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Metric-Affine Configurations in Metric Affine Planes — Part I

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Summary. We introduce several configurational axioms for metric affine planes such as theorem on three perpendiculars, orthogonalization of major Desargues Axiom, orthogonalization of the trapezium variant of Desargues Axiom, axiom on parallel projection together with its indirect forms. For convenience we also consider affine Major Desargues Axiom. The aim is to prove logical relationships which hold between the introduced statements.

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The articles [3], [2], and [1] provide the notation and terminology for this paper.

We follow the rules: X denotes a metric affine plane and o, a, a_1, b, b_1, c, c_1, d denote elements of X.

Let us consider X. We say that X satisfies Desargues Axiom if and only if the condition (Def. 1) is satisfied.

(Def. 1) Let given o, a, a_1, b, b_1, c, c_1. Suppose $o \neq a$ and $o \neq a_1$ and $o \neq b$ and $o \neq b_1$ and $o \neq c$ and $o \neq c_1$ and not $\mathbf{L}(b,b_1,a)$ and not $\mathbf{L}(a,a_1,c)$ and $\mathbf{L}(o,a,a_1)$ and $\mathbf{L}(o,b,b_1)$ and $\mathbf{L}(o,c,c_1)$ and $a,b \parallel a_1,b_1$ and $a,c \parallel a_1,c_1$. Then $b,c \parallel b_1,c_1$.

We introduce Desargues Axiom holds in X as a synonym of X satisfies Desargues Axiom. We say that X satisfies AH if and only if the condition (Def. 2) is satisfied.

(Def. 2) Let given o, a, a_1, b, b_1, c, c_1. Suppose $o,a \perp o,a_1$ and $o,b \perp o,b_1$ and $o,c \perp o,c_1$ and $a,b \perp a_1,b_1$ and $o,a \parallel b,c$ and $a,c \perp a_1,c_1$ and $o,c \nparallel o,a$ and $o,a \nparallel o,b$. Then $b,c \perp b_1,c_1$.

We introduce AH holds in X as a synonym of X satisfies AH. We say that X satisfies theorem on three perpendiculars if and only if:

(Def. 3) For all a, b, c such that not $\mathbf{L}(a,b,c)$ there exists d such that $d,a \perp b,c$ and $d,b \perp a,c$ and $d,c \perp a,b$.

We introduce theorem on three perpendiculars holds in X as a synonym of X satisfies theorem on three perpendiculars. We say that X satisfies orthogonal version of Desargues Axiom if and only if:

(Def. 4) For all o, a, a_1, b, b_1, c, c_1 such that $o,a \perp o,a_1$ and $o,b \perp o,b_1$ and $o,c \perp o,c_1$ and $a,b \perp a_1,b_1$ and $a,c \perp a_1,c_1$ and $o,c \nparallel o,a$ and $o,a \nparallel o,b$ holds $b,c \perp b_1,c_1$.

We introduce orthogonal version of Desargues Axiom holds in X as a synonym of X satisfies orthogonal version of Desargues Axiom. We say that X satisfies LIN if and only if the condition (Def. 5) is satisfied.

(Def. 5) Let given o, a, a_1, b, b_1, c, c_1. Suppose that $o \neq a$ and $o \neq a_1$ and $o \neq b$ and $o \neq b_1$ and $o \neq c$ and $o \neq c_1$ and $a \neq b$ and $o,c \perp o,c_1$ and $o,a \perp o,a_1$ and $o,b \perp o,b_1$ and not $\mathbf{L}(o,c,a)$ and $\mathbf{L}(o,a,b)$ and $\mathbf{L}(o,a_1,b_1)$ and $a,c \perp a_1,c_1$ and $b,c \perp b_1,c_1$. Then $a,a_1 \upharpoonleft\!\upharpoonright b,b_1$.

We introduce LIN holds in X as a synonym of X satisfies LIN. We say that X satisfies first indirect form of LIN if and only if the condition (Def. 6) is satisfied.

(Def. 6) Let given o, a, a_1, b, b_1, c, c_1. Suppose that $o \neq a$ and $o \neq a_1$ and $o \neq b$ and $o \neq b_1$ and $o \neq c$ and $o \neq c_1$ and $a \neq b$ and $o,c \perp o,c_1$ and $o,a \perp o,a_1$ and $o,b \perp o,b_1$ and not $\mathbf{L}(o,c,a)$ and $\mathbf{L}(o,a,b)$ and $\mathbf{L}(o,a_1,b_1)$ and $a,c \perp a_1,c_1$ and $a,a_1 \upharpoonleft\!\upharpoonright b,b_1$. Then $b,c \perp b_1,c_1$.

We introduce first indirect form of LIN holds in X as a synonym of X satisfies first indirect form of LIN. We say that X satisfies second indirect form of LIN if and only if the condition (Def. 7) is satisfied.

(Def. 7) Let given o, a, a_1, b, b_1, c, c_1. Suppose that $o \neq a$ and $o \neq a_1$ and $o \neq b$ and $o \neq b_1$ and $o \neq c$ and $o \neq c_1$ and $a \neq b$ and $a,a_1 \upharpoonleft\!\upharpoonright b,b_1$ and $o,a \perp o,a_1$ and $o,b \perp o,b_1$ and not $\mathbf{L}(o,c,a)$ and $\mathbf{L}(o,a,b)$ and $\mathbf{L}(o,a_1,b_1)$ and $a,c \perp a_1,c_1$ and $b,c \perp b_1,c_1$. Then $o,c \perp o,c_1$.

We introduce second indirect form of LIN holds in X as a synonym of X satisfies second indirect form of LIN.

Next we state several propositions:

(1) If orthogonal version of Desargues Axiom holds in X, then Desargues Axiom holds in X.

(2) If orthogonal version of Desargues Axiom holds in X, then AH holds in X.

(3) If LIN holds in X, then first indirect form of LIN holds in X.

(4) If first indirect form of LIN holds in X, then second indirect form of LIN holds in X.

(5) If LIN holds in X, then orthogonal version of Desargues Axiom holds in X.

(6) If LIN holds in X, then theorem on three perpendiculars holds in X.

References

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