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Properties of Subsets

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Summary. The text includes theorems concerning properties of subsets, and some operations on sets. The functions yielding improper subsets of a set, i.e. the empty set and the set itself are introduced. Functions and predicates introduced for sets are redefined. Some theorems about enumerated sets are proved.

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The articles [3], [2], and [1] provide the notation and terminology for this paper.

In this paper E, X, x, y are sets.

Let X be a set. Observe that 2^X is non empty.

Let us consider x. Note that $\{x\}$ is non empty. Let us consider y. Note that $\{x,y\}$ is non empty.

Let us consider X. Element of X is defined as follows:

(Def. 2)[1](i) It $\in X$ if X is non empty,

(ii) it is empty, otherwise.

Let us consider X. A subset of X is an element of 2^X.

Let X be a non empty set. Observe that there exists a subset of X which is non empty.

Let X_1, X_2 be non empty sets. Observe that $[: X_1, X_2 :]$ is non empty.

Let X_1, X_2, X_3 be non empty sets. One can verify that $[: X_1, X_2, X_3 :]$ is non empty.

Let X_1, X_2, X_3, X_4 be non empty sets. Observe that $[: X_1, X_2, X_3, X_4 :]$ is non empty.

Let D be a non empty set and let X be a non empty subset of D. We see that the element of X is an element of D.

Let us consider E. One can check that there exists a subset of E which is empty.

Let us consider E. The functor $\emptyset_E$ yielding an empty subset of E is defined by:

(Def. 3) $\emptyset_E = \emptyset$.

The functor Ω_E yields a subset of E and is defined by:

(Def. 4) $\Omega_E = E$.

Next we state the proposition

(4)[2] $\emptyset$ is a subset of X.

In the sequel A, B, C are subsets of E.

Next we state three propositions:

[1] The definition (Def. 1) has been removed.
[2] The propositions (1)–(3) have been removed.

(7)[3] If for every element x of E such that $x \in A$ holds $x \in B$, then $A \subseteq B$.

(8) If for every element x of E holds $x \in A$ iff $x \in B$, then $A = B$.

(10)[4] If $A \neq \emptyset$, then there exists an element x of E such that $x \in A$.

Let us consider E, A. The functor $A^{\rm c}$ yielding a subset of E is defined by:

(Def. 5) $A^{\rm c} = E \setminus A$.

Let us notice that the functor $A^{\rm c}$ is involutive. Let us consider B. Then $A \cup B$ is a subset of E. Then $A \cap B$ is a subset of E. Then $A \setminus B$ is a subset of E. Then $A \dot{-} B$ is a subset of E.

We now state a number of propositions:

(15)[5] If for every element x of E holds $x \in A$ iff $x \in B$ or $x \in C$, then $A = B \cup C$.

(16) If for every element x of E holds $x \in A$ iff $x \in B$ and $x \in C$, then $A = B \cap C$.

(17) If for every element x of E holds $x \in A$ iff $x \in B$ and $x \notin C$, then $A = B \setminus C$.

(18) If for every element x of E holds $x \in A$ iff $x \in B$ iff $x \notin C$, then $A = B \dot{-} C$.

(21)[6] $\emptyset_E = (\Omega_E)^{\rm c}$.

(22) $\Omega_E = (\emptyset_E)^{\rm c}$.

(25)[7] $A \cup A^{\rm c} = \Omega_E$.

(26) A misses $A^{\rm c}$.

(28)[8] $A \cup \Omega_E = \Omega_E$.

(29) $(A \cup B)^{\rm c} = A^{\rm c} \cap B^{\rm c}$.

(30) $(A \cap B)^{\rm c} = A^{\rm c} \cup B^{\rm c}$.

(31) $A \subseteq B$ iff $B^{\rm c} \subseteq A^{\rm c}$.

(32) $A \setminus B = A \cap B^{\rm c}$.

(33) $(A \setminus B)^{\rm c} = A^{\rm c} \cup B$.

(34) $(A \dot{-} B)^{\rm c} = A \cap B \cup A^{\rm c} \cap B^{\rm c}$.

(35) If $A \subseteq B^{\rm c}$, then $B \subseteq A^{\rm c}$.

(36) If $A^{\rm c} \subseteq B$, then $B^{\rm c} \subseteq A$.

(38)[9] $A \subseteq A^{\rm c}$ iff $A = \emptyset_E$.

(39) $A^{\rm c} \subseteq A$ iff $A = \Omega_E$.

(40) If $X \subseteq A$ and $X \subseteq A^{\rm c}$, then $X = \emptyset$.

(41) $(A \cup B)^{\rm c} \subseteq A^{\rm c}$.

(42) $A^{\rm c} \subseteq (A \cap B)^{\rm c}$.

(43) A misses B iff $A \subseteq B^{\rm c}$.

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[3] The propositions (5) and (6) have been removed.
[4] The proposition (9) has been removed.
[5] The propositions (11)–(14) have been removed.
[6] The propositions (19) and (20) have been removed.
[7] The propositions (23) and (24) have been removed.
[8] The proposition (27) has been removed.
[9] The proposition (37) has been removed.

(44) A misses $B^{\rm c}$ iff $A \subseteq B$.

(46)[10] If A misses B and $A^{\rm c}$ misses $B^{\rm c}$, then $A = B^{\rm c}$.

(47) If $A \subseteq B$ and C misses B, then $A \subseteq C^{\rm c}$.

(48) If for every element a of A holds $a \in B$, then $A \subseteq B$.

(49) If for every element x of E holds $x \in A$, then $E = A$.

(50) If $E \neq \emptyset$, then for every B and for every element x of E such that $x \notin B$ holds $x \in B^{\rm c}$.

(51) For all A, B such that for every element x of E holds $x \in A$ iff $x \notin B$ holds $A = B^{\rm c}$.

(52) For all A, B such that for every element x of E holds $x \notin A$ iff $x \in B$ holds $A = B^{\rm c}$.

(53) For all A, B such that for every element x of E holds $x \in A$ iff $x \notin B$ holds $A = B^{\rm c}$.

(54) If $x \in A^{\rm c}$, then $x \notin A$.

In the sequel x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 are elements of X.
One can prove the following propositions:

(55) If $X \neq \emptyset$, then $\{x_1\}$ is a subset of X.

(56) If $X \neq \emptyset$, then $\{x_1, x_2\}$ is a subset of X.

(57) If $X \neq \emptyset$, then $\{x_1, x_2, x_3\}$ is a subset of X.

(58) If $X \neq \emptyset$, then $\{x_1, x_2, x_3, x_4\}$ is a subset of X.

(59) If $X \neq \emptyset$, then $\{x_1, x_2, x_3, x_4, x_5\}$ is a subset of X.

(60) If $X \neq \emptyset$, then $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ is a subset of X.

(61) If $X \neq \emptyset$, then $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$ is a subset of X.

(62) If $X \neq \emptyset$, then $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ is a subset of X.

(63) If $x \in X$, then $\{x\}$ is a subset of X.

In this article we present several logical schemes. The scheme *Subset Ex* deals with a set $\mathcal{A}$ and a unary predicate $\mathcal{P}$, and states that:

There exists a subset X of $\mathcal{A}$ such that for every x holds $x \in X$ iff $x \in \mathcal{A}$ and $\mathcal{P}[x]$

for all values of the parameters.

The scheme *Subset Eq* deals with a set $\mathcal{A}$ and a unary predicate $\mathcal{P}$, and states that:

Let X_1, X_2 be subsets of $\mathcal{A}$. Suppose for every element y of $\mathcal{A}$ holds $y \in X_1$ iff $\mathcal{P}[y]$ and for every element y of $\mathcal{A}$ holds $y \in X_2$ iff $\mathcal{P}[y]$. Then $X_1 = X_2$

for all values of the parameters.

Let X, Y be non empty sets. Let us note that the predicate X misses Y is irreflexive. We introduce X meets Y as an antonym of X misses Y.

Let S be a set. Let us assume that *contradiction*.[11]

(Def. 6) $\operatorname{choose}(S)$ is an element of S.

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[10] The proposition (45) has been removed.

[11] This definition is absolutely permissive, i.e. we assume a *contradiction*, but we are interested only in the type of the functor 'choose'.

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