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Some Properties for Convex Combinations

Noboru Endou
Gifu National College of Technology

Yasumasa Suzuki
Miyagi University

Yasunari Shidama
Shinshu University
Nagano

Summary. This is a continuation of [6]. In this article, we proved that convex combination on convex family is convex.

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The articles [12], [18], [13], [8], [2], [19], [3], [5], [1], [10], [4], [16], [15], [14], [17], [11], [7], [6], and [9] provide the notation and terminology for this paper.

1. CONVEX COMBINATIONS ON CONVEX FAMILY

The following propositions are true:

(1) For every non empty RLS structure V and for all convex subsets M, N of V holds $M \cap N$ is convex.

(2) Let V be a real unitary space-like non empty unitary space structure, M be a subset of V, F be a finite sequence of elements of the carrier of V, and B be a finite sequence of elements of $\mathbb{R}$. Suppose $M = \{u; u$ ranges over vectors of V: $\bigwedge_{i:\text{natural number}} (i \in \operatorname{dom} F \cap \operatorname{dom} B \Rightarrow \bigvee_{v:\text{vector of } V} (v = F(i) \wedge (u|v) \leq B(i)))\}$. Then M is convex.

(3) Let V be a real unitary space-like non empty unitary space structure, M be a subset of V, F be a finite sequence of elements of the carrier of V, and B be a finite sequence of elements of $\mathbb{R}$. Suppose $M = \{u; u$ ranges over vectors of V: $\bigwedge_{i:\text{natural number}} (i \in \operatorname{dom} F \cap \operatorname{dom} B \Rightarrow \bigvee_{v:\text{vector of } V} (v = F(i) \wedge (u|v) < B(i)))\}$. Then M is convex.

(4) Let V be a real unitary space-like non empty unitary space structure, M be a subset of V, F be a finite sequence of elements of the carrier of V, and B be a finite sequence of elements of $\mathbb{R}$. Suppose $M = \{u; u$ ranges over vectors of V: $\bigwedge_{i:\text{natural number}} (i \in \operatorname{dom} F \cap \operatorname{dom} B \Rightarrow \bigvee_{v:\text{vector of } V} (v = F(i) \wedge (u|v) \geq B(i)))\}$. Then M is convex.

(5) Let V be a real unitary space-like non empty unitary space structure, M be a subset of V, F be a finite sequence of elements of the carrier of V, and B be a finite sequence of elements of $\mathbb{R}$. Suppose $M = \{u; u$ ranges over vectors of V: $\bigwedge_{i:\text{natural number}} (i \in \operatorname{dom} F \cap \operatorname{dom} B \Rightarrow \bigvee_{v:\text{vector of } V} (v = F(i) \wedge (u|v) > B(i)))\}$. Then M is convex.

(6) Let V be a real linear space and M be a subset of V. Then for every subset N of V and for every linear combination L of N such that L is convex and $N \subseteq M$ holds $\sum L \in M$ if and only if M is convex.

Let V be a real linear space and let M be a subset of V. The functor LC_M yielding a set is defined by:

(Def. 1) For every set L holds $L \in \mathrm{LC}_M$ iff L is a linear combination of M.

Let V be a real linear space. One can verify that there exists a linear combination of V which is convex.

Let V be a real linear space. A convex combination of V is a convex linear combination of V.

Let V be a real linear space and let M be a non empty subset of V. One can check that there exists a linear combination of M which is convex.

Let V be a real linear space and let M be a non empty subset of V. A convex combination of M is a convex linear combination of M.

One can prove the following propositions:

(7) For every real linear space V and for every subset M of V holds Convex-Family $M \neq \emptyset$.

(8) For every real linear space V and for every subset M of V holds $M \subseteq \mathrm{conv}\, M$.

(9) Let V be a real linear space, L_1, L_2 be convex combinations of V, and r be a real number. If $0 < r$ and $r < 1$, then $r \cdot L_1 + (1 - r) \cdot L_2$ is a convex combination of V.

(10) Let V be a real linear space, M be a non empty subset of V, L_1, L_2 be convex combinations of M, and r be a real number. If $0 < r$ and $r < 1$, then $r \cdot L_1 + (1 - r) \cdot L_2$ is a convex combination of M.

(11) For every real linear space V holds there exists a linear combination of V which is convex.

(12) For every real linear space V and for every non empty subset M of V holds there exists a linear combination of M which is convex.

2. MISCELLANEOUS

The following propositions are true:

(13) For every real linear space V and for all subspaces W_1, W_2 of V holds $\mathrm{Up}(W_1 + W_2) = \mathrm{Up}(W_1) + \mathrm{Up}(W_2)$.

(14) For every real linear space V and for all subspaces W_1, W_2 of V holds $\mathrm{Up}(W_1 \cap W_2) = \mathrm{Up}(W_1) \cap \mathrm{Up}(W_2)$.

(15) Let V be a real linear space, L_1, L_2 be convex combinations of V, and a, b be real numbers. Suppose $a \cdot b > 0$. Then the support of $a \cdot L_1 + b \cdot L_2 =$ (the support of $a \cdot L_1$) $\cup$ (the support of $b \cdot L_2$).

(16) Let F, G be functions. Suppose F and G are fiberwise equipotent. Let x_1, x_2 be sets. Suppose $x_1 \in \mathrm{dom}\, F$ and $x_2 \in \mathrm{dom}\, F$ and $x_1 \neq x_2$. Then there exist sets z_1, z_2 such that $z_1 \in \mathrm{dom}\, G$ and $z_2 \in \mathrm{dom}\, G$ and $z_1 \neq z_2$ and $F(x_1) = G(z_1)$ and $F(x_2) = G(z_2)$.

(17) Let V be a real linear space, L be a linear combination of V, and A be a subset of V. Suppose $A \subseteq$ the support of L. Then there exists a linear combination L_1 of V such that the support of $L_1 = A$.

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