JOURNAL OF FORMALIZED MATHEMATICS
Volume 10, Released 1998, Published 2003
Inst. of Computer Science, Univ. of Białystok

Natural Numbers

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MML Identifier: NAT_2.

WWW: http://mizar.org/JFM/Vol10/nat_2.html

The articles [11], [14], [2], [5], [12], [1], [8], [10], [9], [6], [4], [13], [7], and [3] provide the notation and terminology for this paper.

1. Preliminaries

The scheme *NonUniqPiFinRecExD* deals with a non empty set $\mathcal{A}$, an element $\mathcal{B}$ of $\mathcal{A}$, a natural number $\mathcal{C}$, and a ternary predicate $\mathcal{P}$, and states that:

> There exists a finite sequence p of elements of $\mathcal{A}$ such that $\operatorname{len} p = \mathcal{C}$ but $p_1 = \mathcal{B}$ or $\mathcal{C} = 0$ but for every natural number n such that $1 \leq n$ and $n < \mathcal{C}$ holds $\mathcal{P}[n, p_n, p_{n+1}]$

provided the following condition is satisfied:

- Let n be a natural number. Suppose $1 \leq n$ and $n < \mathcal{C}$. Let x be an element of $\mathcal{A}$. Then there exists an element y of $\mathcal{A}$ such that $\mathcal{P}[n,x,y]$.

The following propositions are true:

(1) For every real number x holds $x < \lfloor x \rfloor + 1$.

(2) For all real numbers x, y such that $x \geq 0$ and $y > 0$ holds $\frac{x}{\lfloor \frac{x}{y} \rfloor + 1} < y$.

2. Division and Rest of Division

One can prove the following propositions:

(4)[1] For every natural number n holds $0 \div n = 0$.

(5) For every non empty natural number n holds $n \div n = 1$.

(6) For every natural number n holds $n \div 1 = n$.

(7) For all natural numbers i, j, k, l such that $i \leq j$ and $k \leq j$ holds if $i = (j -' k) + l$, then $k = (j -' i) + l$.

(8) For all natural numbers i, n such that $i \in \operatorname{Seg} n$ holds $(n -' i) + 1 \in \operatorname{Seg} n$.

(9) For all natural numbers i, j such that $j < i$ holds $(i -' (j+1)) + 1 = i -' j$.

(10) For all natural numbers i, j such that $i \geq j$ holds $j -' i = 0$.

(11) For all non empty natural numbers i, j holds $i -' j < i$.

[1] The proposition (3) has been removed.

(12) For all natural numbers n, k such that $k \leq n$ holds $2^n = 2^k \cdot 2^{n -' k}$.

(13) For all natural numbers n, k such that $k \leq n$ holds $2^k \mid 2^n$.

(14) For all natural numbers n, k such that $k > 0$ and $n \div k = 0$ holds $n < k$.

In the sequel n, k, i denote natural numbers.
Next we state a number of propositions:

(15) If $k > 0$ and $k \leq n$, then $n \div k \geq 1$.

(16) If $k \neq 0$, then $(n+k) \div k = (n \div k) + 1$.

(17) If $k \mid n$ and $1 \leq n$ and $1 \leq i$ and $i \leq k$, then $(n -' i) \div k = (n \div k) - 1$.

(18) For all natural numbers n, k such that $k \leq n$ holds $2^n \div 2^k = 2^{n -' k}$.

(19) For every natural number n such that $n > 0$ holds $2^n \bmod 2 = 0$.

(20) For every natural number n such that $n > 0$ holds $n \bmod 2 = 0$ iff $(n -' 1) \bmod 2 = 1$.

(21) For every non empty natural number n such that $n \neq 1$ holds $n > 1$.

(22) For all natural numbers n, k such that $n \leq k$ and $k < n+n$ holds $k \div n = 1$.

(23) For every natural number n holds n is even iff $n \bmod 2 = 0$.

(24) For every natural number n holds n is odd iff $n \bmod 2 = 1$.

(25) For all natural numbers n, k, t such that $1 \leq t$ and $k \leq n$ and $2 \cdot t \mid k$ holds $n \div t$ is even iff $(n -' k) \div t$ is even.

(26) For all natural numbers n, m, k such that $n \leq m$ holds $n \div k \leq m \div k$.

(27) For all natural numbers n, k such that $k \leq 2 \cdot n$ holds $(k+1) \div 2 \leq n$.

(28) For every even natural number n holds $n \div 2 = (n+1) \div 2$.

(29) For all natural numbers n, k, i holds $n \div k \div i = n \div k \cdot i$.

Let n be a natural number. Let us observe that n is trivial if and only if:

(Def. 1) $n = 0$ or $n = 1$.

Let us note that there exists a natural number which is non trivial and there exists a number which is non trivial and natural.

Next we state two propositions:

(30) For every natural number k holds k is non trivial iff k is non empty and $k \neq 1$.

(31) For every non trivial natural number k holds $k \geq 2$.

The scheme *Ind from 2* concerns a unary predicate $\mathcal{P}$, and states that:

For every non trivial natural number k holds $\mathcal{P}[k]$

provided the following conditions are satisfied:

- $\mathcal{P}[2]$, and
- For every non trivial natural number k such that $\mathcal{P}[k]$ holds $\mathcal{P}[k+1]$.

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Received February 23, 1998

Published January 2, 2004