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Binary Arithmetics. Binary Sequences

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The articles [14], [8], [1], [11], [12], [15], [3], [5], [2], [13], [6], [4], [10], [9], and [7] provide the notation and terminology for this paper.

1. Binary Arithmetics

One can prove the following propositions:

(1) For every non empty natural number n and for every n-tuple F of *Boolean* holds $\mathrm{Absval}(F) < 2^n$.

(2) For every non empty natural number n and for all n-tuples F_1, F_2 of *Boolean* such that $\mathrm{Absval}(F_1) = \mathrm{Absval}(F_2)$ holds $F_1 = F_2$.

(3) For all finite sequences t_1, t_2 such that $\mathrm{Rev}(t_1) = \mathrm{Rev}(t_2)$ holds $t_1 = t_2$.

(4) For every natural number n holds $\underbrace{\langle 0, \ldots, 0 \rangle}_{n+1} = \underbrace{\langle 0, \ldots, 0 \rangle}_{n} \frown \langle 0 \rangle$.

(5) For every natural number n holds $\underbrace{\langle 0, \ldots, 0 \rangle}_{n} \in \mathit{Boolean}^*$.

(6) For every natural number n and for every n-tuple y of *Boolean* such that $y = \underbrace{\langle 0, \ldots, 0 \rangle}_{n}$ holds $\neg y = n \mapsto 1$.

(7) For every non empty natural number n and for every n-tuple F of *Boolean* such that $F = \underbrace{\langle 0, \ldots, 0 \rangle}_{n}$ holds $\mathrm{Absval}(F) = 0$.

(8) For every non empty natural number n and for every n-tuple F of *Boolean* such that $F = \underbrace{\langle 0, \ldots, 0 \rangle}_{n}$ holds $\mathrm{Absval}(\neg F) = 2^n - 1$.

(9) For every natural number n holds $\mathrm{Rev}(\underbrace{\langle 0, \ldots, 0 \rangle}_{n}) = \underbrace{\langle 0, \ldots, 0 \rangle}_{n}$.

(10) For every natural number n and for every n-tuple y of *Boolean* such that $y = \underbrace{\langle 0, \ldots, 0 \rangle}_{n}$ holds $\mathrm{Rev}(\neg y) = \neg y$.

(11) $\mathrm{Bin1}(1) = \langle \mathit{true} \rangle$.

(12) For every non empty natural number n holds $\mathrm{Absval}(\mathrm{Bin1}(n)) = 1$.

(13) For all elements x, y of *Boolean* holds $x \lor y = \mathit{true}$ iff $x = \mathit{true}$ or $y = \mathit{true}$ and $x \lor y = \mathit{false}$ iff $x = \mathit{false}$ and $y = \mathit{false}$.

(14) For all elements x, y of *Boolean* holds $\mathrm{add\_ovfl}(\langle x \rangle, \langle y \rangle) = \mathit{true}$ iff $x = \mathit{true}$ and $y = \mathit{true}$.

(15) $\neg \langle \mathit{false} \rangle = \langle \mathit{true} \rangle$.

(16) $\neg \langle \mathit{true} \rangle = \langle \mathit{false} \rangle$.

(17) $\langle \mathit{false} \rangle + \langle \mathit{false} \rangle = \langle \mathit{false} \rangle$.

(18) $\langle \mathit{false} \rangle + \langle \mathit{true} \rangle = \langle \mathit{true} \rangle$ and $\langle \mathit{true} \rangle + \langle \mathit{false} \rangle = \langle \mathit{true} \rangle$.

(19) $\langle \mathit{true} \rangle + \langle \mathit{true} \rangle = \langle \mathit{false} \rangle$.

(20) Let n be a non empty natural number and x, y be n-tuples of *Boolean*. Suppose $x_n = \mathit{true}$ and $(\mathrm{carry}(x, \mathrm{Bin1}(n)))_n = \mathit{true}$. Let k be a non empty natural number. If $k \neq 1$ and $k \leq n$, then $x_k = \mathit{true}$ and $(\mathrm{carry}(x, \mathrm{Bin1}(n)))_k = \mathit{true}$.

(21) For every non empty natural number n and for every n-tuple x of *Boolean* such that $x_n = \mathit{true}$ and $(\mathrm{carry}(x, \mathrm{Bin1}(n)))_n = \mathit{true}$ holds $\mathrm{carry}(x, \mathrm{Bin1}(n)) = \neg \mathrm{Bin1}(n)$.

(22) Let n be a non empty natural number and x, y be n-tuples of *Boolean*. If $y = \langle \underbrace{0, \ldots, 0}_{n} \rangle$ and $x_n = \mathit{true}$ and $(\mathrm{carry}(x, \mathrm{Bin1}(n)))_n = \mathit{true}$, then $x = \neg y$.

(23) For every non empty natural number n and for every n-tuple y of *Boolean* such that $y = \langle \underbrace{0, \ldots, 0}_{n} \rangle$ holds $\mathrm{carry}(\neg y, \mathrm{Bin1}(n)) = \neg \mathrm{Bin1}(n)$.

(24) Let n be a non empty natural number and x, y be n-tuples of *Boolean*. If $y = \langle \underbrace{0, \ldots, 0}_{n} \rangle$, then $\mathrm{add\_ovfl}(x, \mathrm{Bin1}(n)) = \mathit{true}$ iff $x = \neg y$.

(25) For every non empty natural number n and for every n-tuple z of *Boolean* such that $z = \langle \underbrace{0, \ldots, 0}_{n} \rangle$ holds $\neg z + \mathrm{Bin1}(n) = z$.

2. Binary Sequences

Let n, k be natural numbers. The functor n-BinarySequence(k) yields a n-tuple of *Boolean* and is defined as follows:

(Def. 1) For every natural number i such that $i \in \mathrm{Seg}\, n$ holds $(n\text{-BinarySequence}(k))_i = ((k \div 2^{i -' 1}) \bmod 2 = 0 \rightarrow \mathit{false}, \mathit{true})$.

The following propositions are true:

(26) For every natural number n holds n-BinarySequence$(0) = \langle \underbrace{0, \ldots, 0}_{n} \rangle$.

(27) For all natural numbers n, k such that $k < 2^n$ holds $((n+1)\text{-BinarySequence}(k))(n+1) = \mathit{false}$.

(28) For every non empty natural number n and for every natural number k such that $k < 2^n$ holds $(n+1)\text{-BinarySequence}(k) = (n\text{-BinarySequence}(k)) \frown \langle \mathit{false} \rangle$.

(29) For every non empty natural number n holds $(n+1)$-BinarySequence$(2^n) = \langle\underbrace{0,\ldots,0}_{n}\rangle \frown \langle true\rangle$.

(30) For every non empty natural number n and for every natural number k such that $2^n \leq k$ and $k < 2^{n+1}$ holds $((n+1)$-BinarySequence$(k))(n+1) = true$.

(31) Let n be a non empty natural number and k be a natural number. If $2^n \leq k$ and $k < 2^{n+1}$, then $(n+1)$-BinarySequence$(k) = (n$-BinarySequence$(k -' 2^n)) \frown \langle true\rangle$.

(32) Let n be a non empty natural number and k be a natural number. Suppose $k < 2^n$. Let x be a n-tuple of *Boolean*. If $x = \langle\underbrace{0,\ldots,0}_{n}\rangle$, then n-BinarySequence$(k) = \neg x$ iff $k = 2^n - 1$.

(33) For every non empty natural number n and for every natural number k such that $k+1 < 2^n$ holds add_ovfl$(n$-BinarySequence$(k), \mathrm{Bin1}(n)) = false$.

(34) For every non empty natural number n and for every natural number k such that $k+1 < 2^n$ holds n-BinarySequence$(k+1) = (n$-BinarySequence$(k)) + \mathrm{Bin1}(n)$.

(35) For all natural numbers n, i holds $(n+1)$-BinarySequence$(i) = \langle i \bmod 2\rangle \frown (n$-BinarySequence$(i \div 2))$.

(36) For every non empty natural number n and for every natural number k such that $k < 2^n$ holds Absval$(n$-BinarySequence$(k)) = k$.

(37) For every non empty natural number n and for every n-tuple x of *Boolean* holds n-BinarySequence$(\mathrm{Absval}(x)) = x$.

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