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Predicate Calculus for Boolean Valued Functions. Part III

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Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of usual predicate logic.

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The articles [9], [2], [11], [14], [13], [12], [7], [1], [10], [8], [3], [5], [4], and [6] provide the notation and terminology for this paper.

1. PRELIMINARIES

In this paper Y denotes a non empty set.

Next we state several propositions:

(1) For every element z of Y and for all partitions P_1, P_2 of Y such that $P_1 \Subset P_2$ holds $\mathrm{EqClass}(z, P_1) \subseteq \mathrm{EqClass}(z, P_2)$.

(2) For every element z of Y and for all partitions P_1, P_2 of Y holds $\mathrm{EqClass}(z, P_1) \subseteq \mathrm{EqClass}(z, P_1 \vee P_2)$.

(3) For every element z of Y and for all partitions P_1, P_2 of Y holds $\mathrm{EqClass}(z, P_1 \wedge P_2) \subseteq \mathrm{EqClass}(z, P_1)$.

(4) For every element z of Y and for every partition P_1 of Y holds $\mathrm{EqClass}(z, P_1) \subseteq \mathrm{EqClass}(z, O(Y))$ and $\mathrm{EqClass}(z, I(Y)) \subseteq \mathrm{EqClass}(z, P_1)$.

(5) Let G be a subset of $\mathrm{PARTITIONS}(Y)$ and A, B be partitions of Y. Suppose G is independent and $G = \{A, B\}$ and $A \neq B$. Let a, b be sets. If $a \in A$ and $b \in B$, then a meets b.

(6) Let G be a subset of $\mathrm{PARTITIONS}(Y)$ and A, B be partitions of Y. If G is independent and $G = \{A, B\}$ and $A \neq B$, then $\bigwedge G = A \wedge B$.

(7) For every subset G of $\mathrm{PARTITIONS}(Y)$ and for all partitions A, B of Y such that $G = \{A, B\}$ and $A \neq B$ holds $\mathrm{CompF}(A, G) = B$.

2. Predicate Calculus

We now state a number of propositions:

(11)[1] Let a be an element of $Boolean^Y$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is independent, then $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\forall_{a,B}G,A}G$.

(12) For every element a of $Boolean^Y$ and for every subset G of PARTITIONS(Y) and for all partitions A, B of Y holds $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.

(13) Let a be an element of $Boolean^Y$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is independent, then $\forall_{\forall_{a,A}G,B}G \Subset \forall_{\exists_{a,B}G,A}G$.

(14) For every element a of $Boolean^Y$ and for every subset G of PARTITIONS(Y) and for all partitions A, B of Y holds $\forall_{\exists_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.

(15) For every element a of $Boolean^Y$ and for every subset G of PARTITIONS(Y) and for all partitions A, B of Y holds $\neg\exists_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(16) Let a be an element of $Boolean^Y$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is independent, then $\exists_{\neg\forall_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(17) Let a be an element of $Boolean^Y$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is independent, then $\neg\forall_{\forall_{a,A}G,B}G = \exists_{\neg\forall_{a,B}G,A}G$.

(18) Let a be an element of $Boolean^Y$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is independent, then $\forall_{\neg\forall_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(19) Let a be an element of $Boolean^Y$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is independent, then $\neg\forall_{\forall_{a,A}G,B}G = \exists_{\exists_{\neg a,B}G,A}G$.

(20) Let a be an element of $Boolean^Y$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is independent, then $\neg\forall_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,A}G,B}G$.

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[1] The propositions (8)–(10) have been removed.

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