JOURNAL OF FORMALIZED MATHEMATICS
Volume 13, Released 2001, Published 2003
Inst. of Computer Science, Univ. of Białystok

Circuit Generated by Terms and Circuit Calculating Terms

Grzegorz Bancerek
University of Białystok
Shinshu University, Nagano

Summary. In the paper we investigate the dependence between the structure of circuits and sets of terms. Circuits in our terminology (see [19]) are treated as locally-finite many sorted algebras over special signatures. Such approach enables to formalize every real circuit. The goal of this investigation is to specify circuits by terms and, enentualy, to have methods of formal verification of real circuits. The following notation is introduced in this paper:

- structural equivalence of circuits, i.e. equivalence of many sorted signatures,
- embedding of a circuit into another one,
- similarity of circuits (a concept narrower than isomorphism of many sorted algebras over equivalent signatures),
- calculation of terms by a circuit according to an algebra,
- specification of circuits by terms and an algebra.

MML Identifier: CIRCTRM1.

WWW: http://mizar.org/JFM/Vol13/circtrm1.html

The articles [23], [15], [28], [27], [1], [29], [12], [13], [10], [17], [14], [24], [16], [3], [25], [26], [20], [21], [19], [22], [18], [11], [2], [4], [5], [6], [7], [8], and [9] provide the notation and terminology for this paper.

1. Circuit Structure Generated by Terms

One can prove the following proposition

(1) Let S be a non empty non void many sorted signature, A be a non-empty algebra over S, V be a variables family of A, t be a term of S over V, and T be a term of A over V. If $T = t$, then the sort of $T =$ the sort of t.

Let D be a non empty set and let X be a subset of D. Then id_X is a function from X into D.

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, and let X be a non empty subset of S-Terms(V). The functor X-CircuitStr yielding a non empty strict many sorted signature is defined by the condition (Def. 1).

(Def. 1) X-CircuitStr $= \langle \mathrm{Subtrees}(X), [:\text{the operation symbols of } S, \{\text{the carrier of } S\}:]\text{-Subtrees}(X), [:\text{the operation symbols of } S, \{\text{the carrier of } S\}:]\text{-ImmediateSubtrees}(X), \mathrm{id}_{[:\text{the operation symbols of } S, \{\text{the carrier of } S\}:]\text{-Subtrees}(X)} \rangle$.

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, and let X be a non empty subset of S-Terms(V). Note that X-CircuitStr is unsplit.

In the sequel S denotes a non empty non void many sorted signature, V denotes a non-empty many sorted set indexed by the carrier of S, A denotes a non-empty algebra over S, and X denotes a non empty subset of S-Terms(V).

We now state two propositions:

(2) X-CircuitStr is void if and only if for every element t of X holds t is root and $t(\emptyset) \notin [:$the operation symbols of S, $\{$the carrier of $S\}:]$.

(3) X is a set with a compound term of S over V iff X-CircuitStr is non void.

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, and let X be a set with a compound term of S over V. Observe that X-CircuitStr is non void.

One can prove the following three propositions:

(4)(i) Every vertex of X-CircuitStr is a term of S over V, and

(ii) for every set s such that $s \in$ the operation symbols of X-CircuitStr holds s is a compound term of S over V.

(5) Let t be a vertex of X-CircuitStr. Then $t \in$ the operation symbols of X-CircuitStr if and only if t is a compound term of S over V.

(6) Let X be a set with a compound term of S over V and g be a gate of X-CircuitStr. Then (the result sort of X-CircuitStr$)(g) = g$ and the result sort of $g = g$.

Let us consider S, V, let X be a set with a compound term of S over V, and let g be a gate of X-CircuitStr. Note that Arity(g) is decorated tree yielding.

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, and let X be a non empty subset of S-Terms(V). Observe that every vertex of X-CircuitStr is finite, function-like, and relation-like.

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, and let X be a non empty subset of S-Terms(V). Observe that every vertex of X-CircuitStr is decorated tree-like.

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, and let X be a set with a compound term of S over V. Observe that every gate of X-CircuitStr is finite, function-like, and relation-like.

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, and let X be a set with a compound term of S over V. Note that every gate of X-CircuitStr is decorated tree-like.

Next we state the proposition

(7) Let X_1, X_2 be non empty subsets of S-Terms(V). Then the arity of X_1-CircuitStr $\approx$ the arity of X_2-CircuitStr and the result sort of X_1-CircuitStr $\approx$ the result sort of X_2-CircuitStr.

Let X, Y be constituted of decorated trees sets. One can check that $X \cup Y$ is constituted of decorated trees.

We now state several propositions:

(8) For all constituted of decorated trees non empty sets X_1, X_2 holds Subtrees$(X_1 \cup X_2) =$ Subtrees$(X_1) \cup$ Subtrees(X_2).

(9) For all constituted of decorated trees non empty sets X_1, X_2 and for every set C holds C-Subtrees$(X_1 \cup X_2) = (C$-Subtrees$(X_1)) \cup (C$-Subtrees$(X_2))$.

(10) Let X_1, X_2 be constituted of decorated trees non empty sets. If every element of X_1 is finite and every element of X_2 is finite, then for every set C holds C-ImmediateSubtrees$(X_1 \cup X_2) =$ $(C$-ImmediateSubtrees$(X_1)) {+}\cdot (C$-ImmediateSubtrees$(X_2))$.

(11) For all non empty subsets X_1, X_2 of S-Terms(V) holds $(X_1 \cup X_2)$-CircuitStr $=$ $(X_1$-CircuitStr$) {+}\cdot (X_2$-CircuitStr$)$.

(12) Let x be a set. Then $x \in \mathrm{InputVertices}(X\text{-CircuitStr})$ if and only if the following conditions are satisfied:

(i) $x \in \mathrm{Subtrees}(X)$, and

(ii) there exists a sort symbol s of S and there exists an element v of $V(s)$ such that $x =$ the root tree of $\langle v, s \rangle$.

(13) For every set X with a compound term of S over V and for every gate g of X-CircuitStr holds $g = g(\emptyset)\text{-tree}(\mathrm{Arity}(g))$.

2. Circuit Generated by Terms

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, let X be a non empty subset of S-Terms(V), let v be a vertex of X-CircuitStr, and let A be an algebra over S. The sort of v w.r.t. A is defined as follows:

(Def. 2) For every term u of S over V such that $u = v$ holds the sort of v w.r.t. $A =$ (the sorts of A)(the sort of u).

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, let X be a non empty subset of S-Terms(V), let v be a vertex of X-CircuitStr, and let A be a non-empty algebra over S. Note that the sort of v w.r.t. A is non empty.

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, and let X be a non empty subset of S-Terms(V). Let us assume that X is a set with a compound term of S over V. Let o be a gate of X-CircuitStr and let A be an algebra over S. The action of o w.r.t A is a function and is defined by the condition (Def. 3).

(Def. 3) Let X' be a set with a compound term of S over V. Suppose $X' = X$. Let o' be a gate of X'-CircuitStr. Suppose $o' = o$. Then the action of o w.r.t $A =$ (the characteristics of A)$(o'(\emptyset)_{\mathbf{1}})$.

The scheme *MSFuncEx* deals with a non empty set $\mathcal{A}$, non-empty many sorted sets $\mathcal{B}$, $\mathcal{C}$ indexed by $\mathcal{A}$, and a ternary predicate $\mathcal{P}$, and states that:

There exists a many sorted function f from $\mathcal{B}$ into $\mathcal{C}$ such that for every element i of $\mathcal{A}$ and for every element a of $\mathcal{B}(i)$ holds $\mathcal{P}[i, a, f(i)(a)]$

provided the following condition is met:

- For every element i of $\mathcal{A}$ and for every element a of $\mathcal{B}(i)$ there exists an element b of $\mathcal{C}(i)$ such that $\mathcal{P}[i, a, b]$.

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, let X be a non empty subset of S-Terms(V), and let A be an algebra over S. The functor X-CircuitSorts(A) yielding a many sorted set indexed by the carrier of X-CircuitStr is defined as follows:

(Def. 4) For every vertex v of X-CircuitStr holds $(X\text{-CircuitSorts}(A))(v) =$ the sort of v w.r.t. A.

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, let X be a non empty subset of S-Terms(V), and let A be a non-empty algebra over S. One can check that X-CircuitSorts(A) is non-empty.

One can prove the following proposition

(14) Let X be a set with a compound term of S over V, g be a gate of X-CircuitStr, and o be an operation symbol of S. If $g(\emptyset) = \langle o, \text{the carrier of } S \rangle$, then $(X\text{-CircuitSorts}(A)) \cdot \mathrm{Arity}(g) =$ (the sorts of A) $\cdot\, \mathrm{Arity}(o)$.

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, let X be a non empty subset of S-Terms(V), and let A be a non-empty algebra over S. The functor X-CircuitCharact(A) yielding a many sorted function from $(X\text{-CircuitSorts}(A))^{\#} \cdot$ the arity of X-CircuitStr into $(X\text{-CircuitSorts}(A)) \cdot$ the result sort of X-CircuitStr is defined as follows:

(Def. 5) For every gate g of X-CircuitStr such that $g \in$ the operation symbols of X-CircuitStr holds $(X\text{-CircuitCharact}(A))(g) =$ the action of g w.r.t A.

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, let X be a non empty subset of S-Terms(V), and let A be a non-empty algebra over S. The functor X-Circuit(A) yielding a non-empty strict algebra over X-CircuitStr is defined by:

(Def. 6) $X\text{-Circuit}(A) = \langle X\text{-CircuitSorts}(A), X\text{-CircuitCharact}(A)\rangle$.

One can prove the following four propositions:

(15) For every vertex v of X-CircuitStr holds (the sorts of X-Circuit$(A))(v) =$ the sort of v w.r.t. A.

(16) Let A be a locally-finite non-empty algebra over S, X be a set with a compound term of S over V, and g be an operation symbol of X-CircuitStr. Then $\mathrm{Den}(g, X\text{-Circuit}(A)) =$ the action of g w.r.t A.

(17) Let A be a locally-finite non-empty algebra over S, X be a set with a compound term of S over V, g be an operation symbol of X-CircuitStr, and o be an operation symbol of S. If $g(\emptyset) = \langle o$, the carrier of $S\rangle$, then $\mathrm{Den}(g, X\text{-Circuit}(A)) = \mathrm{Den}(o, A)$.

(18) Let A be a locally-finite non-empty algebra over S and X be a non empty subset of S-Terms(V). Then X-Circuit(A) is locally-finite.

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, let X be a set with a compound term of S over V, and let A be a locally-finite non-empty algebra over S. Observe that X-Circuit(A) is locally-finite.

Next we state two propositions:

(19) Let S be a non empty non void many sorted signature, V be a non-empty many sorted set indexed by the carrier of S, X_1, X_2 be sets with compound terms of S over V, and A be a non-empty algebra over S. Then $X_1\text{-Circuit}(A) \approx X_2\text{-Circuit}(A)$.

(20) Let S be a non empty non void many sorted signature, V be a non-empty many sorted set indexed by the carrier of S, X_1, X_2 be sets with compound terms of S over V, and A be a non-empty algebra over S. Then $(X_1 \cup X_2)\text{-Circuit}(A) = (X_1\text{-Circuit}(A)) +\cdot (X_2\text{-Circuit}(A))$.

3. Correctness of a Term Circuit

In the sequel S is a non empty non void many sorted signature, A is a non-empty locally-finite algebra over S, V is a variables family of A, and X is a set with a compound term of S over V.

Let S be a non empty non void many sorted signature, let A be a non-empty algebra over S, let V be a variables family of A, and let t be a decorated tree. Let us assume that t is a term of S over V. Let f be a many sorted function from V into the sorts of A. The functor $[\![t]\!]_A(f)$ is defined as follows:

(Def. 7) There exists a term t' of A over V such that $t' = t$ and $[\![t]\!]_A(f) = t'\,{}^{@}\,f$.

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, let X be a set with a compound term of S over V, let A be a non-empty locally-finite algebra over S, and let s be a state of X-Circuit(A). A many sorted function from V into the sorts of A is said to be a valuation compatible with s if it satisfies the condition (Def. 8).

(Def. 8) Let x be a vertex of S and v be an element of $V(x)$. If the root tree of $\langle v, x\rangle \in \mathrm{Subtrees}(X)$, then $\mathrm{it}(x)(v) = s($the root tree of $\langle v, x\rangle)$.

We now state the proposition

(21) Let s be a state of X-Circuit(A), f be a valuation compatible with s, and n be a natural number. Then f is a valuation compatible with Following(s,n).

Let x be a set, let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, and let p be a finite sequence of elements of S-Terms(V). Observe that x-tree(p) is finite.

Next we state two propositions:

(22) Let s be a state of X-Circuit(A), f be a valuation compatible with s, and t be a term of S over V. If $t \in$ Subtrees(X), then Following$(s, 1 + \mathrm{height}\,\mathrm{dom}\,t)$ is stable at t and (Following$(s, 1 + \mathrm{height}\,\mathrm{dom}\,t)$)$(t) = [\![t]\!]_A(f)$.

(23) Suppose that it is not true that there exists a term t of S over V and there exists an operation symbol o of S such that $t \in$ Subtrees(X) and $t(\emptyset) = \langle o$, the carrier of $S\rangle$ and Arity$(o) = \emptyset$. Let s be a state of X-Circuit(A), f be a valuation compatible with s, and t be a term of S over V. If $t \in$ Subtrees(X), then Following$(s, \mathrm{height}\,\mathrm{dom}\,t)$ is stable at t and (Following$(s, \mathrm{height}\,\mathrm{dom}\,t)$)$(t) = [\![t]\!]_A(f)$.

4. Circuit Similarity

Let X be a set. Note that id_X is one-to-one.

Let f be an one-to-one function. One can check that f^{-1} is one-to-one. Let g be an one-to-one function. Observe that $g \cdot f$ is one-to-one.

Let S_1, S_2 be non empty many sorted signatures and let f, g be functions. We say that S_1 and S_2 are equivalent w.r.t. f and g if and only if the conditions (Def. 9) are satisfied.

(Def. 9)(i) f is one-to-one,

(ii) g is one-to-one,

(iii) f and g form morphism between S_1 and S_2, and

(iv) f^{-1} and g^{-1} form morphism between S_2 and S_1.

Next we state several propositions:

(24) Let S_1, S_2 be non empty many sorted signatures and f, g be functions. Suppose S_1 and S_2 are equivalent w.r.t. f and g. Then the carrier of $S_2 = f^\circ$(the carrier of S_1) and the operation symbols of $S_2 = g^\circ$(the operation symbols of S_1).

(25) Let S_1, S_2 be non empty many sorted signatures and f, g be functions. Suppose S_1 and S_2 are equivalent w.r.t. f and g. Then $\mathrm{rng}\, f =$ the carrier of S_2 and $\mathrm{rng}\, g =$ the operation symbols of S_2.

(26) Let S be a non empty many sorted signature. Then S and S are equivalent w.r.t. $\mathrm{id}_{\text{the carrier of } S}$ and $\mathrm{id}_{\text{the operation symbols of } S}$.

(27) Let S_1, S_2 be non empty many sorted signatures and f, g be functions. Suppose S_1 and S_2 are equivalent w.r.t. f and g. Then S_2 and S_1 are equivalent w.r.t. f^{-1} and g^{-1}.

(28) Let S_1, S_2, S_3 be non empty many sorted signatures and f_1, g_1, f_2, g_2 be functions. Suppose S_1 and S_2 are equivalent w.r.t. f_1 and g_1 and S_2 and S_3 are equivalent w.r.t. f_2 and g_2. Then S_1 and S_3 are equivalent w.r.t. $f_2 \cdot f_1$ and $g_2 \cdot g_1$.

(29) Let S_1, S_2 be non empty many sorted signatures and f, g be functions. Suppose S_1 and S_2 are equivalent w.r.t. f and g. Then f° InputVertices$(S_1) =$ InputVertices(S_2) and f° InnerVertices$(S_1) =$ InnerVertices(S_2).

Let S_1, S_2 be non empty many sorted signatures. We say that S_1 and S_2 are equivalent if and only if:

(Def. 10) There exist one-to-one functions f, g such that S_1 and S_2 are equivalent w.r.t. f and g.

Let us notice that the predicate S_1 and S_2 are equivalent is reflexive and symmetric.

The following proposition is true

(30) Let S_1, S_2, S_3 be non empty many sorted signatures. Suppose S_1 and S_2 are equivalent and S_2 and S_3 are equivalent. Then S_1 and S_3 are equivalent.

Let S_1, S_2 be non empty many sorted signatures and let f be a function. We say that f preserves inputs of S_1 in S_2 if and only if:

(Def. 11) $f^\circ \mathrm{InputVertices}(S_1) \subseteq \mathrm{InputVertices}(S_2)$.

Next we state four propositions:

(31) Let S_1, S_2 be non empty many sorted signatures and f, g be functions. Suppose f and g form morphism between S_1 and S_2. Let v be a vertex of S_1. Then $f(v)$ is a vertex of S_2.

(32) Let S_1, S_2 be non empty non void many sorted signatures and f, g be functions. Suppose f and g form morphism between S_1 and S_2. Let v be a gate of S_1. Then $g(v)$ is a gate of S_2.

(33) Let S_1, S_2 be non empty many sorted signatures and f, g be functions. If f and g form morphism between S_1 and S_2, then $f^\circ \mathrm{InnerVertices}(S_1) \subseteq \mathrm{InnerVertices}(S_2)$.

(34) Let S_1, S_2 be circuit-like non void non empty many sorted signatures and f, g be functions. Suppose f and g form morphism between S_1 and S_2. Let v_1 be a vertex of S_1. Suppose $v_1 \in \mathrm{InnerVertices}(S_1)$. Let v_2 be a vertex of S_2. If $v_2 = f(v_1)$, then the action at $v_2 = g($the action at $v_1)$.

Let S_1, S_2 be non empty many sorted signatures, let f, g be functions, let C_1 be a non-empty algebra over S_1, and let C_2 be a non-empty algebra over S_2. We say that f and g form embedding of C_1 into C_2 if and only if the conditions (Def. 12) are satisfied.

(Def. 12)(i) f is one-to-one,

(ii) g is one-to-one,

(iii) f and g form morphism between S_1 and S_2,

(iv) the sorts of $C_1 =$ (the sorts of $C_2) \cdot f$, and

(v) the characteristics of $C_1 =$ (the characteristics of $C_2) \cdot g$.

One can prove the following two propositions:

(35) Let S be a non empty many sorted signature and C be a non-empty algebra over S. Then $\mathrm{id}_{\text{the carrier of } S}$ and $\mathrm{id}_{\text{the operation symbols of } S}$ form embedding of C into C.

(36) Let S_1, S_2, S_3 be non empty many sorted signatures, f_1, g_1, f_2, g_2 be functions, C_1 be a non-empty algebra over S_1, C_2 be a non-empty algebra over S_2, and C_3 be a non-empty algebra over S_3. Suppose f_1 and g_1 form embedding of C_1 into C_2 and f_2 and g_2 form embedding of C_2 into C_3. Then $f_2 \cdot f_1$ and $g_2 \cdot g_1$ form embedding of C_1 into C_3.

Let S_1, S_2 be non empty many sorted signatures, let f, g be functions, let C_1 be a non-empty algebra over S_1, and let C_2 be a non-empty algebra over S_2. We say that C_1 and C_2 are similar w.r.t. f and g if and only if:

(Def. 13) f and g form embedding of C_1 into C_2 and f^{-1} and g^{-1} form embedding of C_2 into C_1.

One can prove the following propositions:

(37) Let S_1, S_2 be non empty many sorted signatures, f, g be functions, C_1 be a non-empty algebra over S_1, and C_2 be a non-empty algebra over S_2. Suppose C_1 and C_2 are similar w.r.t. f and g. Then S_1 and S_2 are equivalent w.r.t. f and g.

(38) Let S_1, S_2 be non empty many sorted signatures, f, g be functions, C_1 be a non-empty algebra over S_1, and C_2 be a non-empty algebra over S_2. Then C_1 and C_2 are similar w.r.t. f and g if and only if the following conditions are satisfied:

(i) S_1 and S_2 are equivalent w.r.t. f and g,

(ii) the sorts of $C_1 =$ (the sorts of $C_2) \cdot f$, and

(iii) the characteristics of $C_1 =$ (the characteristics of $C_2) \cdot g$.

(39) Let S be a non empty many sorted signature and C be a non-empty algebra over S. Then C and C are similar w.r.t. $\mathrm{id}_{\text{the carrier of } S}$ and $\mathrm{id}_{\text{the operation symbols of } S}$.

(40) Let S_1, S_2 be non empty many sorted signatures, f, g be functions, C_1 be a non-empty algebra over S_1, and C_2 be a non-empty algebra over S_2. Suppose C_1 and C_2 are similar w.r.t. f and g. Then C_2 and C_1 are similar w.r.t. f^{-1} and g^{-1}.

(41) Let S_1, S_2, S_3 be non empty many sorted signatures, f_1, g_1, f_2, g_2 be functions, C_1 be a non-empty algebra over S_1, C_2 be a non-empty algebra over S_2, and C_3 be a non-empty algebra over S_3. Suppose C_1 and C_2 are similar w.r.t. f_1 and g_1 and C_2 and C_3 are similar w.r.t. f_2 and g_2. Then C_1 and C_3 are similar w.r.t. $f_2 \cdot f_1$ and $g_2 \cdot g_1$.

Let S_1, S_2 be non empty many sorted signatures, let C_1 be a non-empty algebra over S_1, and let C_2 be a non-empty algebra over S_2. We say that C_1 and C_2 are similar if and only if:

(Def. 14) There exist functions f, g such that C_1 and C_2 are similar w.r.t. f and g.

For simplicity, we follow the rules: G_1, G_2 are circuit-like non void non empty many sorted signatures, f, g are functions, C_1 is a non-empty circuit of G_1, and C_2 is a non-empty circuit of G_2.

Next we state a number of propositions:

(42) Suppose f and g form embedding of C_1 into C_2. Then

(i) $\operatorname{dom} f =$ the carrier of G_1,

(ii) $\operatorname{rng} f \subseteq$ the carrier of G_2,

(iii) $\operatorname{dom} g =$ the operation symbols of G_1, and

(iv) $\operatorname{rng} g \subseteq$ the operation symbols of G_2.

(43) Suppose f and g form embedding of C_1 into C_2. Let o_1 be a gate of G_1 and o_2 be a gate of G_2. If $o_2 = g(o_1)$, then $\mathrm{Den}(o_2, C_2) = \mathrm{Den}(o_1, C_1)$.

(44) Suppose f and g form embedding of C_1 into C_2. Let o_1 be a gate of G_1 and o_2 be a gate of G_2. Suppose $o_2 = g(o_1)$. Let s_1 be a state of C_1 and s_2 be a state of C_2. If $s_1 = s_2 \cdot f$, then $o_2\,\text{depends-on-in}\,s_2 = o_1\,\text{depends-on-in}\,s_1$.

(45) If f and g form embedding of C_1 into C_2, then for every state s of C_2 holds $s \cdot f$ is a state of C_1.

(46) Suppose f and g form embedding of C_1 into C_2. Let s_2 be a state of C_2 and s_1 be a state of C_1. Suppose $s_1 = s_2 \cdot f$ and for every vertex v of G_1 such that $v \in \mathrm{InputVertices}(G_1)$ holds s_2 is stable at $f(v)$. Then $\mathrm{Following}(s_1) = \mathrm{Following}(s_2) \cdot f$.

(47) Suppose f and g form embedding of C_1 into C_2 and f preserves inputs of G_1 in G_2. Let s_2 be a state of C_2 and s_1 be a state of C_1. If $s_1 = s_2 \cdot f$, then $\mathrm{Following}(s_1) = \mathrm{Following}(s_2) \cdot f$.

(48) Suppose f and g form embedding of C_1 into C_2 and f preserves inputs of G_1 in G_2. Let s_2 be a state of C_2 and s_1 be a state of C_1. If $s_1 = s_2 \cdot f$, then for every natural number n holds $\mathrm{Following}(s_1, n) = \mathrm{Following}(s_2, n) \cdot f$.

(49) Suppose f and g form embedding of C_1 into C_2 and f preserves inputs of G_1 in G_2. Let s_2 be a state of C_2 and s_1 be a state of C_1. If $s_1 = s_2 \cdot f$, then if s_2 is stable, then s_1 is stable.

(50) Suppose f and g form embedding of C_1 into C_2 and f preserves inputs of G_1 in G_2. Let s_2 be a state of C_2 and s_1 be a state of C_1. Suppose $s_1 = s_2 \cdot f$. Let v_1 be a vertex of G_1. Then s_1 is stable at v_1 if and only if s_2 is stable at $f(v_1)$.

(51) If C_1 and C_2 are similar w.r.t. f and g, then for every state s of C_2 holds $s \cdot f$ is a state of C_1.

(52) Suppose C_1 and C_2 are similar w.r.t. f and g. Let s_1 be a state of C_1 and s_2 be a state of C_2. Then $s_1 = s_2 \cdot f$ if and only if $s_2 = s_1 \cdot f^{-1}$.

(53) If C_1 and C_2 are similar w.r.t. f and g, then $f^\circ \mathrm{InputVertices}(G_1) = \mathrm{InputVertices}(G_2)$ and $f^\circ \mathrm{InnerVertices}(G_1) = \mathrm{InnerVertices}(G_2)$.

(54) If C_1 and C_2 are similar w.r.t. f and g, then f preserves inputs of G_1 in G_2.

(55) Suppose C_1 and C_2 are similar w.r.t. f and g. Let s_1 be a state of C_1 and s_2 be a state of C_2. If $s_1 = s_2 \cdot f$, then $\mathrm{Following}(s_1) = \mathrm{Following}(s_2) \cdot f$.

(56) Suppose C_1 and C_2 are similar w.r.t. f and g. Let s_1 be a state of C_1 and s_2 be a state of C_2. If $s_1 = s_2 \cdot f$, then for every natural number n holds $\mathrm{Following}(s_1, n) = \mathrm{Following}(s_2, n) \cdot f$.

(57) Suppose C_1 and C_2 are similar w.r.t. f and g. Let s_1 be a state of C_1 and s_2 be a state of C_2. If $s_1 = s_2 \cdot f$, then s_1 is stable iff s_2 is stable.

(58) Suppose C_1 and C_2 are similar w.r.t. f and g. Let s_1 be a state of C_1 and s_2 be a state of C_2. Suppose $s_1 = s_2 \cdot f$. Let v_1 be a vertex of G_1. Then s_1 is stable at v_1 if and only if s_2 is stable at $f(v_1)$.

5. Term Specification

Let S be a non empty non void many sorted signature, let A be a non-empty algebra over S, let V be a non-empty many sorted set indexed by the carrier of S, let X be a non empty subset of S-Terms(V), let G be a circuit-like non void non empty many sorted signature, and let C be a non-empty circuit of G. We say that C calculates X in A if and only if:

(Def. 15) There exist f, g such that f and g form embedding of X-Circuit(A) into C and f preserves inputs of X-CircuitStr in G.

We say that X and A specify C if and only if:

(Def. 16) C and X-Circuit(A) are similar.

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, let A be a non-empty algebra over S, let X be a non empty subset of S-Terms(V), let G be a circuit-like non void non empty many sorted signature, and let C be a non-empty circuit of G. Let us assume that C calculates X in A. An one-to-one function is said to be a sort map from X and A into C if:

(Def. 17) It preserves inputs of X-CircuitStr in G and there exists g such that it and g form embedding of X-Circuit(A) into C.

Let S be a non empty non void many sorted signature, let V be a non-empty many sorted set indexed by the carrier of S, let A be a non-empty algebra over S, let X be a non empty subset of S-Terms(V), let G be a circuit-like non void non empty many sorted signature, and let C be a non-empty circuit of G. Let us assume that C calculates X in A. Let f be a sort map from X and A into C. An one-to-one function is said to be an operation map from X and A into C obeying f if:

(Def. 18) f and it form embedding of X-Circuit(A) into C.

Next we state several propositions:

(59) Let G be a circuit-like non void non empty many sorted signature and C be a non-empty circuit of G. If X and A specify C, then C calculates X in A.

(60) Let G be a circuit-like non void non empty many sorted signature and C be a non-empty circuit of G. Suppose C calculates X in A. Let f be a sort map from X and A into C and t be a term of S over V. Suppose $t \in \text{Subtrees}(X)$. Let s be a state of C. Then

(i) $\text{Following}(s, 1 + \text{height}\,\text{dom}\, t)$ is stable at $f(t)$, and

(ii) for every state s' of X-Circuit(A) such that $s' = s \cdot f$ and for every valuation h compatible with s' holds $(\text{Following}(s, 1 + \text{height}\,\text{dom}\, t))(f(t)) = [\![t]\!]_A(h)$.

(61) Let G be a circuit-like non void non empty many sorted signature and C be a non-empty circuit of G. Suppose C calculates X in A. Let t be a term of S over V. Suppose $t \in \text{Subtrees}(X)$. Then there exists a vertex v of G such that for every state s of C holds

(i) $\text{Following}(s, 1 + \text{height}\,\text{dom}\, t)$ is stable at v, and

(ii) there exists a sort map f from X and A into C such that for every state s' of X-Circuit(A) such that $s' = s \cdot f$ and for every valuation h compatible with s' holds $(\text{Following}(s, 1 + \text{height}\,\text{dom}\, t))(v) = [\![t]\!]_A(h)$.

(62) Let G be a circuit-like non void non empty many sorted signature and C be a non-empty circuit of G. Suppose X and A specify C. Let f be a sort map from X and A into C, s be a state of C, and t be a term of S over V. Suppose $t \in \text{Subtrees}(X)$. Then

(i) $\text{Following}(s, 1 + \text{height}\,\text{dom}\, t)$ is stable at $f(t)$, and

(ii) for every state s' of X-Circuit(A) such that $s' = s \cdot f$ and for every valuation h compatible with s' holds $(\text{Following}(s, 1 + \text{height}\,\text{dom}\, t))(f(t)) = [\![t]\!]_A(h)$.

(63) Let G be a circuit-like non void non empty many sorted signature and C be a non-empty circuit of G. Suppose X and A specify C. Let t be a term of S over V. Suppose $t \in \text{Subtrees}(X)$. Then there exists a vertex v of G such that for every state s of C holds

(i) $\text{Following}(s, 1 + \text{height}\,\text{dom}\, t)$ is stable at v, and

(ii) there exists a sort map f from X and A into C such that for every state s' of X-Circuit(A) such that $s' = s \cdot f$ and for every valuation h compatible with s' holds $(\text{Following}(s, 1 + \text{height}\,\text{dom}\, t))(v) = [\![t]\!]_A(h)$.

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Received April 10, 2001

Published January 2, 2004