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# Replacement of Subtrees in a Tree

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**Summary.** This paper is based on previous works [1], [2] in which the operation replacement of subtree in a tree has been defined. We extend this notion for arbitrary non empty antichain.

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The articles [5], [7], [6], [8], [4], [3], [1], and [2] provide the notation and terminology for this paper.

For simplicity, we follow the rules: $T$, $T_1$ denote trees, $P$ denotes an antichain of prefixes of $T$, $p$, $q$, $r$ denote finite sequences of elements of $\mathbb{N}$, and $t$ denotes an element of $T$.

The following proposition is true

(1) For all finite sequences $p$, $q$, $r$, $s$ such that $p \frown q = s \frown r$ holds $p$ and $s$ are $\subseteq$-comparable.

Let us consider $T$, $T_1$ and let us consider $P$. Let us assume that $P \neq \emptyset$. The functor $\overbrace{T,P,T_1}$ yielding a tree is defined as follows:

(Def. 1) $q \in \overbrace{T,P,T_1}$ iff $q \in T$ and for every $p$ such that $p \in P$ holds $p \not\prec q$ or there exist $p$, $r$ such that $p \in P$ and $r \in T_1$ and $q = p \frown r$.

We now state several propositions:

(2) Suppose $P \neq \emptyset$. Then $\overbrace{T,P,T_1} = \{t_1; t_1$ ranges over elements of $T$: $\bigwedge_p (p \in P \Rightarrow p \not\prec t_1)\} \cup \{p \frown s; p$ ranges over elements of $T$, $s$ ranges over elements of $T_1$: $p \in P\}$.

(3) $\{t_1; t_1$ ranges over elements of $T$: $\bigwedge_p (p \in P \Rightarrow p \not\preceq t_1)\} \subseteq \{t_1; t_1$ ranges over elements of $T$: $\bigwedge_p (p \in P \Rightarrow p \not\prec t_1)\}$.

(4) $P \subseteq \{t_1; t_1$ ranges over elements of $T$: $\bigwedge_p (p \in P \Rightarrow p \not\prec t_1)\}$.

(5) $\{t_1; t_1$ ranges over elements of $T$: $\bigwedge_p (p \in P \Rightarrow p \not\prec t_1)\} \setminus \{t_1; t_1$ ranges over elements of $T$: $\bigwedge_p (p \in P \Rightarrow p \not\preceq t_1)\} = P$.

(6) For all $T$, $T_1$, $P$ holds $P \subseteq \{p \frown s; p$ ranges over elements of $T$, $s$ ranges over elements of $T_1$: $p \in P\}$.

(7) Suppose $P \neq \emptyset$. Then $\overbrace{T,P,T_1} = \{t_1; t_1$ ranges over elements of $T$: $\bigwedge_p (p \in P \Rightarrow p \not\preceq t_1)\} \cup \{p \frown s; p$ ranges over elements of $T$, $s$ ranges over elements of $T_1$: $p \in P\}$.

 

(9)[1] If $p \in P$, then $T_1 = \overbrace{T, P, T_1} \upharpoonright p$.

Let us consider $T$. Observe that there exists an antichain of prefixes of $T$ which is non empty.

Let us consider $T$ and let $t$ be an element of $T$. Then $\{t\}$ is a non empty antichain of prefixes of $T$.

One can prove the following proposition

(10) $\overbrace{T, \{t\}, T_1} = T$ with-replacement$(t, T_1)$.

In the sequel $T$, $T_1$ are decorated trees, $P$ is an antichain of prefixes of dom $T$, and $t$ is an element of dom $T$.

Let us consider $T$, $P$, $T_1$. Let us assume that $P \neq \emptyset$. The functor $\overbrace{T, P, T_1}$ yielding a decorated tree is defined by the conditions (Def. 2).

(Def. 2)(i) $\operatorname{dom}\overbrace{T, P, T_1} = \overbrace{\operatorname{dom} T, P, \operatorname{dom} T_1}$, and

(ii) for every $q$ such that $q \in \overbrace{\operatorname{dom} T, P, \operatorname{dom} T_1}$ holds for every $p$ such that $p \in P$ holds $p \not\preceq q$ and $\overbrace{T, P, T_1}(q) = T(q)$ or there exist $p$, $r$ such that $p \in P$ and $r \in \operatorname{dom} T_1$ and $q = p \frown r$ and $\overbrace{T, P, T_1}(q) = T_1(r)$.

One can prove the following propositions:

(13)[2] Suppose $P \neq \emptyset$. Let given $q$. Suppose $q \in \operatorname{dom}\overbrace{T, P, T_1}$. Then for every $p$ such that $p \in P$ holds $p \not\preceq q$ and $\overbrace{T, P, T_1}(q) = T(q)$ or there exist $p$, $r$ such that $p \in P$ and $r \in \operatorname{dom} T_1$ and $q = p \frown r$ and $\overbrace{T, P, T_1}(q) = T_1(r)$.

(14) Suppose $p \in \operatorname{dom} T$. Let given $q$. Suppose $q \in \operatorname{dom}(T$ with-replacement$(p, T_1))$. Then $p \not\preceq q$ and $(T$ with-replacement$(p, T_1))(q) = T(q)$ or there exists $r$ such that $r \in \operatorname{dom} T_1$ and $q = p \frown r$ and $(T$ with-replacement$(p, T_1))(q) = T_1(r)$.

(15) Suppose $P \neq \emptyset$. Let given $q$. Suppose $q \in \operatorname{dom}\overbrace{T, P, T_1}$ and $q \in \{t_1; t_1$ ranges over elements of dom $T$ : $\bigwedge_p (p \in P \Rightarrow p \not\preceq t_1)\}$. Then $\overbrace{T, P, T_1}(q) = T(q)$.

(16) If $p \in \operatorname{dom} T$, then for every $q$ such that $q \in \operatorname{dom}(T$ with-replacement$(p, T_1))$ and $q \in \{t_1; t_1$ ranges over elements of dom $T$ : $p \not\preceq t_1\}$ holds $(T$ with-replacement$(p, T_1))(q) = T(q)$.

(17) Let given $q$. Suppose $q \in \operatorname{dom}\overbrace{T, P, T_1}$ and $q \in \{p \frown s; p$ ranges over elements of dom $T, s$ ranges over elements of dom $T_1$ : $p \in P\}$. Then there exists an element $p'$ of dom $T$ and there exists an element $r$ of dom $T_1$ such that $q = p' \frown r$ and $p' \in P$ and $\overbrace{T, P, T_1}(q) = T_1(r)$.

(18) Suppose $p \in \operatorname{dom} T$. Let given $q$. Suppose $q \in \operatorname{dom}(T$ with-replacement$(p, T_1))$ and $q \in \{p \frown s; s$ ranges over elements of dom $T_1$ : $s = s\}$. Then there exists an element $r$ of dom $T_1$ such that $q = p \frown r$ and $(T$ with-replacement$(p, T_1))(q) = T_1(r)$.

(19) $\overbrace{T, \{t\}, T_1} = T$ with-replacement$(t, T_1)$.

In the sequel $D$ denotes a non empty set, $T$, $T_1$ denote trees decorated with elements of $D$, and $P$ denotes an antichain of prefixes of dom $T$.

Let us consider $D$, $T$, $P$, $T_1$. Let us assume that $P \neq \emptyset$. The functor $\overbrace{T, P, T_1}$ yields a tree decorated with elements of $D$ and is defined by:

(Def. 3) $\overbrace{T, P, T_1} = \overbrace{T, P, T_1}$.

[1] The proposition (8) has been removed.

[2] The propositions (11) and (12) have been removed.

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