = \{x_2,x_3,x_1\}$.

(102)[8] $\{x_1,x_2,x_3\} = \{x_3,x_2,x_1\}$.

(103) $\{x_1,x_2,x_3,x_4\} = \{x_1,x_2,x_4,x_3\}$.

(104) $\{x_1,x_2,x_3,x_4\} = \{x_1,x_3,x_2,x_4\}$.

(105) $\{x_1,x_2,x_3,x_4\} = \{x_1,x_3,x_4,x_2\}$.

(107)[9] $\{x_1,x_2,x_3,x_4\} = \{x_1,x_4,x_3,x_2\}$.

(108) $\{x_1,x_2,x_3,x_4\} = \{x_2,x_1,x_3,x_4\}$.

(109) $\{x_1,x_2,x_3,x_4\} = \{x_2,x_1,x_4,x_3\}$.

(110) $\{x_1,x_2,x_3,x_4\} = \{x_2,x_3,x_1,x_4\}$.

(111) $\{x_1,x_2,x_3,x_4\} = \{x_2,x_3,x_4,x_1\}$.

(112) $\{x_1,x_2,x_3,x_4\} = \{x_2,x_4,x_1,x_3\}$.

(113) $\{x_1,x_2,x_3,x_4\} = \{x_2,x_4,x_3,x_1\}$.

(116)[10] $\{x_1,x_2,x_3,x_4\} = \{x_3,x_2,x_1,x_4\}$.

(117) $\{x_1,x_2,x_3,x_4\} = \{x_3,x_2,x_4,x_1\}$.

(118) $\{x_1,x_2,x_3,x_4\} = \{x_3,x_4,x_1,x_2\}$.

(119) $\{x_1,x_2,x_3,x_4\} = \{x_3,x_4,x_2,x_1\}$.

(123)[11] $\{x_1,x_2,x_3,x_4\} = \{x_4,x_2,x_3,x_1\}$.

(125)[12] $\{x_1,x_2,x_3,x_4\} = \{x_4,x_3,x_2,x_1\}$.

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[7] The proposition (97) has been removed.
[8] The proposition (101) has been removed.
[9] The proposition (106) has been removed.
[10] The propositions (114) and (115) have been removed.
[11] The propositions (120)–(122) have been removed.
[12] The proposition (124) has been removed.

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Received January 8, 1989

Published January 2, 2004