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Moore-Smith Convergence[1]

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Summary. The paper introduces the concept of a net (a generalized sequence). The goal is to enable the continuation of the translation of [14].

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The articles [23], [12], [27], [24], [28], [29], [10], [11], [9], [1], [3], [19], [2], [4], [20], [17], [13], [21], [25], [18], [26], [5], [22], [6], [7], [15], [16], and [8] provide the notation and terminology for this paper.

1. PRELIMINARIES

The scheme *SubsetEq* deals with a non empty set $\mathcal{A}$, subsets $\mathcal{B}$, $\mathcal{C}$ of $\mathcal{A}$, and a unary predicate $\mathcal{P}$, and states that:

$\mathcal{B} = \mathcal{C}$

provided the following requirements are met:

- For every element y of $\mathcal{A}$ holds $y \in \mathcal{B}$ iff $\mathcal{P}[y]$, and
- For every element y of $\mathcal{A}$ holds $y \in \mathcal{C}$ iff $\mathcal{P}[y]$.

Let f be a function. Let us assume that f is non empty and constant. The value of f is defined as follows:

(Def. 1) There exists a set x such that $x \in \mathrm{dom}\, f$ and the value of $f = f(x)$.

Let us mention that there exists a function which is non empty and constant.

The following propositions are true:

(2)[1] For every non empty set X and for every set x holds the value of $X \longmapsto x = x$.

(3) For every function f holds $\overline{\overline{\mathrm{rng}\, f}} \subseteq \overline{\overline{\mathrm{dom}\, f}}$.

Let us note that every set which is universal is also transitive and a Tarski class and every set which is transitive and a Tarski class is also universal.

In the sequel x, X denote sets and T denotes a universal class.

Let us consider X. The universe of X is defined by:

(Def. 3)[2] The universe of $X = \mathbf{T}(X^{*\in})$.

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[1] The proposition (1) has been removed.
[2] The definition (Def. 2) has been removed.

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Let us consider X. One can verify that the universe of X is transitive and a Tarski class.
Let us consider X. One can check that the universe of X is universal and non empty.
The following proposition is true

(5)[3] For every function f such that $\operatorname{dom} f \in T$ and $\operatorname{rng} f \subseteq T$ holds $\prod f \in T$.

2. Topological spaces

Next we state the proposition

(6) Let T be a non empty topological space, A be a subset of T, and p be a point of T. Then $p \in \overline{A}$ if and only if for every neighbourhood G of p holds G meets A.

Let T be a non empty topological space. We introduce T is Hausdorff as a synonym of T is T_2.
One can check that there exists a non empty topological space which is Hausdorff.
Next we state two propositions:

(7) For every non empty topological space X and for every subset A of X holds Ω_X is a neighbourhood of A.

(8) For every non empty topological space X and for every subset A of X and for every neighbourhood Y of A holds $A \subseteq Y$.

3. 1-sorted structures

The following proposition is true

(9) Let Y be a non empty set, J be a 1-sorted yielding many sorted set indexed by Y, and i be an element of Y. Then (the support of $J)(i) =$ the carrier of $J(i)$.

One can check that there exists a function which is non empty, constant, and 1-sorted yielding.
Let J be a 1-sorted yielding function. Let us observe that J is nonempty if and only if:

(Def. 4) For every set i such that $i \in \operatorname{rng} J$ holds i is a non empty 1-sorted structure.

We introduce J is yielding non-empty carriers as a synonym of J is nonempty.

Let X be a set and let L be a 1-sorted structure. Observe that $X \longmapsto L$ is 1-sorted yielding.

Let I be a set. Observe that there exists a 1-sorted yielding many sorted set indexed by I which is yielding non-empty carriers.

Let I be a non empty set and let J be a relational structure yielding many sorted set indexed by I. Note that the carrier of $\prod J$ is functional.

Let I be a set and let J be a yielding non-empty carriers 1-sorted yielding many sorted set indexed by I. Note that the support of J is non-empty.

The following proposition is true

(10) Let T be a non empty 1-sorted structure, S be a subset of T, and p be an element of T. Then $p \notin S$ if and only if $p \in S^{\rm c}$.

4. Relational structures

Let T be a non empty relational structure and let A be a lower subset of T. One can check that $A^{\rm c}$ is upper.

Let T be a non empty relational structure and let A be an upper subset of T. Observe that $A^{\rm c}$ is lower.

Let N be a non empty relational structure. Let us observe that N is directed if and only if:

(Def. 5) For all elements x, y of N there exists an element z of N such that $x \leq z$ and $y \leq z$.

[3] The proposition (4) has been removed.

Let X be a set. Note that $2^X_\subseteq$ is directed.

One can verify that there exists a relational structure which is non empty, directed, transitive, and strict.

Let M be a non empty set, let N be a non empty relational structure, let f be a function from M into the carrier of N, and let m be an element of M. Then $f(m)$ is an element of N.

Let I be a set. Note that there exists a relational structure yielding many sorted set indexed by I which is yielding non-empty carriers.

Let I be a non empty set and let J be a yielding non-empty carriers relational structure yielding many sorted set indexed by I. Note that $\prod J$ is non empty.

One can prove the following proposition

(11) For all relational structures R_1, R_2 holds $\Omega_{[: R_1, R_2 :]} = [: \Omega_{(R_1)}, \Omega_{(R_2)} :]$.

Let Y_1, Y_2 be directed relational structures. Observe that $[: Y_1, Y_2 :]$ is directed.

Next we state the proposition

(12) For every relational structure R holds the carrier of $R =$ the carrier of $R^\smile$.

Let S be a 1-sorted structure and let N be a net structure over S. We say that N is constant if and only if:

(Def. 6) The mapping of N is constant.

Let R be a relational structure, let T be a non empty 1-sorted structure, and let p be an element of T. The functor $R \longmapsto p$ yields a strict net structure over T and is defined by the conditions (Def. 7).

(Def. 7)(i) The relational structure of $(R \longmapsto p) =$ the relational structure of R, and

(ii) the mapping of $(R \longmapsto p) =$ (the carrier of $(R \longmapsto p)) \longmapsto p$.

Let R be a relational structure, let T be a non empty 1-sorted structure, and let p be an element of T. Observe that $R \longmapsto p$ is constant.

Let R be a non empty relational structure, let T be a non empty 1-sorted structure, and let p be an element of T. Observe that $R \longmapsto p$ is non empty.

Let R be a non empty directed relational structure, let T be a non empty 1-sorted structure, and let p be an element of T. Observe that $R \longmapsto p$ is directed.

Let R be a non empty transitive relational structure, let T be a non empty 1-sorted structure, and let p be an element of T. Observe that $R \longmapsto p$ is transitive.

We now state two propositions:

(13) Let R be a relational structure, T be a non empty 1-sorted structure, and p be an element of T. Then the carrier of $(R \longmapsto p) =$ the carrier of R.

(14) Let R be a non empty relational structure, T be a non empty 1-sorted structure, p be an element of T, and q be an element of $R \longmapsto p$. Then $(R \longmapsto p)(q) = p$.

Let T be a non empty 1-sorted structure and let N be a non empty net structure over T. Note that the mapping of N is non empty.

5. SUBSTRUCTURES OF NETS

One can prove the following propositions:

(15) Every relational structure R is a full relational substructure of R.

(16) Let R be a relational structure and S be a relational substructure of R. Then every relational substructure of S is a relational substructure of R.

Let S be a 1-sorted structure and let N be a net structure over S. A net structure over S is said to be a structure of a subnet of N if:

(Def. 8) It is a relational substructure of N and the mapping of it $=$ (the mapping of N)$\restriction$(the carrier of it).

One can prove the following two propositions:

(17) For every 1-sorted structure S holds every net structure N over S is a structure of a subnet of N.

(18) Let Q be a 1-sorted structure, R be a net structure over Q, and S be a structure of a subnet of R. Then every structure of a subnet of S is a structure of a subnet of R.

Let S be a 1-sorted structure, let N be a net structure over S, and let M be a structure of a subnet of N. We say that M is full if and only if:

(Def. 9) M is a full relational substructure of N.

Let S be a 1-sorted structure and let N be a net structure over S. Note that there exists a structure of a subnet of N which is full and strict.

Let S be a 1-sorted structure and let N be a non empty net structure over S. Note that there exists a structure of a subnet of N which is full, non empty, and strict.

We now state three propositions:

(19) Let S be a 1-sorted structure, N be a net structure over S, and M be a structure of a subnet of N. Then the carrier of $M \subseteq$ the carrier of N.

(20) Let S be a 1-sorted structure, N be a net structure over S, M be a structure of a subnet of N, x, y be elements of N, and i, j be elements of M. If $x = i$ and $y = j$ and $i \leq j$, then $x \leq y$.

(21) Let S be a 1-sorted structure, N be a non empty net structure over S, M be a non empty full structure of a subnet of N, x, y be elements of N, and i, j be elements of M. If $x = i$ and $y = j$ and $x \leq y$, then $i \leq j$.

6. More about nets

Let T be a non empty 1-sorted structure. Observe that there exists a net in T which is constant and strict.

Let T be a non empty 1-sorted structure and let N be a constant net structure over T. Observe that the mapping of N is constant.

Let T be a non empty 1-sorted structure and let N be a net structure over T. Let us assume that N is constant and non empty. The value of N yielding an element of T is defined by:

(Def. 10) The value of $N =$ the value of the mapping of N.

We now state the proposition

(22) Let R be a non empty relational structure, T be a non empty 1-sorted structure, and p be an element of T. Then the value of $R \longmapsto p = p$.

Let T be a non empty 1-sorted structure and let N be a net in T. A net in T is called a subnet of N if it satisfies the condition (Def. 12).

(Def. 12)[4] There exists a map f from it into N such that

(i) the mapping of it $=$ (the mapping of N) $\cdot f$, and

(ii) for every element m of N there exists an element n of it such that for every element p of it such that $n \leq p$ holds $m \leq f(p)$.

The following propositions are true:

[4] The definition (Def. 11) has been removed.

(23) For every non empty 1-sorted structure T holds every net N in T is a subnet of N.

(24) Let T be a non empty 1-sorted structure and N_1, N_2, N_3 be nets in T. Suppose N_1 is a subnet of N_2 and N_2 is a subnet of N_3. Then N_1 is a subnet of N_3.

(25) Let T be a non empty 1-sorted structure, N be a constant net in T, and i be an element of N. Then $N(i) =$ the value of N.

(26) Let L be a non empty 1-sorted structure, N be a net in L, and X, Y be sets. If N is eventually in X and eventually in Y, then X meets Y.

(27) Let S be a non empty 1-sorted structure, N be a net in S, M be a subnet of N, and given X. If M is often in X, then N is often in X.

(28) Let S be a non empty 1-sorted structure, N be a net in S, and given X. If N is eventually in X, then N is often in X.

(29) For every non empty 1-sorted structure S holds every net in S is eventually in the carrier of S.

7. The restriction of a net

Let S be a 1-sorted structure, let N be a net structure over S, and let us consider X. The functor $N^{-1}(X)$ yields a strict structure of a subnet of N and is defined by:

(Def. 13) $N^{-1}(X)$ is a full relational substructure of N and the carrier of $N^{-1}(X) =$ (the mapping of $N)^{-1}(X)$.

Let S be a 1-sorted structure, let N be a transitive net structure over S, and let us consider X. One can verify that $N^{-1}(X)$ is transitive and full.

Next we state three propositions:

(30) Let S be a non empty 1-sorted structure, N be a net in S, and given X. If N is often in X, then $N^{-1}(X)$ is non empty and directed.

(31) Let S be a non empty 1-sorted structure, N be a net in S, and given X. If N is often in X, then $N^{-1}(X)$ is a subnet of N.

(32) Let S be a non empty 1-sorted structure, N be a net in S, given X, and M be a subnet of N. If $M = N^{-1}(X)$, then M is eventually in X.

8. The universe of nets

Let X be a non empty 1-sorted structure. The functor $\mathrm{NetUniv}(X)$ is defined by the condition (Def. 14).

(Def. 14) Let given x. Then $x \in \mathrm{NetUniv}(X)$ if and only if there exists a strict net N in X such that $N = x$ and the carrier of $N \in$ the universe of the carrier of X.

Let X be a non empty 1-sorted structure. One can verify that $\mathrm{NetUniv}(X)$ is non empty.

9. Parametrized families of nets, iteration

Let X be a set and let T be a 1-sorted structure. A many sorted set indexed by X is said to be a net set of X, T if:

(Def. 15) For every set i such that $i \in \mathrm{rng}\,\mathrm{it}$ holds i is a net in T.

We now state the proposition

(33) Let X be a set, T be a 1-sorted structure, and F be a many sorted set indexed by X. Then F is a net set of X, T if and only if for every set i such that $i \in X$ holds $F(i)$ is a net in T.

Let X be a non empty set, let T be a 1-sorted structure, let J be a net set of X, T, and let i be an element of X. Then $J(i)$ is a net in T.

Let X be a set and let T be a 1-sorted structure. Observe that every net set of X, T is relational structure yielding.

Let T be a 1-sorted structure and let Y be a net in T. One can verify that every net set of the carrier of Y, T is yielding non-empty carriers.

Let T be a non empty 1-sorted structure, let Y be a net in T, and let J be a net set of the carrier of Y, T. One can verify that $\prod J$ is directed and transitive.

Let X be a set and let T be a 1-sorted structure. Note that every net set of X, T is yielding non-empty carriers.

Let X be a set and let T be a 1-sorted structure. Note that there exists a net set of X, T which is yielding non-empty carriers.

Let T be a non empty 1-sorted structure, let Y be a net in T, and let J be a net set of the carrier of Y, T. The functor Iterated(J) yields a strict net in T and is defined by the conditions (Def. 16).

(Def. 16)(i) The relational structure of Iterated$(J) = [: Y, \prod J :]$, and

(ii) for every element i of Y and for every function f such that $i \in$ the carrier of Y and $f \in$ the carrier of $\prod J$ holds (the mapping of Iterated$(J))(i, f) =$ (the mapping of $J(i))(f(i))$.

We now state four propositions:

(34) Let T be a non empty 1-sorted structure, Y be a net in T, and J be a net set of the carrier of Y, T. If $Y \in$ NetUniv(T) and for every element i of Y holds $J(i) \in$ NetUniv(T), then Iterated$(J) \in$ NetUniv(T).

(35) Let T be a non empty 1-sorted structure, N be a net in T, and J be a net set of the carrier of N, T. Then the carrier of Iterated$(J) = [:$ the carrier of N, $\prod$(the support of J)$:]$.

(36) Let T be a non empty 1-sorted structure, N be a net in T, J be a net set of the carrier of N, T, i be an element of N, f be an element of $\prod J$, and x be an element of Iterated(J). If $x = \langle i, f \rangle$, then (Iterated$(J))(x) =$ (the mapping of $J(i))(f(i))$.

(37) Let T be a non empty 1-sorted structure, Y be a net in T, and J be a net set of the carrier of Y, T. Then rng(the mapping of Iterated$(J)) \subseteq \bigcup\{$rng(the mapping of $J(i))$: i ranges over elements of $Y\}$.

10. Poset of Open Neighbourhoods

Let T be a non empty topological space and let p be a point of T. The open neighbourhoods of p constitute a relational structure defined as follows:

(Def. 17) The open neighbourhoods of $p = (\langle\{V; V$ ranges over subsets of T: $p \in V \wedge V$is open$\}, \subseteq \rangle)^{\smile}$.

Let T be a non empty topological space and let p be a point of T. Note that the open neighbourhoods of p is non empty.

We now state three propositions:

(38) Let T be a non empty topological space, p be a point of T, and x be an element of the open neighbourhoods of p. Then there exists a subset W of T such that $W = x$ and $p \in W$ and W is open.

(39) Let T be a non empty topological space, p be a point of T, and x be a subset of T. Then $x \in$ the carrier of the open neighbourhoods of p if and only if $p \in x$ and x is open.

(40) Let T be a non empty topological space, p be a point of T, and x, y be elements of the open neighbourhoods of p. Then $x \leq y$ if and only if $y \subseteq x$.

Let T be a non empty topological space and let p be a point of T. One can verify that the open neighbourhoods of p is transitive and directed.

11. Nets in topological spaces

Let T be a non empty topological space and let N be a net in T. The functor $\mathrm{Lim}\, N$ yields a subset of T and is defined as follows:

(Def. 18) For every point p of T holds $p \in \mathrm{Lim}\, N$ iff for every neighbourhood V of p holds N is eventually in V.

One can prove the following four propositions:

(41) For every non empty topological space T and for every net N in T and for every subnet Y of N holds $\mathrm{Lim}\, N \subseteq \mathrm{Lim}\, Y$.

(42) For every non empty topological space T and for every constant net N in T holds the value of $N \in \mathrm{Lim}\, N$.

(43) Let T be a non empty topological space, N be a net in T, and p be a point of T. Suppose $p \in \mathrm{Lim}\, N$. Let d be an element of N. Then there exists a subset S of T such that $S = \{N(c); c$ ranges over elements of N: $d \leq c\}$ and $p \in \overline{S}$.

(44) Let T be a non empty topological space. Then T is Hausdorff if and only if for every net N in T and for all points p, q of T such that $p \in \mathrm{Lim}\, N$ and $q \in \mathrm{Lim}\, N$ holds $p = q$.

Let T be a Hausdorff non empty topological space and let N be a net in T. Observe that $\mathrm{Lim}\, N$ is trivial.

Let T be a non empty topological space and let N be a net in T. We say that N is convergent if and only if:

(Def. 19) $\mathrm{Lim}\, N \neq \emptyset$.

Let T be a non empty topological space. One can verify that every net in T which is constant is also convergent.

Let T be a non empty topological space. Note that there exists a net in T which is convergent and strict.

Let T be a Hausdorff non empty topological space and let N be a convergent net in T. The functor $\lim N$ yielding an element of T is defined as follows:

(Def. 20) $\lim N \in \mathrm{Lim}\, N$.

We now state four propositions:

(45) For every Hausdorff non empty topological space T and for every constant net N in T holds $\lim N =$ the value of N.

(46) Let T be a non empty topological space, N be a net in T, and p be a point of T. Suppose $p \notin \mathrm{Lim}\, N$. Then it is not true that there exists a subnet Y of N and there exists a subnet Z of Y such that $p \in \mathrm{Lim}\, Z$.

(47) Let T be a non empty topological space and N be a net in T. Suppose $N \in \mathrm{NetUniv}(T)$. Let p be a point of T. Suppose $p \notin \mathrm{Lim}\, N$. Then there exists a subnet Y of N such that $Y \in \mathrm{NetUniv}(T)$ and it is not true that there exists a subnet Z of Y such that $p \in \mathrm{Lim}\, Z$.

(48) Let T be a non empty topological space, N be a net in T, and p be a point of T. Suppose $p \in \mathrm{Lim}\, N$. Let J be a net set of the carrier of N, T. If for every element i of N holds $N(i) \in \mathrm{Lim}\, J(i)$, then $p \in \mathrm{Lim}\, \mathrm{Iterated}(J)$.

12. Convergence classes

Let S be a non empty 1-sorted structure. Convergence class of S is defined by:

(Def. 21) It $\subseteq$ [: NetUniv(S), the carrier of S :].

Let S be a non empty 1-sorted structure. Observe that every convergence class of S is relation-like.

Let T be a non empty topological space. The functor Convergence(T) yielding a convergence class of T is defined by:

(Def. 22) For every net N in T and for every point p of T holds $\langle N, p\rangle \in$ Convergence(T) iff $N \in$ NetUniv(T) and $p \in \mathrm{Lim}\, N$.

Let T be a non empty 1-sorted structure and let C be a convergence class of T. We say that C has (CONSTANTS) property if and only if:

(Def. 23) For every constant net N in T such that $N \in$ NetUniv(T) holds $\langle N$, the value of $N\rangle \in C$.

We say that C has (SUBNETS) property if and only if the condition (Def. 24) is satisfied.

(Def. 24) Let N be a net in T and Y be a subnet of N. If $Y \in$ NetUniv(T), then for every element p of T such that $\langle N, p\rangle \in C$ holds $\langle Y, p\rangle \in C$.

We say that C has (DIVERGENCE) property if and only if the condition (Def. 25) is satisfied.

(Def. 25) Let X be a net in T and p be an element of T. Suppose $X \in$ NetUniv(T) and $\langle X, p\rangle \notin C$. Then there exists a subnet Y of X such that $Y \in$ NetUniv(T) and it is not true that there exists a subnet Z of Y such that $\langle Z, p\rangle \in C$.

We say that C has (ITERATED LIMITS) property if and only if the condition (Def. 26) is satisfied.

(Def. 26) Let X be a net in T and p be an element of T. Suppose $\langle X, p\rangle \in C$. Let J be a net set of the carrier of X, T. If for every element i of X holds $\langle J(i), X(i)\rangle \in C$, then $\langle$ Iterated(J), $p\rangle \in C$.

Let T be a non empty topological space. One can check that Convergence(T) has (CONSTANTS) property, (SUBNETS) property, (DIVERGENCE) property, and (ITERATED LIMITS) property.

Let S be a non empty 1-sorted structure and let C be a convergence class of S. The functor ConvergenceSpace(C) yielding a strict topological structure is defined by the conditions (Def. 27).

(Def. 27)(i) The carrier of ConvergenceSpace(C) = the carrier of S, and

(ii) the topology of ConvergenceSpace(C) $= \{V; V$ ranges over subsets of S: $\bigwedge_{p\,:\,\text{element of } S} (p \in V \Rightarrow \bigwedge_{N\,:\,\text{net in } S} (\langle N, p\rangle \in C \Rightarrow N$ is eventually in $V))\}$.

Let S be a non empty 1-sorted structure and let C be a convergence class of S. Observe that ConvergenceSpace(C) is non empty.

Let S be a non empty 1-sorted structure and let C be a convergence class of S. One can check that ConvergenceSpace(C) is topological space-like.

One can prove the following proposition

(49) For every non empty 1-sorted structure S and for every convergence class C of S holds $C \subseteq$ Convergence(ConvergenceSpace(C)).

Let T be a non empty 1-sorted structure and let C be a convergence class of T. We say that C is topological if and only if:

(Def. 28) C has (CONSTANTS) property, (SUBNETS) property, (DIVERGENCE) property, and (ITERATED LIMITS) property.

Let T be a non empty 1-sorted structure. One can check that there exists a convergence class of T which is non empty and topological.

Let T be a non empty 1-sorted structure. Observe that every convergence class of T which is topological has also (CONSTANTS) property, (SUBNETS) property, (DIVERGENCE) property, and (ITERATED LIMITS) property and every convergence class of T which has (CONSTANTS) property, (SUBNETS) property, (DIVERGENCE) property, and (ITERATED LIMITS) property is also topological.

Next we state four propositions:

(50) Let T be a non empty 1-sorted structure, C be a topological convergence class of T, and S be a subset of ConvergenceSpace(C) **qua** non empty topological space. Then S is open if and only if for every element p of T such that $p \in S$ and for every net N in T such that $\langle N, p\rangle \in C$ holds N is eventually in S.

(51) Let T be a non empty 1-sorted structure, C be a topological convergence class of T, and S be a subset of ConvergenceSpace(C) **qua** non empty topological space. Then S is closed if and only if for every element p of T and for every net N in T such that $\langle N, p\rangle \in C$ and N is often in S holds $p \in S$.

(52) Let T be a non empty 1-sorted structure, C be a topological convergence class of T, S be a subset of ConvergenceSpace(C), and p be a point of ConvergenceSpace(C). Suppose $p \in \overline{S}$. Then there exists a net N in ConvergenceSpace(C) such that $\langle N, p\rangle \in C$ and rng(the mapping of $N) \subseteq S$ and $p \in \mathrm{Lim}\, N$.

(53) Let T be a non empty 1-sorted structure and C be a convergence class of T. Then Convergence(ConvergenceSpace$(C)) = C$ if and only if C is topological.

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