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Basic Concepts for Petri Nets with Boolean Markings

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Summary. Contains basic concepts for Petri nets with Boolean markings and the firability/firing of single transitions as well as sequences of transitions [6]. The concept of a Boolean marking is introduced as a mapping of a Boolean TRUE/FALSE to each of the places in a place/transition net. This simplifies the conventional definitions of the firability and firing of a transition. One note of caution in this article - the definition of firing a transition does not require that the transition be firable. Therefore, it is advisable to check that transitions ARE firable before firing them.

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WWW: http://mizar.org/JFM/Vol5/boolmark.html

The articles [10], [13], [1], [14], [3], [4], [9], [11], [8], [2], [12], [5], [15], and [7] provide the notation and terminology for this paper.

1. PRELIMINARIES

The following three propositions are true:

(1) Let A, B be non empty sets, f be a function from A into B, C be a subset of A, and v be an element of B. Then $f{+}\cdot(C \longmapsto v)$ is a function from A into B.

(2) Let X, Y be non empty sets, A, B be subsets of X, and f be a function from X into Y. If $f^\circ A$ misses $f^\circ B$, then A misses B.

(3) For all sets A, B and for every function f and for every set x such that A misses B holds $(f{+}\cdot(A \longmapsto x))^\circ B = f^\circ B$.

2. BOOLEAN MARKING AND FIRABILITY/FIRING OF TRANSITIONS

Let P_1 be a place/transition net structure. The functor Bool_marks_ofP_1 yields a non empty set of functions from the places of P_1 to *Boolean* and is defined as follows:

(Def. 1) $\text{Bool\_marks\_of}\,P_1 = \mathit{Boolean}^{\text{the places of } P_1}$.

Let P_1 be a place/transition net structure. A Boolean marking of P_1 is an element of Bool_marks_ofP_1.

Let P_1 be a place/transition net structure, let M_0 be a Boolean marking of P_1, and let t be a transition of P_1. We say that t is firable on M_0 if and only if:

(Def. 2) $M_0{}^\circ({}^*\{t\}) \subseteq \{\mathit{true}\}$.

Let P_1 be a place/transition net structure, let M_0 be a Boolean marking of P_1, and let t be a transition of P_1. The functor Firing(t, M_0) yields a Boolean marking of P_1 and is defined by:

(Def. 3) $\mathrm{Firing}(t,M_0) = M_0 {+\cdot} ({}^*\{t\} \longmapsto \mathit{false}) {+\cdot} (\overline{\{t\}} \longmapsto \mathit{true})$.

Let P_1 be a place/transition net structure, let M_0 be a Boolean marking of P_1, and let Q be a finite sequence of elements of the transitions of P_1. We say that Q is firable on M_0 if and only if the conditions (Def. 4) are satisfied.

(Def. 4)(i) $Q = \emptyset$, or

(ii) there exists a finite sequence M of elements of Bool_marks_ofP_1 such that $\mathrm{len}\,Q = \mathrm{len}\,M$ and Q_1 is firable on M_0 and $M_1 = \mathrm{Firing}(Q_1, M_0)$ and for every natural number i such that $i < \mathrm{len}\,Q$ and $i > 0$ holds Q_{i+1} is firable on M_i and $M_{i+1} = \mathrm{Firing}(Q_{i+1}, M_i)$.

Let P_1 be a place/transition net structure, let M_0 be a Boolean marking of P_1, and let Q be a finite sequence of elements of the transitions of P_1. The functor $\mathrm{Firing}(Q,M_0)$ yields a Boolean marking of P_1 and is defined as follows:

(Def. 5)(i) $\mathrm{Firing}(Q,M_0) = M_0$ if $Q = \emptyset$,

(ii) there exists a finite sequence M of elements of Bool_marks_ofP_1 such that $\mathrm{len}\,Q = \mathrm{len}\,M$ and $\mathrm{Firing}(Q,M_0) = M_{\mathrm{len}\,M}$ and $M_1 = \mathrm{Firing}(Q_1, M_0)$ and for every natural number i such that $i < \mathrm{len}\,Q$ and $i > 0$ holds $M_{i+1} = \mathrm{Firing}(Q_{i+1}, M_i)$, otherwise.

We now state several propositions:

(5)[1] For every non empty set A and for every set y and for every function f holds $(f {+\cdot} (A \longmapsto y))^{\circ}A = \{y\}$.

(6) Let P_1 be a place/transition net structure, M_0 be a Boolean marking of P_1, t be a transition of P_1, and s be a place of P_1. If $s \in \overline{\{t\}}$, then $(\mathrm{Firing}(t,M_0))(s) = \mathit{true}$.

(7) Let P_1 be a place/transition net structure and S_1 be a non empty subset of the places of P_1. Then S_1 is deadlock-like if and only if for every Boolean marking M_0 of P_1 such that $M_0{}^{\circ}S_1 = \{\mathit{false}\}$ and for every transition t of P_1 such that t is firable on M_0 holds $(\mathrm{Firing}(t,M_0))^{\circ}S_1 = \{\mathit{false}\}$.

(8) Let D be a non empty set, Q_0, Q_1 be finite sequences of elements of D, and i be a natural number. If $1 \leq i$ and $i \leq \mathrm{len}\,Q_0$, then $(Q_0 \frown Q_1)_i = (Q_0)_i$.

(10)[2] Let P_1 be a place/transition net structure, M_0 be a Boolean marking of P_1, and Q_0, Q_1 be finite sequences of elements of the transitions of P_1. Then $\mathrm{Firing}(Q_0 \frown Q_1, M_0) = \mathrm{Firing}(Q_1, \mathrm{Firing}(Q_0, M_0))$.

(11) Let P_1 be a place/transition net structure, M_0 be a Boolean marking of P_1, and Q_0, Q_1 be finite sequences of elements of the transitions of P_1. If $Q_0 \frown Q_1$ is firable on M_0, then Q_1 is firable on $\mathrm{Firing}(Q_0, M_0)$ and Q_0 is firable on M_0.

(12) Let P_1 be a place/transition net structure, M_0 be a Boolean marking of P_1, and t be a transition of P_1. Then t is firable on M_0 if and only if $\langle t \rangle$ is firable on M_0.

(13) Let P_1 be a place/transition net structure, M_0 be a Boolean marking of P_1, and t be a transition of P_1. Then $\mathrm{Firing}(t, M_0) = \mathrm{Firing}(\langle t \rangle, M_0)$.

(14) Let P_1 be a place/transition net structure and S_1 be a non empty subset of the places of P_1. Then S_1 is deadlock-like if and only if for every Boolean marking M_0 of P_1 such that $M_0{}^{\circ}S_1 = \{\mathit{false}\}$ and for every finite sequence Q of elements of the transitions of P_1 such that Q is firable on M_0 holds $(\mathrm{Firing}(Q,M_0))^{\circ}S_1 = \{\mathit{false}\}$.

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[1] The proposition (4) has been removed.
[2] The proposition (9) has been removed.

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