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More on Segments on a Go-Board

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Summary. We continue the preparatory work for the Jordan Curve Theorem.

MML Identifier: GOBOARD8.

WWW: http://mizar.org/JFM/Vol7/goboard8.html

The articles [12], [5], [1], [10], [2], [13], [6], [11], [14], [3], [4], [7], [8], and [9] provide the notation and terminology for this paper.

We adopt the following rules: i, j, k are natural numbers, p is a point of $\mathcal{E}^2_{\rm T}$, and f is a non constant standard special circular sequence.

We now state a number of propositions:

(1) Let given k. Suppose $1 \leq k$ and $k+2 \leq \operatorname{len} f$. Let given i, j. Suppose that

(i) $1 \leq i$,

(ii) $i+1 \leq \operatorname{len}$ the Go-board of f,

(iii) $1 \leq j$,

(iv) $j+2 \leq \operatorname{width}$ the Go-board of f,

(v) $f_{k+1} =$ the Go-board of $f \circ (i+1, j+1)$, and

(vi) $f_k =$ the Go-board of $f \circ (i+1, j)$ and $f_{k+2} =$ the Go-board of $f \circ (i+1, j+2)$ or $f_{k+2} =$ the Go-board of $f \circ (i+1, j)$ and $f_k =$ the Go-board of $f \circ (i+1, j+2)$.

Then $\mathcal{L}(\frac{1}{2} \cdot ((\text{the Go-board of } f \circ (i, j)) + (\text{the Go-board of } f \circ (i+1, j+1))), \frac{1}{2} \cdot ((\text{the Go-board of } f \circ (i, j+1)) + (\text{the Go-board of } f \circ (i+1, j+2))))$ misses $\widetilde{\mathcal{L}}(f)$.

(2) Let given k. Suppose $1 \leq k$ and $k+2 \leq \operatorname{len} f$. Let given i, j. Suppose that

(i) $1 \leq i$,

(ii) $i+2 \leq \operatorname{len}$ the Go-board of f,

(iii) $1 \leq j$,

(iv) $j+2 \leq \operatorname{width}$ the Go-board of f,

(v) $f_{k+1} =$ the Go-board of $f \circ (i+1, j+1)$, and

(vi) $f_k =$ the Go-board of $f \circ (i+2, j+1)$ and $f_{k+2} =$ the Go-board of $f \circ (i+1, j+2)$ or $f_{k+2} =$ the Go-board of $f \circ (i+2, j+1)$ and $f_k =$ the Go-board of $f \circ (i+1, j+2)$.

Then $\mathcal{L}(\frac{1}{2} \cdot ((\text{the Go-board of } f \circ (i, j)) + (\text{the Go-board of } f \circ (i+1, j+1))), \frac{1}{2} \cdot ((\text{the Go-board of } f \circ (i, j+1)) + (\text{the Go-board of } f \circ (i+1, j+2))))$ misses $\widetilde{\mathcal{L}}(f)$.

(3) Let given k. Suppose $1 \leq k$ and $k+2 \leq \operatorname{len} f$. Let given i, j. Suppose that
(i) $1 \leq i$,
(ii) $i+2 \leq \operatorname{len}$ the Go-board of f,
(iii) $1 \leq j$,
(iv) $j+2 \leq \operatorname{width}$ the Go-board of f,
(v) $f_{k+1} =$ the Go-board of $f \circ (i+1, j+1)$, and
(vi) $f_k =$ the Go-board of $f \circ (i+2, j+1)$ and $f_{k+2} =$ the Go-board of $f \circ (i+1, j)$ or $f_{k+2} =$ the Go-board of $f \circ (i+2, j+1)$ and $f_k =$ the Go-board of $f \circ (i+1, j)$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f \circ (i, j)) + ($the Go-board of $f \circ (i+1, j+1))), \frac{1}{2} \cdot (($the Go-board of $f \circ (i, j+1)) + ($the Go-board of $f \circ (i+1, j+2))))$ misses $\widetilde{\mathcal{L}}(f)$.

(4) Let given k. Suppose $1 \leq k$ and $k+2 \leq \operatorname{len} f$. Let given i, j. Suppose that
(i) $1 \leq i$,
(ii) $i+1 \leq \operatorname{len}$ the Go-board of f,
(iii) $1 \leq j$,
(iv) $j+2 \leq \operatorname{width}$ the Go-board of f,
(v) $f_{k+1} =$ the Go-board of $f \circ (i, j+1)$, and
(vi) $f_k =$ the Go-board of $f \circ (i, j)$ and $f_{k+2} =$ the Go-board of $f \circ (i, j+2)$ or $f_{k+2} =$ the Go-board of $f \circ (i, j)$ and $f_k =$ the Go-board of $f \circ (i, j+2)$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f \circ (i, j)) + ($the Go-board of $f \circ (i+1, j+1))), \frac{1}{2} \cdot (($the Go-board of $f \circ (i, j+1)) + ($the Go-board of $f \circ (i+1, j+2))))$ misses $\widetilde{\mathcal{L}}(f)$.

(5) Let given k. Suppose $1 \leq k$ and $k+2 \leq \operatorname{len} f$. Let given i, j. Suppose that
(i) $1 \leq i$,
(ii) $i+2 \leq \operatorname{len}$ the Go-board of f,
(iii) $1 \leq j$,
(iv) $j+2 \leq \operatorname{width}$ the Go-board of f,
(v) $f_{k+1} =$ the Go-board of $f \circ (i+1, j+1)$, and
(vi) $f_k =$ the Go-board of $f \circ (i, j+1)$ and $f_{k+2} =$ the Go-board of $f \circ (i+1, j+2)$ or $f_{k+2} =$ the Go-board of $f \circ (i, j+1)$ and $f_k =$ the Go-board of $f \circ (i+1, j+2)$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f \circ (i+1, j)) + ($the Go-board of $f \circ (i+2, j+1))), \frac{1}{2} \cdot (($the Go-board of $f \circ (i+1, j+1)) + ($the Go-board of $f \circ (i+2, j+2))))$ misses $\widetilde{\mathcal{L}}(f)$.

(6) Let given k. Suppose $1 \leq k$ and $k+2 \leq \operatorname{len} f$. Let given i, j. Suppose that
(i) $1 \leq i$,
(ii) $i+2 \leq \operatorname{len}$ the Go-board of f,
(iii) $1 \leq j$,
(iv) $j+2 \leq \operatorname{width}$ the Go-board of f,
(v) $f_{k+1} =$ the Go-board of $f \circ (i+1, j+1)$, and
(vi) $f_k =$ the Go-board of $f \circ (i, j+1)$ and $f_{k+2} =$ the Go-board of $f \circ (i+1, j)$ or $f_{k+2} =$ the Go-board of $f \circ (i, j+1)$ and $f_k =$ the Go-board of $f \circ (i+1, j)$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f \circ (i+1, j)) + ($the Go-board of $f \circ (i+2, j+1))), \frac{1}{2} \cdot (($the Go-board of $f \circ (i+1, j+1)) + ($the Go-board of $f \circ (i+2, j+2))))$ misses $\widetilde{\mathcal{L}}(f)$.

(7) Let given k. Suppose $1 \leq k$ and $k+2 \leq \mathrm{len}\, f$. Let given i. Suppose that

(i) $1 \leq i$,

(ii) $i+2 \leq \mathrm{len}$ the Go-board of f,

(iii) $f_{k+1} =$ the Go-board of $f \circ (i+1,1)$, and

(iv) $f_k =$ the Go-board of $f \circ (i+2,1)$ and $f_{k+2} =$ the Go-board of $f \circ (i+1,2)$ or $f_{k+2} =$ the Go-board of $f \circ (i+2,1)$ and $f_k =$ the Go-board of $f \circ (i+1,2)$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f \circ (i,1)) + ($the Go-board of $f \circ (i+1,1))) - [0,1], \frac{1}{2} \cdot (($the Go-board of $f \circ (i,1)) + ($the Go-board of $f \circ (i+1,2))))$ misses $\widetilde{\mathcal{L}}(f)$.

(8) Let given k. Suppose $1 \leq k$ and $k+2 \leq \mathrm{len}\, f$. Let given i. Suppose that

(i) $1 \leq i$,

(ii) $i+2 \leq \mathrm{len}$ the Go-board of f,

(iii) $f_{k+1} =$ the Go-board of $f \circ (i+1,1)$, and

(iv) $f_k =$ the Go-board of $f \circ (i,1)$ and $f_{k+2} =$ the Go-board of $f \circ (i+1,2)$ or $f_{k+2} =$ the Go-board of $f \circ (i,1)$ and $f_k =$ the Go-board of $f \circ (i+1,2)$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f \circ (i+1,1)) + ($the Go-board of $f \circ (i+2,1))) - [0,1], \frac{1}{2} \cdot (($the Go-board of $f \circ (i+1,1)) + ($the Go-board of $f \circ (i+2,2))))$ misses $\widetilde{\mathcal{L}}(f)$.

(9) Let given k. Suppose $1 \leq k$ and $k+2 \leq \mathrm{len}\, f$. Let given i. Suppose that

(i) $1 \leq i$,

(ii) $i+2 \leq \mathrm{len}$ the Go-board of f,

(iii) $f_{k+1} =$ the Go-board of $f \circ (i+1, \mathrm{width}$ the Go-board of $f)$, and

(iv) $f_k =$ the Go-board of $f \circ (i+2, \mathrm{width}$ the Go-board of $f)$ and $f_{k+2} =$ the Go-board of $f \circ (i+1, \mathrm{width}$ the Go-board of $f -' 1)$ or $f_{k+2} =$ the Go-board of $f \circ (i+2, \mathrm{width}$ the Go-board of $f)$ and $f_k =$ the Go-board of $f \circ (i+1, \mathrm{width}$ the Go-board of $f -' 1)$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f \circ (i, \mathrm{width}$ the Go-board of $f -' 1)) + ($the Go-board of $f \circ (i+1, \mathrm{width}$ the Go-board of $f))), \frac{1}{2} \cdot (($the Go-board of $f \circ (i, \mathrm{width}$ the Go-board of $f)) + ($the Go-board of $f \circ (i+1, \mathrm{width}$ the Go-board of $f))) + [0,1])$ misses $\widetilde{\mathcal{L}}(f)$.

(10) Let given k. Suppose $1 \leq k$ and $k+2 \leq \mathrm{len}\, f$. Let given i. Suppose that

(i) $1 \leq i$,

(ii) $i+2 \leq \mathrm{len}$ the Go-board of f,

(iii) $f_{k+1} =$ the Go-board of $f \circ (i+1, \mathrm{width}$ the Go-board of $f)$, and

(iv) $f_k =$ the Go-board of $f \circ (i, \mathrm{width}$ the Go-board of $f)$ and $f_{k+2} =$ the Go-board of $f \circ (i+1, \mathrm{width}$ the Go-board of $f -' 1)$ or $f_{k+2} =$ the Go-board of $f \circ (i, \mathrm{width}$ the Go-board of $f)$ and $f_k =$ the Go-board of $f \circ (i+1, \mathrm{width}$ the Go-board of $f -' 1)$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f \circ (i+1, \mathrm{width}$ the Go-board of $f -' 1)) + ($the Go-board of $f \circ (i+2, \mathrm{width}$ the Go-board of $f))), \frac{1}{2} \cdot (($the Go-board of $f \circ (i+1, \mathrm{width}$ the Go-board of $f)) + ($the Go-board of $f \circ (i+2, \mathrm{width}$ the Go-board of $f))) + [0,1])$ misses $\widetilde{\mathcal{L}}(f)$.

(11) Let given k. Suppose $1 \leq k$ and $k+2 \leq \mathrm{len}\, f$. Let given i, j. Suppose that

(i) $1 \leq j$,

(ii) $j+1 \leq \mathrm{width}$ the Go-board of f,

(iii) $1 \leq i$,

(iv) $i+2 \leq \mathrm{len}$ the Go-board of f,

(v) $f_{k+1} =$ the Go-board of $f \circ (i+1, j+1)$, and

(vi) $f_k =$ the Go-board of $f \circ (i, j+1)$ and $f_{k+2} =$ the Go-board of $f \circ (i+2, j+1)$ or $f_{k+2} =$ the Go-board of $f \circ (i, j+1)$ and $f_k =$ the Go-board of $f \circ (i+2, j+1)$.

Then $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of } f\circ(i,j))+(\text{the Go-board of } f\circ(i+1,j+1))),\frac{1}{2}\cdot((\text{the Go-board of } f\circ(i+1,j))+(\text{the Go-board of } f\circ(i+2,j+1))))$ misses $\widetilde{\mathcal{L}}(f)$.

(12) Let given k. Suppose $1\leq k$ and $k+2\leq \operatorname{len} f$. Let given j, i. Suppose that

(i) $1\leq j$,

(ii) $j+2\leq \operatorname{width}$ the Go-board of f,

(iii) $1\leq i$,

(iv) $i+2\leq \operatorname{len}$ the Go-board of f,

(v) $f_{k+1}=$ the Go-board of $f\circ(i+1,j+1)$, and

(vi) $f_{k}=$ the Go-board of $f\circ(i+1,j+2)$ and $f_{k+2}=$ the Go-board of $f\circ(i+2,j+1)$ or $f_{k+2}=$ the Go-board of $f\circ(i+1,j+2)$ and $f_{k}=$ the Go-board of $f\circ(i+2,j+1)$.

Then $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of } f\circ(i,j))+(\text{the Go-board of } f\circ(i+1,j+1))),\frac{1}{2}\cdot((\text{the Go-board of } f\circ(i+1,j))+(\text{the Go-board of } f\circ(i+2,j+1))))$ misses $\widetilde{\mathcal{L}}(f)$.

(13) Let given k. Suppose $1\leq k$ and $k+2\leq \operatorname{len} f$. Let given j, i. Suppose that

(i) $1\leq j$,

(ii) $j+2\leq \operatorname{width}$ the Go-board of f,

(iii) $1\leq i$,

(iv) $i+2\leq \operatorname{len}$ the Go-board of f,

(v) $f_{k+1}=$ the Go-board of $f\circ(i+1,j+1)$, and

(vi) $f_{k}=$ the Go-board of $f\circ(i+1,j+2)$ and $f_{k+2}=$ the Go-board of $f\circ(i,j+1)$ or $f_{k+2}=$ the Go-board of $f\circ(i+1,j+2)$ and $f_{k}=$ the Go-board of $f\circ(i,j+1)$.

Then $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of } f\circ(i,j))+(\text{the Go-board of } f\circ(i+1,j+1))),\frac{1}{2}\cdot((\text{the Go-board of } f\circ(i+1,j))+(\text{the Go-board of } f\circ(i+2,j+1))))$ misses $\widetilde{\mathcal{L}}(f)$.

(14) Let given k. Suppose $1\leq k$ and $k+2\leq \operatorname{len} f$. Let given j, i. Suppose that

(i) $1\leq j$,

(ii) $j+1\leq \operatorname{width}$ the Go-board of f,

(iii) $1\leq i$,

(iv) $i+2\leq \operatorname{len}$ the Go-board of f,

(v) $f_{k+1}=$ the Go-board of $f\circ(i+1,j)$, and

(vi) $f_{k}=$ the Go-board of $f\circ(i,j)$ and $f_{k+2}=$ the Go-board of $f\circ(i+2,j)$ or $f_{k+2}=$ the Go-board of $f\circ(i,j)$ and $f_{k}=$ the Go-board of $f\circ(i+2,j)$.

Then $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of } f\circ(i,j))+(\text{the Go-board of } f\circ(i+1,j+1))),\frac{1}{2}\cdot((\text{the Go-board of } f\circ(i+1,j))+(\text{the Go-board of } f\circ(i+2,j+1))))$ misses $\widetilde{\mathcal{L}}(f)$.

(15) Let given k. Suppose $1\leq k$ and $k+2\leq \operatorname{len} f$. Let given j, i. Suppose that

(i) $1\leq j$,

(ii) $j+2\leq \operatorname{width}$ the Go-board of f,

(iii) $1\leq i$,

(iv) $i+2\leq \operatorname{len}$ the Go-board of f,

(v) $f_{k+1}=$ the Go-board of $f\circ(i+1,j+1)$, and

(vi) $f_{k}=$ the Go-board of $f\circ(i+1,j)$ and $f_{k+2}=$ the Go-board of $f\circ(i+2,j+1)$ or $f_{k+2}=$ the Go-board of $f\circ(i+1,j)$ and $f_{k}=$ the Go-board of $f\circ(i+2,j+1)$.

Then $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of } f\circ(i,j+1))+(\text{the Go-board of } f\circ(i+1,j+2))),\frac{1}{2}\cdot((\text{the Go-board of } f\circ(i+1,j+1))+(\text{the Go-board of } f\circ(i+2,j+2))))$ misses $\widetilde{\mathcal{L}}(f)$.

(16) Let given k. Suppose $1 \leq k$ and $k+2 \leq \text{len}\, f$. Let given j, i. Suppose that

(i) $1 \leq j$,

(ii) $j+2 \leq$ width the Go-board of f,

(iii) $1 \leq i$,

(iv) $i+2 \leq$ len the Go-board of f,

(v) $f_{k+1} =$ the Go-board of $f \circ (i+1, j+1)$, and

(vi) $f_k =$ the Go-board of $f \circ (i+1, j)$ and $f_{k+2} =$ the Go-board of $f \circ (i, j+1)$ or $f_{k+2} =$ the Go-board of $f \circ (i+1, j)$ and $f_k =$ the Go-board of $f \circ (i, j+1)$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f \circ (i, j+1)) + ($the Go-board of $f \circ (i+1, j+2))), \frac{1}{2} \cdot (($the Go-board of $f \circ (i+1, j+1)) + ($the Go-board of $f \circ (i+2, j+2))))$ misses $\widetilde{\mathcal{L}}(f)$.

(17) Let given k. Suppose $1 \leq k$ and $k+2 \leq \text{len}\, f$. Let given j. Suppose that

(i) $1 \leq j$,

(ii) $j+2 \leq$ width the Go-board of f,

(iii) $f_{k+1} =$ the Go-board of $f \circ (1, j+1)$, and

(iv) $f_k =$ the Go-board of $f \circ (1, j+2)$ and $f_{k+2} =$ the Go-board of $f \circ (2, j+1)$ or $f_{k+2} =$ the Go-board of $f \circ (1, j+2)$ and $f_k =$ the Go-board of $f \circ (2, j+1)$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f \circ (1, j)) + ($the Go-board of $f \circ (1, j+1))) - [1,0], \frac{1}{2} \cdot (($the Go-board of $f \circ (1, j)) + ($the Go-board of $f \circ (2, j+1))))$ misses $\widetilde{\mathcal{L}}(f)$.

(18) Let given k. Suppose $1 \leq k$ and $k+2 \leq \text{len}\, f$. Let given j. Suppose that

(i) $1 \leq j$,

(ii) $j+2 \leq$ width the Go-board of f,

(iii) $f_{k+1} =$ the Go-board of $f \circ (1, j+1)$, and

(iv) $f_k =$ the Go-board of $f \circ (1, j)$ and $f_{k+2} =$ the Go-board of $f \circ (2, j+1)$ or $f_{k+2} =$ the Go-board of $f \circ (1, j)$ and $f_k =$ the Go-board of $f \circ (2, j+1)$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f \circ (1, j+1)) + ($the Go-board of $f \circ (1, j+2))) - [1,0], \frac{1}{2} \cdot (($the Go-board of $f \circ (1, j+1)) + ($the Go-board of $f \circ (2, j+2))))$ misses $\widetilde{\mathcal{L}}(f)$.

(19) Let given k. Suppose $1 \leq k$ and $k+2 \leq \text{len}\, f$. Let given j. Suppose that

(i) $1 \leq j$,

(ii) $j+2 \leq$ width the Go-board of f,

(iii) $f_{k+1} =$ the Go-board of $f \circ ($len the Go-board of f, $j+1)$, and

(iv) $f_k =$ the Go-board of $f \circ ($len the Go-board of f, $j+2)$ and $f_{k+2} =$ the Go-board of $f \circ ($len the Go-board of $f -' 1, j+1)$ or $f_{k+2} =$ the Go-board of $f \circ ($len the Go-board of f, $j+2)$ and $f_k =$ the Go-board of $f \circ ($len the Go-board of $f -' 1, j+1)$.

Then $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f \circ ($len the Go-board of $f -' 1, j)) + ($the Go-board of $f \circ ($len the Go-board of f, $j+1))), \frac{1}{2} \cdot (($the Go-board of $f \circ ($len the Go-board of f, $j)) + ($the Go-board of $f \circ ($len the Go-board of f, $j+1))) + [1,0])$ misses $\widetilde{\mathcal{L}}(f)$.

(20) Let given k. Suppose $1 \leq k$ and $k+2 \leq \text{len}\, f$. Let given j. Suppose that

(i) $1 \leq j$,

(ii) $j+2 \leq$ width the Go-board of f,

(iii) $f_{k+1} =$ the Go-board of $f \circ ($len the Go-board of f, $j+1)$, and

(iv) $f_k =$ the Go-board of $f \circ ($len the Go-board of f, $j)$ and $f_{k+2} =$ the Go-board of $f \circ ($len the Go-board of $f -' 1, j+1)$ or $f_{k+2} =$ the Go-board of $f \circ ($len the Go-board of f, $j)$ and $f_k =$ the Go-board of $f \circ ($len the Go-board of $f -' 1, j+1)$.

Then $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of } f\circ(\text{len the Go-board of } f -' 1, j+1)) + (\text{the Go-board of } f\circ(\text{len the Go-board of } f, j+2))), \frac{1}{2}\cdot((\text{the Go-board of } f\circ(\text{len the Go-board of } f, j+1)) + (\text{the Go-board of } f\circ(\text{len the Go-board of } f, j+2))) + [1,0])$ misses $\widetilde{\mathcal{L}}(f)$.

In the sequel P is a subset of $\mathcal{E}^2_{\mathrm{T}}$.

Next we state a number of propositions:

(21) If for every p such that $p \in P$ holds $p_{\mathbf{1}} < (\text{the Go-board of } f\circ(1,1))_{\mathbf{1}}$, then P misses $\widetilde{\mathcal{L}}(f)$.

(22) If for every p such that $p \in P$ holds $p_{\mathbf{1}} > (\text{the Go-board of } f\circ(\text{len the Go-board of } f, 1))_{\mathbf{1}}$, then P misses $\widetilde{\mathcal{L}}(f)$.

(23) If for every p such that $p \in P$ holds $p_{\mathbf{2}} < (\text{the Go-board of } f\circ(1,1))_{\mathbf{2}}$, then P misses $\widetilde{\mathcal{L}}(f)$.

(24) If for every p such that $p \in P$ holds $p_{\mathbf{2}} > (\text{the Go-board of } f\circ(1, \text{width the Go-board of } f))_{\mathbf{2}}$, then P misses $\widetilde{\mathcal{L}}(f)$.

(25) Let given i. Suppose $1 \leq i$ and $i+2 \leq$ len the Go-board of f. Then $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of } f\circ(i,1)) + (\text{the Go-board of } f\circ(i+1,1))) - [0,1], \frac{1}{2}\cdot((\text{the Go-board of } f\circ(i+1,1)) + (\text{the Go-board of } f\circ(i+2,1))) - [0,1])$ misses $\widetilde{\mathcal{L}}(f)$.

(26) $\mathcal{L}((\text{the Go-board of } f\circ(1,1)) - [1,1], \frac{1}{2}\cdot((\text{the Go-board of } f\circ(1,1)) + (\text{the Go-board of } f\circ(2,1))) - [0,1])$ misses $\widetilde{\mathcal{L}}(f)$.

(27) $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of } f\circ(\text{len the Go-board of } f -' 1, 1)) + (\text{the Go-board of } f\circ(\text{len the Go-board of } f, 1))) - [0,1], (\text{the Go-board of } f\circ(\text{len the Go-board of } f, 1)) + [1,-1])$ misses $\widetilde{\mathcal{L}}(f)$.

(28) Let given i. Suppose $1 \leq i$ and $i+2 \leq$ len the Go-board of f. Then $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of } f\circ(i, \text{width the Go-board of } f)) + (\text{the Go-board of } f\circ(i+1, \text{width the Go-board of } f))) + [0,1], \frac{1}{2}\cdot((\text{the Go-board of } f\circ(i+1, \text{width the Go-board of } f)) + (\text{the Go-board of } f\circ(i+2, \text{width the Go-board of } f))) + [0,1])$ misses $\widetilde{\mathcal{L}}(f)$.

(29) $\mathcal{L}((\text{the Go-board of } f\circ(1, \text{width the Go-board of } f)) + [-1,1], \frac{1}{2}\cdot((\text{the Go-board of } f\circ(1, \text{width the Go-board of } f)) + (\text{the Go-board of } f\circ(2, \text{width the Go-board of } f))) + [0,1])$ misses $\widetilde{\mathcal{L}}(f)$.

(30) $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of } f\circ(\text{len the Go-board of } f -' 1, \text{width the Go-board of } f)) + (\text{the Go-board of } f\circ(\text{len the Go-board of } f, \text{width the Go-board of } f))) + [0,1], (\text{the Go-board of } f\circ(\text{len the Go-board of } f, \text{width the Go-board of } f)) + [1,1])$ misses $\widetilde{\mathcal{L}}(f)$.

(31) Let given j. Suppose $1 \leq j$ and $j+2 \leq$ width the Go-board of f. Then $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of } f\circ(1,j)) + (\text{the Go-board of } f\circ(1,j+1))) - [1,0], \frac{1}{2}\cdot((\text{the Go-board of } f\circ(1,j+1)) + (\text{the Go-board of } f\circ(1,j+2))) - [1,0])$ misses $\widetilde{\mathcal{L}}(f)$.

(32) $\mathcal{L}((\text{the Go-board of } f\circ(1,1)) - [1,1], \frac{1}{2}\cdot((\text{the Go-board of } f\circ(1,1)) + (\text{the Go-board of } f\circ(1,2))) - [1,0])$ misses $\widetilde{\mathcal{L}}(f)$.

(33) $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of } f\circ(1, \text{width the Go-board of } f -' 1)) + (\text{the Go-board of } f\circ(1, \text{width the Go-board of } f))) - [1,0], (\text{the Go-board of } f\circ(1, \text{width the Go-board of } f)) + [-1,1])$ misses $\widetilde{\mathcal{L}}(f)$.

(34) Let given j. Suppose $1 \leq j$ and $j+2 \leq$ width the Go-board of f. Then $\mathcal{L}(\frac{1}{2}\cdot((\text{the Go-board of } f\circ(\text{len the Go-board of } f, j)) + (\text{the Go-board of } f\circ(\text{len the Go-board of } f, j+1))) + [1,0], \frac{1}{2}\cdot((\text{the Go-board of } f\circ(\text{len the Go-board of } f, j+1)) + (\text{the Go-board of } f\circ(\text{len the Go-board of } f, j+2))) + [1,0])$ misses $\widetilde{\mathcal{L}}(f)$.

(35) $\mathcal{L}(($the Go-board of $f \circ ($len the Go-board of f, $1)) + [1,-1], \frac{1}{2} \cdot (($the Go-board of $f \circ ($len the Go-board of f, $1)) + ($the Go-board of $f \circ ($len the Go-board of f, $2))) + [1,0])$ misses $\widetilde{\mathcal{L}}(f)$.

(36) $\mathcal{L}(\frac{1}{2} \cdot (($the Go-board of $f \circ ($len the Go-board of f, width the Go-board of $f -' 1)) + ($the Go-board of $f \circ ($len the Go-board of f, width the Go-board of $f))) + [1,0], ($the Go-board of $f \circ ($len the Go-board of f, width the Go-board of $f)) + [1,1])$ misses $\widetilde{\mathcal{L}}(f)$.

(37) If $1 \leq k$ and $k+1 \leq$ len f, then Int leftcell(f,k) misses $\widetilde{\mathcal{L}}(f)$.

(38) If $1 \leq k$ and $k+1 \leq$ len f, then Int rightcell(f,k) misses $\widetilde{\mathcal{L}}(f)$.

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