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# Some Properties of Functions Modul and Signum

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**Summary.** The article includes definitions and theorems concerning basic properties of the following functions: $|x|$ – modul of real number, sgn $x$ – signum of real number.

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WWW: `http://mizar.org/JFM/Vol1/absvalue.html`

The articles [1] and [2] provide the notation and terminology for this paper.

In this paper $x$, $y$, $z$, $t$ are real numbers.

Let us consider $x$. The functor $|x|$ yielding a real number is defined by:

(Def. 1) $|x| = \begin{cases} x, & \text{if } 0 \leq x, \\ -x, & \text{otherwise.} \end{cases}$

Let us notice that the functor $|x|$ is projective.

Let $x$ be a real number. Then $|x|$ is a real number.

We now state a number of propositions:

(5)[1] $0 \leq |x|$.

(6) If $x \neq 0$, then $0 < |x|$.

(7) $x = 0$ iff $|x| = 0$.

(9)[2] If $|x| = -x$ and $x \neq 0$, then $x < 0$.

(10) $|x \cdot y| = |x| \cdot |y|$.

(11) $-|x| \leq x$ and $x \leq |x|$.

(12) $-y \leq x$ and $x \leq y$ iff $|x| \leq y$.

(13) $|x+y| \leq |x| + |y|$.

(14) If $x \neq 0$, then $|x| \cdot |\frac{1}{x}| = 1$.

(15) $|\frac{1}{x}| = \frac{1}{|x|}$.

(16) $|\frac{x}{y}| = \frac{|x|}{|y|}$.

[1] The propositions (1)–(4) have been removed.
[2] The proposition (8) has been removed.

(17) $|x| = |-x|$.

(18) $|x| - |y| \leq |x - y|$.

(19) $|x - y| \leq |x| + |y|$.

(21)[3] If $|x| \leq z$ and $|y| \leq t$, then $|x + y| \leq z + t$.

(22) $||x| - |y|| \leq |x - y|$.

(24)[4] If $0 \leq x \cdot y$, then $|x + y| = |x| + |y|$.

(25) If $|x + y| = |x| + |y|$, then $0 \leq x \cdot y$.

(26) $\frac{|x+y|}{1+|x+y|} \leq \frac{|x|}{1+|x|} + \frac{|y|}{1+|y|}$.

Let us consider $x$. The functor $\operatorname{sgn} x$ is defined by:

(Def. 2) $$\operatorname{sgn} x = \begin{cases} 1, \text{ if } 0 < x, \\ -1, \text{ if } x < 0, \\ 0, \text{ otherwise.} \end{cases}$$

Let us consider $x$. One can check that $\operatorname{sgn} x$ is real.
Let $x$ be a real number. Then $\operatorname{sgn} x$ is a real number.
We now state a number of propositions:

(31)[5] If $\operatorname{sgn} x = 1$, then $0 < x$.

(32) If $\operatorname{sgn} x = -1$, then $x < 0$.

(33) If $\operatorname{sgn} x = 0$, then $x = 0$.

(34) $x = |x| \cdot \operatorname{sgn} x$.

(35) $\operatorname{sgn}(x \cdot y) = \operatorname{sgn} x \cdot \operatorname{sgn} y$.

(36) $\operatorname{sgn} \operatorname{sgn} x = \operatorname{sgn} x$.

(37) $\operatorname{sgn}(x + y) \leq \operatorname{sgn} x + \operatorname{sgn} y + 1$.

(38) If $x \neq 0$, then $\operatorname{sgn} x \cdot \operatorname{sgn}(\frac{1}{x}) = 1$.

(39) $\frac{1}{\operatorname{sgn} x} = \operatorname{sgn}(\frac{1}{x})$.

(40) $(\operatorname{sgn} x + \operatorname{sgn} y) - 1 \leq \operatorname{sgn}(x + y)$.

(41) $\operatorname{sgn} x = \operatorname{sgn}(\frac{1}{x})$.

(42) $\operatorname{sgn}(\frac{x}{y}) = \frac{\operatorname{sgn} x}{\operatorname{sgn} y}$.

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[3] The proposition (20) has been removed.
[4] The proposition (23) has been removed.
[5] The propositions (27)–(30) have been removed.

## References

[1] Grzegorz Bancerek. The ordinal numbers. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/ordinal1.html`.

[2] Krzysztof Hryniewiecki. Basic properties of real numbers. *Journal of Formalized Mathematics*, 1, 1989. `http://mizar.org/JFM/Vol1/real_1.html`.

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