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Preliminaries to Circuits, I[1]

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Summary. This article is the first in a series of four articles (continued in [23],[22],[24]) about modelling circuits by many-sorted algebras.

Here, we introduce some auxiliary notations and prove auxiliary facts about many sorted sets, many sorted functions and trees.

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The articles [26], [15], [31], [4], [30], [2], [1], [5], [29], [19], [32], [13], [18], [14], [25], [17], [7], [3], [9], [10], [11], [6], [8], [27], [20], [28], [21], [12], and [16] provide the notation and terminology for this paper.

1. Varia

The scheme *FraenkelFinIm* deals with a finite non empty set $\mathcal{A}$, a unary functor $\mathcal{F}$ yielding a set, and a unary predicate $\mathcal{P}$, and states that:

$\{\mathcal{F}(x); x$ ranges over elements of $\mathcal{A} : \mathcal{P}[x]\}$ is finite

for all values of the parameters.

Next we state three propositions:

(2)[1] For every function f and for all sets x, y such that $\operatorname{dom} f = \{x\}$ and $\operatorname{rng} f = \{y\}$ holds $f = \{\langle x, y\rangle\}$.

(3) For all functions f, g, h such that $f \subseteq g$ holds $f {+\cdot} h \subseteq g {+\cdot} h$.

(4) For all functions f, g, h such that $f \subseteq g$ and $\operatorname{dom} f$ misses $\operatorname{dom} h$ holds $f \subseteq g {+\cdot} h$.

Let us note that there exists a set which is finite, non empty, and natural-membered.

Let A be a finite non empty real-membered set. Then $\sup A$ is a real number and it can be characterized by the condition:

(Def. 1) $\sup A \in A$ and for every real number k such that $k \in A$ holds $k \leq \sup A$.

We introduce $\max A$ as a synonym of $\sup A$.

Let X be a finite non empty natural-membered set. One can verify that $\max X$ is natural.

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[1] The proposition (1) has been removed.

2. Many Sorted Sets and Functions

The following proposition is true

(5) For every set I and for every many sorted set M_1 indexed by I holds ${M_1}^{\#}(\varepsilon_I) = \{\emptyset\}$.

The scheme *MSSLambda2Part* deals with a set $\mathcal{A}$, two unary functors $\mathcal{F}$ and $\mathcal{G}$ yielding sets, and a unary predicate $\mathcal{P}$, and states that:

There exists a many sorted set f indexed by $\mathcal{A}$ such that for every element i of $\mathcal{A}$ holds

(i) if $\mathcal{P}[i]$, then $f(i) = \mathcal{F}(i)$, and

(ii) if not $\mathcal{P}[i]$, then $f(i) = \mathcal{G}(i)$

for all values of the parameters.

Let I be a set and let I_1 be a many sorted set indexed by I. We say that I_1 is locally-finite if and only if:

(Def. 3)[2] For every set i such that $i \in I$ holds $I_1(i)$ is finite.

Let I be a set. One can verify that there exists a many sorted set indexed by I which is non-empty and locally-finite.

Let I, A be sets. Then $I \longmapsto A$ is a many sorted set indexed by I.

Let I be a set, let M be a many sorted set indexed by I, and let A be a subset of I. Then $M{\restriction}A$ is a many sorted set indexed by A.

Let M be a non-empty function and let A be a set. Observe that $M{\restriction}A$ is non-empty.

Next we state three propositions:

(6) For every non empty set I and for every non-empty many sorted set B indexed by I holds $\bigcup \operatorname{rng} B$ is non empty.

(7) For every set I holds $\operatorname{uncurry}(I \longmapsto \emptyset) = \emptyset$.

(8) Let I be a non empty set, A be a set, B be a non-empty many sorted set indexed by I, and F be a many sorted function from $I \longmapsto A$ into B. Then $\operatorname{dom} \operatorname{commute}(F) = A$.

Now we present two schemes. The scheme *LambdaRecCorrD* deals with a non empty set $\mathcal{A}$, an element $\mathcal{B}$ of $\mathcal{A}$, and a binary functor $\mathcal{F}$ yielding an element of $\mathcal{A}$, and states that:

(i) There exists a function f from $\mathbb{N}$ into $\mathcal{A}$ such that $f(0) = \mathcal{B}$ and for every natural number i holds $f(i+1) = \mathcal{F}(i, f(i))$, and

(ii) for all functions f_1, f_2 from $\mathbb{N}$ into $\mathcal{A}$ such that $f_1(0) = \mathcal{B}$ and for every natural number i holds $f_1(i+1) = \mathcal{F}(i, f_1(i))$ and $f_2(0) = \mathcal{B}$ and for every natural number i holds $f_2(i+1) = \mathcal{F}(i, f_2(i))$ holds $f_1 = f_2$

for all values of the parameters.

The scheme *LambdaMSFD* deals with a non empty set $\mathcal{A}$, a subset $\mathcal{B}$ of $\mathcal{A}$, many sorted sets $\mathcal{C}$, $\mathcal{D}$ indexed by $\mathcal{B}$, and a unary functor $\mathcal{F}$ yielding a set, and states that:

There exists a many sorted function f from $\mathcal{C}$ into $\mathcal{D}$ such that for every element i of $\mathcal{A}$ such that $i \in \mathcal{B}$ holds $f(i) = \mathcal{F}(i)$

provided the parameters have the following property:

- For every element i of $\mathcal{A}$ such that $i \in \mathcal{B}$ holds $\mathcal{F}(i)$ is a function from $\mathcal{C}(i)$ into $\mathcal{D}(i)$.

Let F be a non-empty function and let f be a function. Observe that $F \cdot f$ is non-empty.

Let I be a set and let M_1 be a non-empty many sorted set indexed by I. One can verify that every element of $\prod M_1$ is function-like and relation-like.

Next we state four propositions:

(9) Let I be a set, f be a non-empty many sorted set indexed by I, g be a function, and s be an element of $\prod f$. Suppose $\operatorname{dom} g \subseteq \operatorname{dom} f$ and for every set x such that $x \in \operatorname{dom} g$ holds $g(x) \in f(x)$. Then $s + \cdot g$ is an element of $\prod f$.

[2] The definition (Def. 2) has been removed.

(10) Let A, B be non empty sets, C be a non-empty many sorted set indexed by A, I_2 be a many sorted function from $A \longmapsto B$ into C, and b be an element of B. Then there exists a many sorted set c indexed by A such that $c = (\text{commute}(I_2))(b)$ and $c \in C$.

(11) Let I be a set, M be a many sorted set indexed by I, and x, g be functions. If $x \in \prod M$, then $x \cdot g \in \prod (M \cdot g)$.

(12) For every natural number n and for every set a holds $\prod (n \mapsto \{a\}) = \{n \mapsto a\}$.

3. TREES

We adopt the following rules: T, T_1 denote finite trees, t, p denote elements of T, and t_1 denotes an element of T_1.

Let D be a non empty set. Observe that every element of FinTrees(D) is finite.

Let T be a finite decorated tree and let t be an element of dom T. One can check that $T \restriction t$ is finite.

One can prove the following proposition

(13) $T \restriction p \approx \{t : p \preceq t\}$.

Let T be a finite decorated tree, let t be an element of dom T, and let T_1 be a finite decorated tree. Observe that T with-replacement(t, T_1) is finite.

Next we state a number of propositions:

(14) T with-replacement$(p, T_1) = \{t : p \not\preceq t\} \cup \{p \frown t_1\}$.

(15) For every finite sequence f of elements of $\mathbb{N}$ such that $f \in T$ with-replacement(p, T_1) and $p \preceq f$ there exists t_1 such that $f = p \frown t_1$.

(16) For every tree yielding finite sequence p and for every natural number k such that $k+1 \in$ dom p holds $\widehat{p} \restriction \langle k \rangle = p(k+1)$.

(17) Let q be a decorated tree yielding finite sequence and k be a natural number. If $k+1 \in$ dom q, then $\langle k \rangle \in \overbrace{\text{dom}_{\kappa}\, q(\kappa)}$.

(18) Let p, q be tree yielding finite sequences and k be a natural number. Suppose len p = len q and $k+1 \in$ dom p and for every natural number i such that $i \in$ dom p and $i \neq k+1$ holds $p(i) = q(i)$. Let t be a tree. If $q(k+1) = t$, then $\widehat{q} = \widehat{p}$ with-replacement$(\langle k \rangle, t)$.

(19) Let e_1, e_2 be finite decorated trees, x be a set, k be a natural number, and p be a decorated tree yielding finite sequence. Suppose $\langle k \rangle \in$ dom e_1 and $e_1 = x$-tree(p). Then there exists a decorated tree yielding finite sequence q such that e_1 with-replacement$(\langle k \rangle, e_2) = x$-tree$(q)$ and len q = len p and $q(k+1) = e_2$ and for every natural number i such that $i \in$ dom p and $i \neq k+1$ holds $q(i) = p(i)$.

(20) For every finite tree T and for every element p of T such that $p \neq \emptyset$ holds card$(T \restriction p) <$ card T.

(21) For every function f holds $\overline{\overline{(f \text{ **qua** set})}} = \overline{\overline{\text{dom} f}}$.

(22) For all finite trees T, T_1 and for every element p of T holds card$(T$ with-replacement$(p, T_1)) +$ card$(T \restriction p) =$ card T + card T_1.

(23) For all finite decorated trees T, T_1 and for every element p of dom T holds card$(T$ with-replacement$(p, T_1)) +$ card$(T \restriction p) =$ card T + card T_1.

Let x be a set. One can check that the root tree of x is finite.

One can prove the following proposition

(24) For every set x holds card(the root tree of x) $= 1$.

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