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Boolean Properties of Sets — Definitions

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The article [1] provides the notation and terminology for this paper.

In this paper X, Y, Z, x are sets.

The scheme *Separation* deals with a set $\mathcal{A}$ and a unary predicate $\mathcal{P}$, and states that:

There exists a set X such that for every set x holds $x \in X$ iff $x \in \mathcal{A}$ and $\mathcal{P}[x]$

for all values of the parameters.

The set $\emptyset$ is defined by:

(Def. 1) It is not true that there exists a set x such that $x \in \emptyset$.

Let X, Y be sets. The functor $X \cup Y$ yields a set and is defined by:

(Def. 2) $x \in X \cup Y$ iff $x \in X$ or $x \in Y$.

Let us note that the functor $X \cup Y$ is commutative and idempotent. The functor $X \cap Y$ yields a set and is defined by:

(Def. 3) $x \in X \cap Y$ iff $x \in X$ and $x \in Y$.

Let us observe that the functor $X \cap Y$ is commutative and idempotent. The functor $X \setminus Y$ yields a set and is defined as follows:

(Def. 4) $x \in X \setminus Y$ iff $x \in X$ and $x \notin Y$.

Let X be a set. We say that X is empty if and only if:

(Def. 5) $X = \emptyset$.

Let Y be a set. The functor $X \dot{-} Y$ yielding a set is defined as follows:

(Def. 6) $X \dot{-} Y = (X \setminus Y) \cup (Y \setminus X)$.

Let us notice that the functor $X \dot{-} Y$ is commutative. We say that X misses Y if and only if:

(Def. 7) $X \cap Y = \emptyset$.

Let us note that the predicate X misses Y is symmetric. We introduce X meets Y as an antonym of X misses Y. The predicate $X \subset Y$ is defined as follows:

(Def. 8) $X \subseteq Y$ and $X \neq Y$.

Let us note that the predicate $X \subset Y$ is irreflexive. We say that X and Y are $\subseteq$-comparable if and only if:

(Def. 9) $X \subseteq Y$ or $Y \subseteq X$.

Let us notice that the predicate X and Y are $\subseteq$-comparable is reflexive and symmetric. Let us observe that $X = Y$ if and only if:

(Def. 10) $X \subseteq Y$ and $Y \subseteq X$.

The following propositions are true:

(1) $x \in X \dot{-} Y$ iff $x \in X$ iff $x \notin Y$.

(2) If for every x holds $x \notin X$ iff $x \in Y$ iff $x \in Z$, then $X = Y \dot{-} Z$.

One can verify the following observations:

* $\emptyset$ is empty,
* there exists a set which is empty, and
* there exists a set which is non empty.

Let D be a non empty set and let X be a set. Observe that $D \cup X$ is non empty and $X \cup D$ is non empty.

One can prove the following three propositions:

(3) X meets Y iff there exists x such that $x \in X$ and $x \in Y$.

(4) X meets Y iff there exists x such that $x \in X \cap Y$.

(5) If X misses Y and $x \in X \cup Y$, then $x \in X$ and $x \notin Y$ or $x \in Y$ and $x \notin X$.

Now we present two schemes. The scheme *Extensionality* deals with sets $\mathcal{A}$, $\mathcal{B}$ and a unary predicate $\mathcal{P}$, and states that:

$\mathcal{A} = \mathcal{B}$

provided the following conditions are satisfied:

- For every x holds $x \in \mathcal{A}$ iff $\mathcal{P}[x]$, and
- For every x holds $x \in \mathcal{B}$ iff $\mathcal{P}[x]$.

The scheme *SetEq* concerns a unary predicate $\mathcal{P}$, and states that:

Let X_1, X_2 be sets. Suppose for every set x holds $x \in X_1$ iff $\mathcal{P}[x]$ and for every set x holds $x \in X_2$ iff $\mathcal{P}[x]$. Then $X_1 = X_2$

for all values of the parameters.

REFERENCES

[1] Andrzej Trybulec. Tarski Grothendieck set theory. *Journal of Formalized Mathematics*, Axiomatics, 1989. http://mizar.org/JFM/Axiomatics/tarski.html.

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