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Affine Localizations of Desargues Axiom[1]

Eugeniusz Kusak
Warsaw University
Białystok

Henryk Oryszczyszyn
Warsaw University
Białystok

Krzysztof Prażmowski
Warsaw University
Białystok

Summary. Several affine localizations of Major Desargues Axiom together with its indirect forms are introduced. Logical relationships between these formulas and between them and the classical Desargues Axiom are demonstrated.

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The articles [1], [3], [4], and [2] provide the notation and terminology for this paper.

We follow the rules: A_1 denotes an affine plane, a, a', b, b', c, c', o, p, q denote elements of A_1, and A, C, P denote subsets of A_1.

Let us consider A_1. We say that A_1 satisfies **DES1** if and only if the condition (Def. 1) is satisfied.

(Def. 1) Let given A, P, C, o, a, a', b, b', c, c', p, q. Suppose that A is a line and P is a line and C is a line and $P \neq A$ and $P \neq C$ and $A \neq C$ and $o \in A$ and $a \in A$ and $a' \in A$ and $o \in P$ and $b \in P$ and $b' \in P$ and $o \in C$ and $c \in C$ and $c' \in C$ and $o \neq a$ and $o \neq b$ and $o \neq c$ and $p \neq q$ and not $\mathbf{L}(b,a,c)$ and not $\mathbf{L}(b',a',c')$ and $a \neq a'$ and $\mathbf{L}(b,a,p)$ and $\mathbf{L}(b',a',p)$ and $\mathbf{L}(b,c,q)$ and $\mathbf{L}(b',c',q)$ and $a,c \upuparrows a',c'$. Then $a,c \upuparrows p,q$.

We introduce A_1 satisfies **DES1** as a synonym of A_1 satisfies **DES1**.

Let us consider A_1. We say that A_1 satisfies $\mathbf{DES1}_1$ if and only if the condition (Def. 2) is satisfied.

(Def. 2) Let given A, P, C, o, a, a', b, b', c, c', p, q. Suppose that A is a line and P is a line and C is a line and $P \neq A$ and $P \neq C$ and $A \neq C$ and $o \in A$ and $a \in A$ and $a' \in A$ and $o \in P$ and $b \in P$ and $b' \in P$ and $o \in C$ and $c \in C$ and $c' \in C$ and $o \neq a$ and $o \neq b$ and $o \neq c$ and $p \neq q$ and $c \neq q$ and not $\mathbf{L}(b,a,c)$ and not $\mathbf{L}(b',a',c')$ and $\mathbf{L}(b,a,p)$ and $\mathbf{L}(b',a',p)$ and $\mathbf{L}(b,c,q)$ and $\mathbf{L}(b',c',q)$ and $a,c \upuparrows p,q$. Then $a,c \upuparrows a',c'$.

We introduce A_1 satisfies $\mathbf{DES1}_1$ as a synonym of A_1 satisfies $\mathbf{DES1}_1$.

Let us consider A_1. We say that A_1 satisfies $\mathbf{DES1}_2$ if and only if the condition (Def. 3) is satisfied.

(Def. 3) Let given A, P, C, o, a, a', b, b', c, c', p, q. Suppose that A is a line and P is a line and C is a line and $P \neq A$ and $P \neq C$ and $A \neq C$ and $o \in A$ and $a \in A$ and $a' \in A$ and $o \in P$ and $b \in P$ and $b' \in P$ and $c \in C$ and $c' \in C$ and $o \neq a$ and $o \neq b$ and $o \neq c$ and $p \neq q$ and not $\mathbf{L}(b,a,c)$ and not $\mathbf{L}(b',a',c')$ and $c \neq c'$ and $\mathbf{L}(b,a,p)$ and $\mathbf{L}(b',a',p)$ and $\mathbf{L}(b,c,q)$ and $\mathbf{L}(b',c',q)$ and $a,c \upuparrows a',c'$ and $a,c \upuparrows p,q$. Then $o \in C$.

We introduce A_1 satisfies $\mathbf{DES1}_2$ as a synonym of A_1 satisfies $\mathbf{DES1}_2$.

Let us consider A_1. We say that A_1 satisfies $\mathbf{DES1}_3$ if and only if the condition (Def. 4) is satisfied.

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(Def. 4) Let given A, P, C, o, a, a', b, b', c, c', p, q. Suppose that A is a line and P is a line and C is a line and $P \neq A$ and $P \neq C$ and $A \neq C$ and $o \in A$ and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $o \in C$ and $c \in C$ and $c' \in C$ and $o \neq a$ and $o \neq b$ and $o \neq c$ and $p \neq q$ and not $\mathbf{L}(b,a,c)$ and not $\mathbf{L}(b',a',c')$ and $b \neq b'$ and $a \neq a'$ and $\mathbf{L}(b,a,p)$ and $\mathbf{L}(b',a',p)$ and $\mathbf{L}(b,c,q)$ and $\mathbf{L}(b',c',q)$ and $a,c \downharpoonleft\!\upharpoonright a',c'$ and $a,c \downharpoonleft\!\upharpoonright p,q$. Then $o \in P$.

We introduce A_1 satisfies $\mathbf{DES1}_3$ as a synonym of A_1 satisfies $\mathbf{DES1}_3$.

Let us consider A_1. We say that A_1 satisfies **DES2** if and only if the condition (Def. 5) is satisfied.

(Def. 5) Let given A, P, C, a, a', b, b', c, c', p, q. Suppose that A is a line and P is a line and C is a line and $A \neq P$ and $A \neq C$ and $P \neq C$ and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $c \in C$ and $c' \in C$ and $A \mathbin{/\!/} P$ and $A \mathbin{/\!/} C$ and not $\mathbf{L}(b,a,c)$ and not $\mathbf{L}(b',a',c')$ and $p \neq q$ and $a \neq a'$ and $\mathbf{L}(b,a,p)$ and $\mathbf{L}(b',a',p)$ and $\mathbf{L}(b,c,q)$ and $\mathbf{L}(b',c',q)$ and $a,c \downharpoonleft\!\upharpoonright a',c'$. Then $a,c \downharpoonleft\!\upharpoonright p,q$.

We introduce A_1 satisfies **DES2** as a synonym of A_1 satisfies **DES2**.

Let us consider A_1. We say that A_1 satisfies $\mathbf{DES2}_1$ if and only if the condition (Def. 6) is satisfied.

(Def. 6) Let given A, P, C, a, a', b, b', c, c', p, q. Suppose that A is a line and P is a line and C is a line and $A \neq P$ and $A \neq C$ and $P \neq C$ and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $c \in C$ and $c' \in C$ and $A \mathbin{/\!/} P$ and $A \mathbin{/\!/} C$ and not $\mathbf{L}(b,a,c)$ and not $\mathbf{L}(b',a',c')$ and $p \neq q$ and $\mathbf{L}(b,a,p)$ and $\mathbf{L}(b',a',p)$ and $\mathbf{L}(b,c,q)$ and $\mathbf{L}(b',c',q)$ and $a,c \downharpoonleft\!\upharpoonright p,q$. Then $a,c \downharpoonleft\!\upharpoonright a',c'$.

We introduce A_1 satisfies $\mathbf{DES2}_1$ as a synonym of A_1 satisfies $\mathbf{DES2}_1$.

Let us consider A_1. We say that A_1 satisfies $\mathbf{DES2}_2$ if and only if the condition (Def. 7) is satisfied.

(Def. 7) Let given A, P, C, a, a', b, b', c, c', p, q. Suppose that A is a line and P is a line and C is a line and $A \neq P$ and $A \neq C$ and $P \neq C$ and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $c \in C$ and $c' \in C$ and $A \mathbin{/\!/} C$ and not $\mathbf{L}(b,a,c)$ and not $\mathbf{L}(b',a',c')$ and $p \neq q$ and $a \neq a'$ and $\mathbf{L}(b,a,p)$ and $\mathbf{L}(b',a',p)$ and $\mathbf{L}(b,c,q)$ and $\mathbf{L}(b',c',q)$ and $a,c \downharpoonleft\!\upharpoonright a',c'$ and $a,c \downharpoonleft\!\upharpoonright p,q$. Then $A \mathbin{/\!/} P$.

We introduce A_1 satisfies $\mathbf{DES2}_2$ as a synonym of A_1 satisfies $\mathbf{DES2}_2$.

Let us consider A_1. We say that A_1 satisfies $\mathbf{DES2}_3$ if and only if the condition (Def. 8) is satisfied.

(Def. 8) Let given A, P, C, a, a', b, b', c, c', p, q. Suppose that A is a line and P is a line and C is a line and $A \neq P$ and $A \neq C$ and $P \neq C$ and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $c \in C$ and $c' \in C$ and $A \mathbin{/\!/} P$ and not $\mathbf{L}(b,a,c)$ and not $\mathbf{L}(b',a',c')$ and $p \neq q$ and $c \neq c'$ and $\mathbf{L}(b,a,p)$ and $\mathbf{L}(b',a',p)$ and $\mathbf{L}(b,c,q)$ and $\mathbf{L}(b',c',q)$ and $a,c \downharpoonleft\!\upharpoonright a',c'$ and $a,c \downharpoonleft\!\upharpoonright p,q$. Then $A \mathbin{/\!/} C$.

We introduce A_1 satisfies $\mathbf{DES2}_3$ as a synonym of A_1 satisfies $\mathbf{DES2}_3$.

Next we state a number of propositions:

(9)[1] If A_1 satisfies **DES1**, then A_1 satisfies $\mathbf{DES1}_1$.

(10) If A_1 satisfies $\mathbf{DES1}_1$, then A_1 satisfies **DES1**.

(11) If A_1 satisfies **DES**, then A_1 satisfies **DES1**.

(12) If A_1 satisfies **DES**, then A_1 satisfies $\mathbf{DES1}_2$.

(13) If A_1 satisfies $\mathbf{DES1}_2$, then A_1 satisfies $\mathbf{DES1}_3$.

(14) If A_1 satisfies $\mathbf{DES1}_2$, then A_1 satisfies **DES**.

(15) If A_1 satisfies $\mathbf{DES2}_1$, then A_1 satisfies **DES2**.

(16) A_1 satisfies $\mathbf{DES2}_1$ iff A_1 satisfies $\mathbf{DES2}_3$.

(17) A_1 satisfies **DES2** iff A_1 satisfies $\mathbf{DES2}_2$.

(18) If A_1 satisfies $\mathbf{DES1}_3$, then A_1 satisfies $\mathbf{DES2}_1$.

[1] The propositions (1)–(8) have been removed.

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