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Complex Numbers — Basic Theorems

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1. Main Part

In this paper a, b, c, d, e denote complex numbers.

Next we state a number of propositions:

(1) $a+(b+c)=(a+b)+c$.

(2) If $a+c=b+c$, then $a=b$.

(3) If $a=a+b$, then $b=0$.

(4) $a\cdot(b\cdot c)=(a\cdot b)\cdot c$.

(5) If $c\neq 0$ and $a\cdot c=b\cdot c$, then $a=b$.

(6) If $a\cdot b=0$, then $a=0$ or $b=0$.

(7) If $b\neq 0$ and $a\cdot b=b$, then $a=1$.

(8) $a\cdot(b+c)=a\cdot b+a\cdot c$.

(9) $(a+b+c)\cdot d=a\cdot d+b\cdot d+c\cdot d$.

(10) $(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d$.

(11) $2\cdot a=a+a$.

(12) $3\cdot a=a+a+a$.

(13) $4\cdot a=a+a+a+a$.

(14) $a-a=0$.

(15) If $a-b=0$, then $a=b$.

(16) If $b-a=b$, then $a=0$.

(17) $a=a-(b-b)$.

(18) $a-(a-b)=b$.

(19) If $a-c=b-c$, then $a=b$.

(20) If $c-a=c-b$, then $a=b$.

(21) $a-b-c=a-c-b$.

(22) $a-c=a-b-(c-b)$.

(23) $c-a-(c-b)=b-a$.

(24) If $a-b=c-d$, then $a-c=b-d$.

(25) $a=a+(b-b)$.

(26) $a=(a+b)-b$.

(27) $a=(a-b)+b$.

(28) $a+c=a+b+(c-b)$.

(29) $(a+b)-c=(a-c)+b$.

(30) $(a-b)+c=(c-b)+a$.

(31) $a+c=(a+b)-(b-c)$.

(32) $a-c=(a+b)-(c+b)$.

(33) If $a+b=c+d$, then $a-c=d-b$.

(34) If $a-c=d-b$, then $a+b=c+d$.

(35) If $a+b=c-d$, then $a+d=c-b$.

(36) $a-(b+c)=a-b-c$.

(37) $a-(b-c)=(a-b)+c$.

(38) $a-(b-c)=a+(c-b)$.

(39) $a-c=(a-b)+(b-c)$.

(40) $a\cdot(b-c)=a\cdot b-a\cdot c$.

(41) $(a-b)\cdot(c-d)=(b-a)\cdot(d-c)$.

(42) $(a-b-c)\cdot d=a\cdot d-b\cdot d-c\cdot d$.

(43) $((a+b)-c)\cdot d=(a\cdot d+b\cdot d)-c\cdot d$.

(44) $((a-b)+c)\cdot d=(a\cdot d-b\cdot d)+c\cdot d$.

(45) $(a+b)\cdot(c-d)=((a\cdot c-a\cdot d)+b\cdot c)-b\cdot d$.

(46) $(a-b)\cdot(c+d)=(a\cdot c+a\cdot d)-b\cdot c-b\cdot d$.

(47) $(a-b)\cdot(e-d)=(a\cdot e-a\cdot d-b\cdot e)+b\cdot d$.

(48) $\frac{\frac{a}{b}}{c}=\frac{\frac{a}{c}}{b}$.

(49) $\frac{a}{0}=0$.

(50) If $a\neq 0$ and $b\neq 0$, then $\frac{a}{b}\neq 0$.

(51) If $b\neq 0$, then $a=\frac{a}{\frac{b}{b}}$.

(52) If $a\neq 0$, then $\frac{a}{\frac{a}{b}}=b$.

(53) If $c\neq 0$ and $\frac{a}{c}=\frac{b}{c}$, then $a=b$.

(54) If $\frac{a}{b} \neq 0$, then $b = \frac{a}{\frac{a}{b}}$.

(55) If $c \neq 0$, then $\frac{a}{b} = \frac{\frac{a}{c}}{\frac{b}{c}}$.

(56) $\frac{1}{\frac{1}{a}} = a$.

(57) $\frac{1}{\frac{a}{b}} = \frac{b}{a}$.

(58) If $\frac{a}{b} = 1$, then $a = b$.

(59) If $\frac{1}{a} = \frac{1}{b}$, then $a = b$.

(60) If $a \neq 0$, then $\frac{a}{a} = 1$.

(61) If $b \neq 0$ and $\frac{b}{a} = b$, then $a = 1$.

(62) If $a \neq 0$, then $\frac{1}{a} \neq 0$.

(63) $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$.

(64) $\frac{a+b+e}{d} = \frac{a}{d} + \frac{b}{d} + \frac{e}{d}$.

(65) $\frac{a+a}{2} = a$.

(66) $\frac{a}{2} + \frac{a}{2} = a$.

(67) If $a = \frac{a+b}{2}$, then $a = b$.

(68) $\frac{a+a+a}{3} = a$.

(69) $\frac{a}{3} + \frac{a}{3} + \frac{a}{3} = a$.

(70) $\frac{a+a+a+a}{4} = a$.

(71) $\frac{a}{4} + \frac{a}{4} + \frac{a}{4} + \frac{a}{4} = a$.

(72) $\frac{a}{4} + \frac{a}{4} = \frac{a}{2}$.

(73) $\frac{a+a}{4} = \frac{a}{2}$.

(74) If $a \cdot b = 1$, then $a = \frac{1}{b}$.

(75) $a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$.

(76) $\frac{a}{b} \cdot e = \frac{e}{b} \cdot a$.

(77) $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$.

(78) $\frac{a}{\frac{b}{c}} = \frac{a \cdot c}{b}$.

(79) $\frac{a}{b \cdot c} = \frac{\frac{a}{b}}{c}$.

(80) $\frac{a}{\frac{b}{c}} = a \cdot \frac{c}{b}$.

(81) $\frac{a}{\frac{b}{c}} = \frac{c}{b} \cdot a$.

(82) $\frac{a}{\frac{b}{e}} = e \cdot \frac{a}{b}$.

(83) $\frac{a}{\frac{b}{c}} = \frac{a}{b} \cdot c$.

(84) $\frac{a \cdot b}{c \cdot d} = \frac{\frac{a}{c} \cdot b}{d}$.

(85) $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}$.

(86) $\frac{a}{c} \cdot \frac{b}{d} = \frac{a}{d} \cdot \frac{b}{c}$.

(87) $\frac{a}{b \cdot c \cdot \frac{d}{e}} = \frac{e}{c} \cdot \frac{a}{b \cdot d}$.

(88) If $b \neq 0$, then $\frac{a}{b} \cdot b = a$.

(89) If $b \neq 0$, then $a = a \cdot \frac{b}{b}$.

(90) If $b \neq 0$, then $a = \frac{a \cdot b}{b}$.

(91) If $b \neq 0$, then $a \cdot c = a \cdot b \cdot \frac{c}{b}$.

(92) If $c \neq 0$, then $\frac{a}{b} = \frac{a \cdot c}{b \cdot c}$.

(93) If $c \neq 0$, then $\frac{a}{b} = \frac{a}{b \cdot c} \cdot c$.

(94) If $b \neq 0$, then $a \cdot c = \frac{a \cdot b}{\frac{b}{c}}$.

(95) If $c \neq 0$ and $d \neq 0$ and $a \cdot c = b \cdot d$, then $\frac{a}{d} = \frac{b}{c}$.

(96) If $c \neq 0$ and $d \neq 0$ and $\frac{a}{d} = \frac{b}{c}$, then $a \cdot c = b \cdot d$.

(97) If $c \neq 0$ and $d \neq 0$ and $a \cdot c = \frac{b}{d}$, then $a \cdot d = \frac{b}{c}$.

(98) If $c \neq 0$, then $\frac{a}{b} = c \cdot \frac{\frac{a}{c}}{b}$.

(99) If $c \neq 0$, then $\frac{a}{b} = \frac{a}{c} \cdot \frac{c}{b}$.

(100) $a \cdot \frac{1}{b} = \frac{a}{b}$.

(101) $\frac{a}{\frac{1}{b}} = a \cdot b$.

(102) $\frac{a}{b} \cdot c = \frac{1}{b} \cdot c \cdot a$.

(103) $\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{a \cdot b}$.

(104) $\frac{1}{c} \cdot \frac{a}{b} = \frac{a}{b \cdot c}$.

(105) $\frac{\frac{a}{b}}{c} = \frac{1}{b} \cdot \frac{a}{c}$.

(106) $\frac{\frac{a}{b}}{c} = \frac{1}{c} \cdot \frac{a}{b}$.

(107) If $a \neq 0$, then $a \cdot \frac{1}{a} = 1$.

(108) If $b \neq 0$, then $a = a \cdot b \cdot \frac{1}{b}$.

(109) If $b \neq 0$, then $a = a \cdot (\frac{1}{b} \cdot b)$.

(110) If $b \neq 0$, then $a = a \cdot \frac{1}{b} \cdot b$.

(111) If $b \neq 0$, then $a = \frac{a}{b \cdot \frac{1}{b}}$.

(112) If $a \neq 0$ and $b \neq 0$, then $\frac{1}{a \cdot b} \neq 0$.

(113) If $a \neq 0$ and $b \neq 0$, then $\frac{a}{b} \cdot \frac{b}{a} = 1$.

(114) If $b \neq 0$, then $\frac{a}{b} + c = \frac{a + b \cdot c}{b}$.

(115) If $c \neq 0$, then $a + b = c \cdot (\frac{a}{c} + \frac{b}{c})$.

(116) If $c \neq 0$, then $a+b = \frac{a \cdot c+b \cdot c}{c}$.

(117) If $b \neq 0$ and $d \neq 0$, then $\frac{a}{b} + \frac{c}{d} = \frac{a \cdot d+c \cdot b}{b \cdot d}$.

(118) If $a \neq 0$, then $a+b = a \cdot (1+\frac{b}{a})$.

(119) $\frac{a}{2 \cdot b} + \frac{a}{2 \cdot b} = \frac{a}{b}$.

(120) $\frac{a}{3 \cdot b} + \frac{a}{3 \cdot b} + \frac{a}{3 \cdot b} = \frac{a}{b}$.

(121) $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$.

(122) $a - \frac{a}{2} = \frac{a}{2}$.

(123) $\frac{a-b-c}{d} = \frac{a}{d} - \frac{b}{d} - \frac{c}{d}$.

(124) If $b \neq 0$ and $d \neq 0$ and $b \neq d$ and $\frac{a}{b} = \frac{e}{d}$, then $\frac{a}{b} = \frac{a-e}{b-d}$.

(125) $\frac{(a+b)-e}{d} = (\frac{a}{d} + \frac{b}{d}) - \frac{e}{d}$.

(126) $\frac{(a-b)+e}{d} = (\frac{a}{d} - \frac{b}{d}) + \frac{e}{d}$.

(127) If $b \neq 0$, then $\frac{a}{b} - e = \frac{a-e \cdot b}{b}$.

(128) If $b \neq 0$, then $c - \frac{a}{b} = \frac{c \cdot b-a}{b}$.

(129) If $c \neq 0$, then $a-b = c \cdot (\frac{a}{c} - \frac{b}{c})$.

(130) If $c \neq 0$, then $a-b = \frac{a \cdot c-b \cdot c}{c}$.

(131) If $b \neq 0$ and $d \neq 0$, then $\frac{a}{b} - \frac{c}{d} = \frac{a \cdot d-c \cdot b}{b \cdot d}$.

(132) If $a \neq 0$, then $a-b = a \cdot (1-\frac{b}{a})$.

(133) If $a \neq 0$, then $c = \frac{(a \cdot c+b)-b}{a}$.

(134) If $-a = -b$, then $a = b$.

(135) If $-a = 0$, then $a = 0$.

(136) If $a+-b = 0$, then $a = b$.

(137) $a = a+b+-b$.

(138) $a = a+(b+-b)$.

(139) $a = -b+a+b$.

(140) $-(a+b) = -a+-b$.

(141) $-(-a+b) = a+-b$.

(142) $a+b = -(-a+-b)$.

(143) $-(a-b) = b-a$.

(144) $-a-b = -b-a$.

(145) $a = -b-(-a-b)$.

(146) $-a-b-c = -a-c-b$.

(147) $-a-b-c = -b-c-a$.

(148) $-a-b-c = -c-b-a$.

(149) $c-a-(c-b)=-(a-b)$.

(150) $0-a=-a$.

(151) $a+b=a--b$.

(152) $a=a-(b+-b)$.

(153) If $a-c=b+-c$, then $a=b$.

(154) If $c-a=c+-b$, then $a=b$.

(155) $(a+b)-c=-c+a+b$.

(156) $(a-b)+c=-b+c+a$.

(157) $a-(-b-c)=a+b+c$.

(158) $a-b-c=(-b-c)+a$.

(159) $a-b-c=(-c+a)-b$.

(160) $a-b-c=(-c-b)+a$.

(161) $-(a+b)=-b-a$.

(162) $-(a-b)=-a+b$.

(163) $-(-a+b)=a-b$.

(164) $a+b=-(-a-b)$.

(165) $(-a+b)-c=(-c+b)-a$.

(166) $(-a+b)-c=(-c-a)+b$.

(167) $-(a+b+c)=-a-b-c$.

(168) $-((a+b)-c)=(-a-b)+c$.

(169) $-((a-b)+c)=(-a+b)-c$.

(170) $-(a-b-c)=-a+b+c$.

(171) $-(-a+b+c)=a-b-c$.

(172) $-((-a+b)-c)=(a-b)+c$.

(173) $-((-a-b)+c)=(a+b)-c$.

(174) $-(-a-b-c)=a+b+c$.

(175) $(-a)\cdot b=-a\cdot b$.

(176) $(-a)\cdot b=a\cdot -b$.

(177) $(-a)\cdot -b=a\cdot b$.

(178) $-a\cdot -b=a\cdot b$.

(179) $-(-a)\cdot b=a\cdot b$.

(180) $(-1)\cdot a=-a$.

(181) $(-a)\cdot -1=a$.

(182) If $b\neq 0$ and $a\cdot b=-b$, then $a=-1$.

(183) If $a \cdot a = 1$, then $a = 1$ or $a = -1$.

(184) $-a + 2 \cdot a = a$.

(185) $(a-b) \cdot c = (b-a) \cdot -c$.

(186) $(a-b) \cdot c = -(b-a) \cdot c$.

(187) $a - 2 \cdot a = -a$.

(188) $-\frac{a}{b} = \frac{-a}{b}$.

(189) $\frac{a}{-b} = -\frac{a}{b}$.

(190) $-\frac{a}{-b} = \frac{a}{b}$.

(191) $-\frac{-a}{b} = \frac{a}{b}$.

(192) $\frac{-a}{-b} = \frac{a}{b}$.

(193) $\frac{-a}{b} = \frac{a}{-b}$.

(194) $-a = \frac{a}{-1}$.

(195) $a = \frac{-a}{-1}$.

(196) If $\frac{a}{b} = -1$, then $a = -b$ and $b = -a$.

(197) If $b \neq 0$ and $\frac{b}{a} = -b$, then $a = -1$.

(198) If $a \neq 0$, then $\frac{-a}{a} = -1$.

(199) If $a \neq 0$, then $\frac{a}{-a} = -1$.

(200) If $a \neq 0$ and $a = \frac{1}{a}$, then $a = 1$ or $a = -1$.

(201) If $b \neq 0$ and $d \neq 0$ and $b \neq -d$ and $\frac{a}{b} = \frac{e}{d}$, then $\frac{a}{b} = \frac{a+e}{b+d}$.

(202) If $a^{-1} = b^{-1}$, then $a = b$.

(203) If $a^{-1} = 0$, then $a = 0$.

(204) If $b \neq 0$, then $a = a \cdot b \cdot b^{-1}$.

(205) $a^{-1} \cdot b^{-1} = (a \cdot b)^{-1}$.

(206) $(a \cdot b^{-1})^{-1} = a^{-1} \cdot b$.

(207) $(a^{-1} \cdot b^{-1})^{-1} = a \cdot b$.

(208) If $a \neq 0$ and $b \neq 0$, then $a \cdot b^{-1} \neq 0$.

(209) If $a \neq 0$ and $b \neq 0$, then $a^{-1} \cdot b^{-1} \neq 0$.

(210) If $a \cdot b^{-1} = 1$, then $a = b$.

(211) If $a \cdot b = 1$, then $a = b^{-1}$.

(213)[1] If $a \neq 0$ and $b \neq 0$, then $a^{-1} + b^{-1} = (a+b) \cdot (a \cdot b)^{-1}$.

(214) If $a \neq 0$ and $b \neq 0$, then $a^{-1} - b^{-1} = (b-a) \cdot (a \cdot b)^{-1}$.

(215) $(\frac{a}{b})^{-1} = \frac{b}{a}$.

(216) $\frac{a^{-1}}{b^{-1}} = \frac{b}{a}$.

[1] The proposition (212) has been removed.

(217) $\frac{1}{a} = a^{-1}$.

(218) $\frac{1}{a^{-1}} = a$.

(219) $(\frac{1}{a})^{-1} = a$.

(220) If $\frac{1}{a} = b^{-1}$, then $a = b$.

(221) $\frac{a}{b^{-1}} = a \cdot b$.

(222) $a^{-1} \cdot \frac{c}{b} = \frac{c}{a \cdot b}$.

(223) $\frac{a^{-1}}{b} = (a \cdot b)^{-1}$.

(224) $(-a)^{-1} = -a^{-1}$.

(225) If $a \neq 0$ and $a = a^{-1}$, then $a = 1$ or $a = -1$.

2. Appendix

One can prove the following propositions:

(226) $(a+b+c)-b = a+c$.

(227) $(a-b)+c+b = a+c$.

(228) $(a+b)-c-b = a-c$.

(229) $(a-b-c)+b = a-c$.

(230) $a-a-b = -b$.

(231) $(-a+a)-b = -b$.

(232) $a-b-a = -b$.

(233) $(-a-b)+a = -b$.

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