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Inst. of Computer Science, Univ. of Białystok

On the Decomposition of the Continuity

Marian Przemski
Warsaw University
Białystok

Summary. This article is devoted to functions of general topological spaces. A function from X to Y is A-continuous if the counterimage of every open set V of Y belongs to A, where A is a collection of subsets of X. We give the following characteristics of the continuity, called decomposition of continuity: A function f is continuous if and only if it is both A-continuous and B-continuous.

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The articles [3], [1], [2], and [4] provide the notation and terminology for this paper.

Let T be a non empty topological space. A subset of T is called an α-set of T if:

(Def. 1) It $\subseteq \mathrm{Int}\,\overline{\mathrm{Int}\,\mathrm{it}}$.

Let I_1 be a subset of T. We say that I_1 is semi-open if and only if:

(Def. 2) $I_1 \subseteq \overline{\mathrm{Int}\,I_1}$.

We say that I_1 is pre-open if and only if:

(Def. 3) $I_1 \subseteq \mathrm{Int}\,\overline{I_1}$.

We say that I_1 is pre-semi-open if and only if:

(Def. 4) $I_1 \subseteq \overline{\mathrm{Int}\,\overline{I_1}}$.

We say that I_1 is semi-pre-open if and only if:

(Def. 5) $I_1 \subseteq \overline{\mathrm{Int}\,I_1} \cup \mathrm{Int}\,\overline{I_1}$.

Let T be a non empty topological space and let B be a subset of T. The functor $\mathrm{sInt}(B)$ yielding a subset of T is defined as follows:

(Def. 6) $\mathrm{sInt}(B) = B \cap \overline{\mathrm{Int}\,B}$.

The functor $\mathrm{pInt}(B)$ yielding a subset of T is defined by:

(Def. 7) $\mathrm{pInt}(B) = B \cap \mathrm{Int}\,\overline{B}$.

The functor $\alpha\mathrm{Int}(B)$ yielding a subset of T is defined as follows:

(Def. 8) $\alpha\mathrm{Int}(B) = B \cap \mathrm{Int}\,\overline{\mathrm{Int}\,B}$.

The functor $\mathrm{psInt}(B)$ yielding a subset of T is defined by:

(Def. 9) $\mathrm{psInt}(B) = B \cap \overline{\mathrm{Int}\overline{B}}$.

Let T be a non empty topological space and let B be a subset of T. The functor $\mathrm{spInt}(B)$ yields a subset of T and is defined as follows:

(Def. 10) $\mathrm{spInt}(B) = \mathrm{sInt}(B) \cup \mathrm{pInt}(B)$.

Let T be a non empty topological space. The functor T^{α} yields a family of subsets of T and is defined by:

(Def. 11) $T^{\alpha} = \{B; B$ ranges over subsets of T: B is an α-set of $T\}$.

The functor $\mathrm{SO}(T)$ yielding a family of subsets of T is defined by:

(Def. 12) $\mathrm{SO}(T) = \{B; B$ ranges over subsets of T: B is semi-open$\}$.

The functor $\mathrm{PO}(T)$ yielding a family of subsets of T is defined by:

(Def. 13) $\mathrm{PO}(T) = \{B; B$ ranges over subsets of T: B is pre-open$\}$.

The functor $\mathrm{SPO}(T)$ yielding a family of subsets of T is defined as follows:

(Def. 14) $\mathrm{SPO}(T) = \{B; B$ ranges over subsets of T: B is semi-pre-open$\}$.

The functor $\mathrm{PSO}(T)$ yields a family of subsets of T and is defined by:

(Def. 15) $\mathrm{PSO}(T) = \{B; B$ ranges over subsets of T: B is pre-semi-open$\}$.

The functor $D(c,\alpha)(T)$ yielding a family of subsets of T is defined by:

(Def. 16) $D(c,\alpha)(T) = \{B; B$ ranges over subsets of T: $\mathrm{Int}B = \alpha\mathrm{Int}(B)\}$.

The functor $D(c,p)(T)$ yielding a family of subsets of T is defined by:

(Def. 17) $D(c,p)(T) = \{B; B$ ranges over subsets of T: $\mathrm{Int}B = \mathrm{pInt}(B)\}$.

The functor $D(c,s)(T)$ yields a family of subsets of T and is defined as follows:

(Def. 18) $D(c,s)(T) = \{B; B$ ranges over subsets of T: $\mathrm{Int}B = \mathrm{sInt}(B)\}$.

The functor $D(c,ps)(T)$ yielding a family of subsets of T is defined by:

(Def. 19) $D(c,ps)(T) = \{B; B$ ranges over subsets of T: $\mathrm{Int}B = \mathrm{psInt}(B)\}$.

The functor $D(\alpha,p)(T)$ yields a family of subsets of T and is defined by:

(Def. 20) $D(\alpha,p)(T) = \{B; B$ ranges over subsets of T: $\alpha\mathrm{Int}(B) = \mathrm{pInt}(B)\}$.

The functor $D(\alpha,s)(T)$ yields a family of subsets of T and is defined by:

(Def. 21) $D(\alpha,s)(T) = \{B; B$ ranges over subsets of T: $\alpha\mathrm{Int}(B) = \mathrm{sInt}(B)\}$.

The functor $D(\alpha,ps)(T)$ yields a family of subsets of T and is defined as follows:

(Def. 22) $D(\alpha,ps)(T) = \{B; B$ ranges over subsets of T: $\alpha\mathrm{Int}(B) = \mathrm{psInt}(B)\}$.

The functor $D(p,sp)(T)$ yields a family of subsets of T and is defined by:

(Def. 23) $D(p,sp)(T) = \{B; B$ ranges over subsets of T: $\mathrm{pInt}(B) = \mathrm{spInt}(B)\}$.

The functor $D(p,ps)(T)$ yields a family of subsets of T and is defined by:

(Def. 24) $D(p,ps)(T) = \{B; B$ ranges over subsets of T: $\mathrm{pInt}(B) = \mathrm{psInt}(B)\}$.

The functor $D(sp,ps)(T)$ yields a family of subsets of T and is defined as follows:

(Def. 25) $D(sp,ps)(T) = \{B; B$ ranges over subsets of T: $\mathrm{spInt}(B) = \mathrm{psInt}(B)\}$.

In the sequel T denotes a non empty topological space and B denotes a subset of T.
The following propositions are true:

(1) $\alpha\mathrm{Int}(B) = \mathrm{pInt}(B)$ iff $\mathrm{sInt}(B) = \mathrm{psInt}(B)$.

(2) B is an α-set of T iff $B = \alpha\mathrm{Int}(B)$.

(3) B is semi-open iff $B = \mathrm{sInt}(B)$.

(4) B is pre-open iff $B = \mathrm{pInt}(B)$.

(5) B is pre-semi-open iff $B = \mathrm{psInt}(B)$.

(6) B is semi-pre-open iff $B = \mathrm{spInt}(B)$.

(7) $T^{\alpha} \cap D(c,\alpha)(T) =$ the topology of T.

(8) $\mathrm{SO}(T) \cap D(c,s)(T) =$ the topology of T.

(9) $\mathrm{PO}(T) \cap D(c,p)(T) =$ the topology of T.

(10) $\mathrm{PSO}(T) \cap D(c,ps)(T) =$ the topology of T.

(11) $\mathrm{PO}(T) \cap D(\alpha,p)(T) = T^{\alpha}$.

(12) $\mathrm{SO}(T) \cap D(\alpha,s)(T) = T^{\alpha}$.

(13) $\mathrm{PSO}(T) \cap D(\alpha,ps)(T) = T^{\alpha}$.

(14) $\mathrm{SPO}(T) \cap D(p,sp)(T) = \mathrm{PO}(T)$.

(15) $\mathrm{PSO}(T) \cap D(p,ps)(T) = \mathrm{PO}(T)$.

(16) $\mathrm{PSO}(T) \cap D(\alpha,p)(T) = \mathrm{SO}(T)$.

(17) $\mathrm{PSO}(T) \cap D(sp,ps)(T) = \mathrm{SPO}(T)$.

Let X, Y be non empty topological spaces and let f be a map from X into Y. We say that f is s-continuous if and only if:

(Def. 26) For every subset G of Y such that G is open holds $f^{-1}(G) \in \mathrm{SO}(X)$.

We say that f is p-continuous if and only if:

(Def. 27) For every subset G of Y such that G is open holds $f^{-1}(G) \in \mathrm{PO}(X)$.

We say that f is α-continuous if and only if:

(Def. 28) For every subset G of Y such that G is open holds $f^{-1}(G) \in X^{\alpha}$.

We say that f is ps-continuous if and only if:

(Def. 29) For every subset G of Y such that G is open holds $f^{-1}(G) \in \mathrm{PSO}(X)$.

We say that f is sp-continuous if and only if:

(Def. 30) For every subset G of Y such that G is open holds $f^{-1}(G) \in \mathrm{SPO}(X)$.

We say that f is (c,α)-continuous if and only if:

(Def. 31) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(c,\alpha)(X)$.

We say that f is (c,s)-continuous if and only if:

(Def. 32) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(c,s)(X)$.

We say that f is (c,p)-continuous if and only if:

(Def. 33) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(c,p)(X)$.

We say that f is (c,ps)-continuous if and only if:

(Def. 34) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(c,ps)(X)$.

We say that f is (α,p)-continuous if and only if:

(Def. 35) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(\alpha,p)(X)$.

We say that f is (α,s)-continuous if and only if:

(Def. 36) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(\alpha,s)(X)$.

We say that f is (α,ps)-continuous if and only if:

(Def. 37) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(\alpha,ps)(X)$.

We say that f is (p,ps)-continuous if and only if:

(Def. 38) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(p,ps)(X)$.

We say that f is (p,sp)-continuous if and only if:

(Def. 39) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(p,sp)(X)$.

We say that f is (sp,ps)-continuous if and only if:

(Def. 40) For every subset G of Y such that G is open holds $f^{-1}(G) \in D(sp,ps)(X)$.

In the sequel X, Y denote non empty topological spaces and f denotes a map from X into Y. One can prove the following propositions:

(18) f is α-continuous iff f is p-continuous and (α,p)-continuous.

(19) f is α-continuous iff f is s-continuous and (α,s)-continuous.

(20) f is α-continuous iff f is ps-continuous and (α,ps)-continuous.

(21) f is p-continuous iff f is sp-continuous and (p,sp)-continuous.

(22) f is p-continuous iff f is ps-continuous and (p,ps)-continuous.

(23) f is s-continuous iff f is ps-continuous and (α,p)-continuous.

(24) f is sp-continuous iff f is ps-continuous and (sp,ps)-continuous.

(25) f is continuous iff f is α-continuous and (c,α)-continuous.

(26) f is continuous iff f is s-continuous and (c,s)-continuous.

(27) f is continuous iff f is p-continuous and (c,p)-continuous.

(28) f is continuous iff f is ps-continuous and (c,ps)-continuous.

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